Electrostatics

by sai sir

Q.1 What is Gauss' Law and what is Gaussian Surface?

Gauss' Law: The flux of the net electric field through a closed surface equal to the net charge enclosed by the surface divided by ε_0.

$$\int \vec{E} \cdot \vec{ds} = \frac{q}{\varepsilon_0}$$

where q is the total charge within the surface.

Mathematically

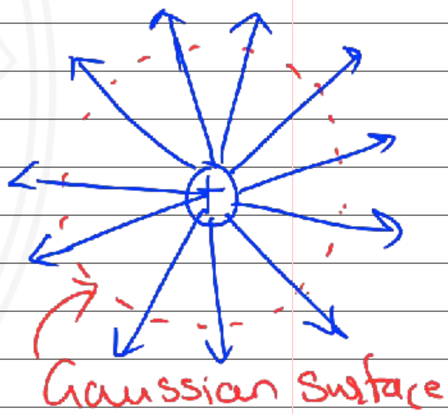
$$\phi = \oint \vec{E} \cdot \vec{ds} = \frac{q}{\varepsilon_0}$$

Where ϕ is the total flux coming out of a closed surface and q is the total charge inside the closed surface.

Gaussion Surface:- All the lines of force originating from a point charge penetrate an imaginary three dimensional surface.
The total flux through Gaussion Surface is

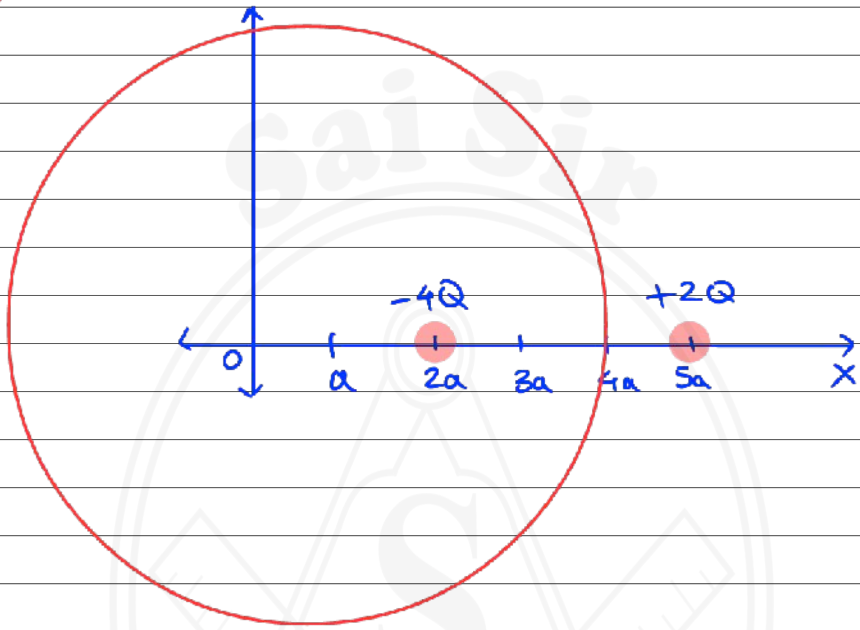
$$\phi_E = \frac{q}{\varepsilon_0}$$

ε_0 = permittivity of free space



A Gaussion Surface is purely imaginary and does not exit physically.

Q.2 Two charges of magnitude $-4Q$ and $+2Q$ are located at points $(2a, 0)$ and $(5a, 0)$ respectively.
What is the electric flux due to these charges through a sphere of radius $4a$ with its centre at the origin



A Gaussian surface is of radius $4a$ and contains charge $-4Q$ inside

As the flux depends only upon charged enclosed by Gaussian surface, required flux according to Gauss' Law would be,

$$\phi = \frac{q}{\varepsilon_0} = \frac{-4Q}{\varepsilon_0}$$

Q.3 State importance of Gauss' Law.
1. Gauss Law gives the relation between the electric charge and its electric field.
2. It also provides methods for finding electric field intensity.

Note :- Electric flux

$$\phi = \vec{E}\cdot\vec{ds} = E\,ds\cos\theta = \frac{q}{\varepsilon_0}.$$

Q.4 Obtain expression for electric field intensity due to uniformly charged spherical shell or hollow sphere.

i) Consider a sphere of radius R with centre 'O'. Let 'σ' be the uniform surface charge density

$$\text{i.e} \quad \sigma = \frac{q}{4\pi R^2} = \frac{\text{Charge}}{\text{Area}}$$

$$\therefore \quad q = \sigma (4\pi R^2)$$

Let the sphere be placed in a dielectric medium of permittivity 'ε', such that

$$\varepsilon = \varepsilon_0 K$$

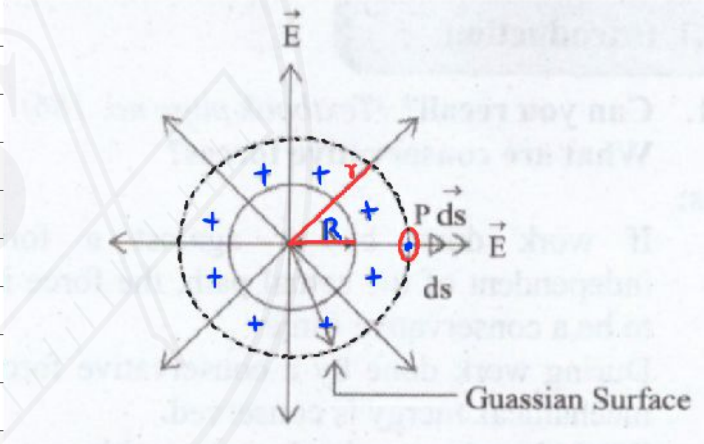
ε_0 = permittivity of free space, K = dielectric constant.

ii) By Gauss' theorem, the net flux through a closed surface is

$$\phi = \frac{q}{\varepsilon} = \frac{q}{\varepsilon_0 K} = \frac{q}{\varepsilon_0} \quad \ldots (1)$$

(for air/vacuum K=1)

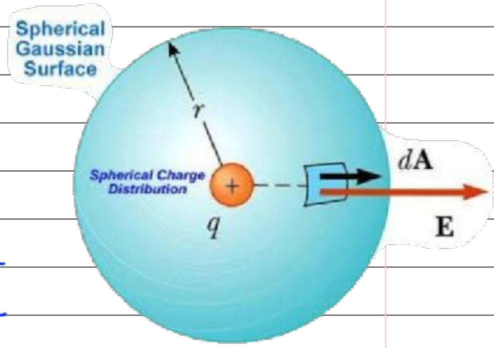
Where q is the total charge inside the closed Gaussian surface



Uniformly charged spherical shell or hollow sphere

iii) To find the Electric field at point 'P', consider a Gaussian sphere of radius 'r' passing from point 'P'. Let ds be a small area around the point 'P' on the Gaussian Surface.

iv) The electric field at each point on Gaussian Surface is same in magnitude and radially outward.
Also the angle between Electric field ($\vec{E}$) and normal to surface of sphere ($\vec{ds}$) is zero.



v) The total electric flux through the Gaussian Surface is

$$\phi = \oint \vec{E} \cdot \vec{ds}$$

$$\phi = \oint E\, ds \cos\theta = \oint E\, ds \cos 0^\circ = \oint E\, ds$$

i.e $\phi = E \oint ds$

$$\phi = E(4\pi r^2) \quad \cdots (2) \qquad (\text{as } \oint ds = 4\pi r^2)$$

Comparing equation (1) and (2)

$$\frac{q}{\varepsilon_0} = E(4\pi r^2)$$

$$\therefore E = \frac{q}{4\pi\varepsilon_0 r^2}$$

is the expression for electric field due to charged Sphere.

Also, $q = \sigma(4\pi R^2)$

$$\therefore E = \frac{\sigma(\cancel{4\pi} R^2)}{\cancel{4\pi}\varepsilon_0 r^2} = \frac{\sigma R^2}{\varepsilon_0 r^2}$$

is the expression for electric field due to a charged sphere in terms of surface charge density.

Case i) If point 'P' lies on the surface of the charged sphere i.e $r = R$

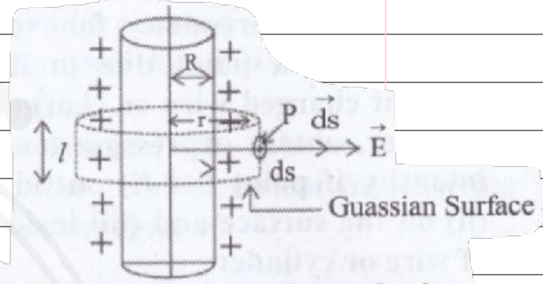
$$\therefore E = \frac{q}{4\pi\varepsilon_0 R^2} = \frac{\sigma R^2}{\varepsilon_0 R^2} = \frac{\sigma}{\varepsilon_0}$$

Case ii) If point 'p' lies inside the charged sphere then $q = 0$

$$\therefore E = 0$$ (Since there is no charged inside sphere i.e $\sigma = 0$)

Q.5 Obtain an expression for electric field intensity due to an infinitely long straight charged wire or charged conducting cylinder.

i) consider a uniformly charged wire of infinite length having a constant linear charged density λ, Kept in a medium of permittivity $\varepsilon = \varepsilon_0 K$



Infinitely long straight charged wire (cylinder)

ii) By Gauss' theorem, the net flux through a closed surface

$$\phi = \frac{q}{\varepsilon} = \frac{q}{\varepsilon_0 K} = \frac{q}{\varepsilon_0} \quad \ldots\ldots (1)$$

(for air / vaccum $K = 1$)

where 'q' is the total charged inside the closed surface.

iii) To find the electric field intensity at Point 'P', at a distance 'r' from the charged conductor, imagine a coaxial gaussian cylinder of length 'l' and radius 'r' passing through point 'P'.

iv) consider a very small area ds at point 'P' on the Gaussian Surface.

v) The Electric field at each point on Gaussian Surface is same in magnitude and radially outward. Also the angle between Electric field ($\vec{E}$) and normal to surface ($\vec{ds}$) is zero.

vi) Therefore, the total electric flux through Gaussian Surface.

$$\phi = \oint \vec{E} \cdot \vec{ds} = \oint E \, ds \cos 0^\circ$$

$$\therefore \phi = \oint E \, ds = E \oint ds = E(2\pi r l) \quad \ldots\ldots (2)$$

($\phi = 2\pi r l$ surface area of Gaussian cylinder)

Comparing eq (1) and eq (2)

$$\frac{q}{\varepsilon_0} = E\,(2\pi r l)$$

$$\therefore\ E = \frac{q}{2\pi\varepsilon_0 r l} \quad \ldots (3)$$

But linear charge density $(\lambda) = \frac{q}{l}$

$$\therefore\ q = \lambda l$$

Hence equation (3) becomes

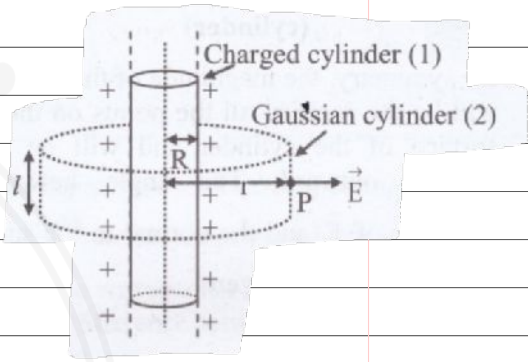
$$E = \frac{\lambda \cancel{l}}{2\pi\varepsilon_0 r \cancel{l}}$$

$$\therefore\ E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

This is the expression for Electric field intensity outside a charged cylinder.

The direction of electric field is outward if λ is positive.
The direction of electric field is inward if λ is Negative.

Q6 Assuming expression for electric field intensity (E) at a point due to infinitely long straight charged wire or conducting cylinder.
Obtain expression for electric field intensity if point lies
(i) outside the surface
(ii) on the surface
(iii) inside the surface of wire or cylinder



If 'σ' is surface charge density
then $\sigma = \dfrac{q}{A}$

$$\therefore \quad \sigma = \frac{\lambda l}{2\pi R l} = \frac{\lambda}{2\pi R}$$

$$\therefore \quad \lambda = 2\pi R \sigma$$

Since $$E = \frac{\lambda}{2\pi \varepsilon_0 r} = \frac{2\pi R \sigma}{2\pi \varepsilon_0 r}$$

$$\therefore \quad E = \frac{R\sigma}{\varepsilon_0 r}$$

This is the expression for electric field intensity for a point lying outside the surface of conducting wire or cylinder.

If point 'P' lies on the conducting wire or cylinder then $r = R$

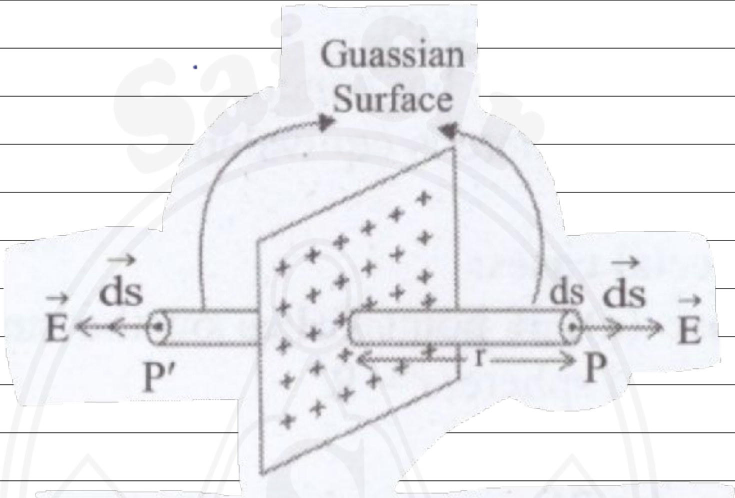
$$\therefore \quad E = \frac{R\sigma}{\varepsilon_0 r} = \frac{R\sigma}{\varepsilon_0 R} = \frac{\sigma}{\varepsilon_0}$$

If point 'P' lies inside the charged cylinder/wire then $q = 0$ i.e $\sigma = 0$

$$\therefore \quad E = 0$$

Q.7 obtain an expression for electric field due to an infinite charged plane sheet.

i) consider a uniformly charged infinite plane sheet with uniform surface charge density 'σ' kept in medium of permittivity ε $(\varepsilon = \varepsilon_0 K)$



Charged infinite plane sheet

ii) By Gauss' theorem, the net flux through a closed surface is

$$\phi = \frac{q}{\varepsilon} = \frac{q}{\varepsilon_0 K} = \frac{q}{\varepsilon_0} \quad \text{(for air/vaccum } K=1) \quad \ldots (1)$$

Where 'q' is the total charge inside the closed surface

iii) Imagine a Gaussian surface around point P in the form of a cylinder having cross sectional area 'A' and length '2r' with its axis perpendicular to the plane sheet.

iv) The flux passing through the curved surface is zero as the electric field is tangential to this surface $(\theta = 90°)$

v) The electric field is at a right angles to the end caps and away from the plane and contributes to the total flux through the closed surface

$$\text{i.e} \quad \phi = \left[\oint E\,ds\right]_P + \left[\oint E\,ds\right]_{P'} \quad \ldots (\theta = 0°, \cos\theta = 1)$$

i.e $\phi = [E \oint ds]_p + [E \oint ds]_{p'}$ (as $\oint ds = A$)

$\phi = EA + EA = 2EA$ ...... (2)

Comparing equation (1) and (2)

$$\frac{q}{\varepsilon_0} = 2EA$$

$$E = \frac{q}{2\varepsilon_0 A}$$

$$E = \frac{\sigma \cancel{A}}{2\varepsilon_0 \cancel{A}} \quad (\because q = \sigma A)$$

This is the required expression for electric field intensity due to infinitly charged plane sheet.

Direction of electric field is outward if sheet is positively charged and inward if it is negatively charged.

Note:-

Permittivity of free space $\varepsilon_0 = 8.85 \times 10^{-12}\ C^2/Nm^2$

$$\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\ Nm^2/C^2$$

Linear charge density $\lambda = \frac{q}{l}$

Surface charge density $\sigma = \frac{q}{A}$

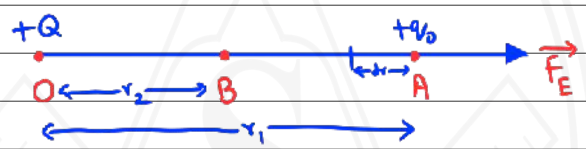
Volume charge density $\rho = \frac{q}{V}$

Q.8 Define electrostatic potential energy? Derive an expression for electrostatic potential energy.

i) Electrostatic potential Energy is the work done against the electrostatic forces to achieve a certain configuration of charges in a given system.

ii) Let us consider the electrostatic field due to a source charge $+Q$ placed at the origin O.

iii) Let a small charge $+q_0$ be brought from point 'A' to point 'B' at respective distances r_1 and r_2 from 'O', against the repulsive forces on it



iv) Work done against the electrostatic force $\vec{F}_E$, in displacing the charge q_0 through a small displacement 'dr' appears as an increase in potential energy of the system

$$\text{i.e} \quad dU = \vec{F}_E \cdot \vec{dr} = -F_E \cdot dr \quad \cdots \quad (1)$$

Negative sign appears because the displacement $\vec{dr}$ is against the electrostatic force $\vec{F}_E$.

v) The electrostatic force between the two charges Q and q_0 separated distance r is

$$F_E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Qq_0}{r^2} \quad \cdots \quad (2)$$

vi) If the charge is displaced from initial position A to final position B, then the change in potential Energy is obtained by integrating equation (1) between r_1 and r_2

i.e $$\Delta U = \int_{r_1}^{r_2} dU = \int_{r_1}^{r_2} -F_E \cdot dr$$

$$\therefore \Delta U = -\int_{r_1}^{r_2} \left(\frac{1}{4\pi\varepsilon_0}\frac{Qq_0}{r^2}\right) dr \quad \text{(from equation (2))}$$

$$\therefore \Delta U = -\frac{Qq_0}{4\pi\varepsilon_0}\int_{r_1}^{r_2} \frac{1}{r^2}\,dr \quad \left(\text{as } \int \frac{1}{r^2}\,dr = \frac{-1}{r}\right)$$

$$\Delta U = \frac{-Qq_0}{4\pi\varepsilon_0}\left[-\frac{1}{r}\right]_{r_1}^{r_2}$$

$$\Delta U = \frac{Qq_0}{4\pi\varepsilon_0}\left[\frac{1}{r}\right]_{r_1}^{r_2} = \frac{Qq_0}{4\pi\varepsilon_0}\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$$

is the expression for electrostatic potential Energy.

Note : Electrostatic potential Energy for the System of two point Charges separated by distance 'r'.

$$U_{(r)} = \frac{q_1 q_2}{4\pi\varepsilon_0}\left[\frac{1}{r}\right] \quad (\text{as } r_2 = r \ \& \ r_1 = \infty)$$

Q.9 Define the following

One joule (1J) : one joule is the energy stored in moving a charge of 1C through a potential difference of 1 volts.

one electron volt (1eV) : It is the change in the Kinetic energy of an electron while crossing two points maintained at a potential difference of 1 volt

$$1\,eV = 1.6 \times 10^{-19} \text{ joule}$$

$$1\,meV = 1.6 \times 10^{-22} \text{ joule}$$

$1\,KeV = 1.6 \times 10^{-16}$ joule and $1\,MeV = 1.6 \times 10^{-13}$ joule

Q.10 Discuss the concept of Electric potential.

i) Electric potential at any point in an electric field is defined as work done in bringing a unit charge from infinity to that point against the direction of electric field intensity.

ii) Potential energy of two particle System separated by distance 'r' is

$$U_{(r)} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{r}\right)$$

i.e $U_{(r)} = \left(\frac{q_1}{4\pi\varepsilon_0 r}\right) q_2 = \left(\frac{q_2}{4\pi\varepsilon_0 r}\right) q_1$

Let $V_{1(r)} = \frac{q_1}{4\pi\varepsilon_0 r}$ and $V_{2(r)} = \frac{q_2}{4\pi\varepsilon_0}$

$$\therefore \quad U_{(r)} = V_{1(r)} q_2 = V_{2(r)} q_1 \quad \ldots (1)$$

Where $V_{1(r)}$ & $V_{2(r)}$ are the respective potentials of Charge q_1 and q_2 at distance 'r' from each other.

Hence, we can write from eq (1)

Electrostatic potential Energy (U) = Electric potential (V) x charge (q)

i.e $U = V \times q$

Thus, we can say $V = \frac{U}{q}$

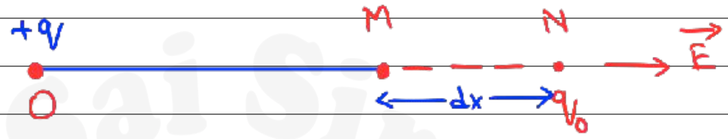
i.e Electric potential is Electrostatic potential Energy per unit charge

Also, Electrostatic potential difference between any two points in an electric field can be written as

$$V_2 - V_1 = \frac{U_2 - U_1}{q} = \frac{dW}{q} \quad \ldots$$ [work done in moving charge from point 2 to 1]

Q.11 Explain relation between electric field and electric potential

i) Consider the electric field produced by a charge $+q$ kept at point 'O'.



ii) A unit positive charge $(+q_0)$ is present in vicinity is moved towards charge $+q$ through a small distance dx.

iii) Hence the work done in moving charge $+q_0$ through distance dx is

$$dW = \vec{F} \cdot dx = -F\,dx \quad \ldots\ (1)$$

Negative sign indicates displacement of charge dx is opposite to electrostatic force F.

iv) By defination of electrostatic potential

$$dV = \frac{dW}{q_0}$$

$$\therefore dW = dV \times q_0 \quad \ldots\ (2)$$

Comparing eq (1) and (2), we get

$$-F\,dx = dV \times q_0 \quad \ldots\ (3)$$

But by defination of electric field intensity (E)

$$E = \frac{F}{q_0}$$

$$F = E \times q_0 \quad \ldots\ (4)$$

Subsituting eq (4) in eq (3)

$$-(E \times \cancel{q_0})\, dx = dV \times \cancel{q_0}$$

$$-E\, dx = dV$$

$$\therefore E = -\frac{dV}{dx}$$

Thus electric field intensity (E) at any point in Electric field is negative gradient of electric potential at that point.

Note :- For electrical circuits the earth is usually taken to be at zero potential

Formulas to remember.

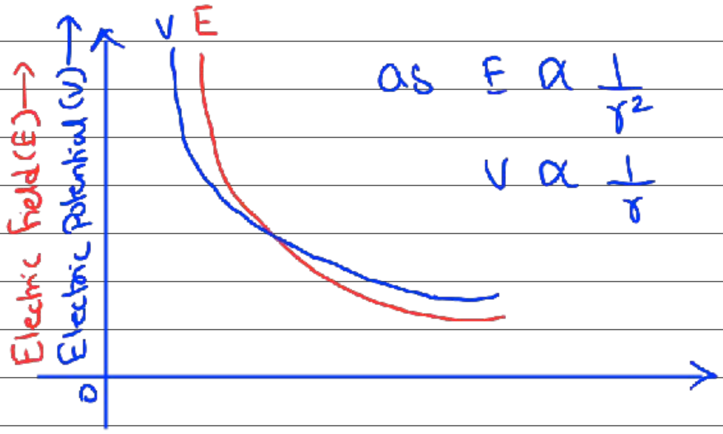
① Electrostatic force $F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}$

② Electrostatic Potential Energy $U = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r}$

③ Electrostatic potential $V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r}$

④ Electrostatic field $E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$

Graphical variation of electric field (E) and electric potential (V)

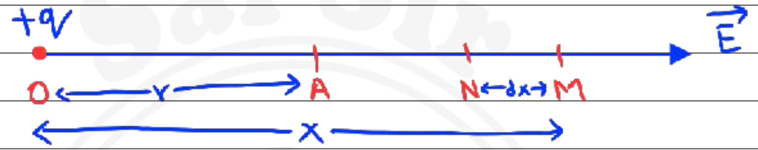


as $E \propto \frac{1}{r^2}$

$V \propto \frac{1}{r}$

Q.12 Derive an expression for electric potential due to a point charge. What can you infer from the expression?

i) Consider a point charge $+q$ located at point O. Let a unit positive charge q_0 be brought from ∞ to point 'A' as shown.



ii) Point 'M' is an intermediate point on this path such that $OM = x$

iii) Magnitude of electrostatic force on a unit positive charge ($q_0 = 1$ units) at 'M' is

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q\,q_0}{x^2} \quad \ldots \quad (1)$$

It is directed away from O, and along OM.

iv) For infinitesimal displacement dx from M to N. the amount of work done is

$$dW = -F\,dx \quad \ldots \quad (2)$$

Negative sign shows displacement is opposite to force.

v) Total work done in displacing unit positive charge from ∞ to A is obtained by integrating above equation between limits ∞ to r

$$\therefore \; W = \int_{\infty}^{r} dw = \int_{\infty}^{r} -F\,dx = -\int_{\infty}^{r} \frac{1}{4\pi\varepsilon_0} \cdot \frac{q \cdot q_0}{x^2}\,dx$$

i.e $$W = -\frac{q\,q_0}{4\pi\varepsilon_0} \int_{\infty}^{r} \frac{1}{x^2}\,dx$$

i.e $$W = \frac{-qq_0}{4\pi\varepsilon_0}\left[-\frac{1}{x}\right]_{\infty}^{r} \qquad \left(\because \int \frac{1}{x^2}dx = \frac{-1}{x}\right)$$

$$W = \frac{qq_0}{4\pi\varepsilon_0}\left[\frac{1}{x}\right]_{\infty}^{r}$$

$$\therefore W = \frac{qq_0}{4\pi\varepsilon_0}\left[\frac{1}{r} - \frac{1}{\infty}\right]$$

$$W = \frac{qq_0}{4\pi\varepsilon_0}\left(\frac{1}{r}\right) \qquad \left(\because \frac{1}{\infty} = 0\right)$$

By defination of Electric potential, it is the work done to bring unit positive charge q_0 from ∞ to a given point.

i.e $$V = \frac{W}{q_0}$$

$$\therefore V = \frac{\frac{q\cancel{q_0}}{4\pi\varepsilon_0}\left(\frac{1}{r}\right)}{\cancel{q_0}}$$

i.e Electric potential $$V = \frac{q}{4\pi\varepsilon_0 r}$$

is the required expression.

Inferences :-

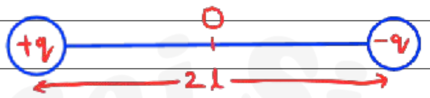
① A positive charged particle produces a positive electric potential and a negatively charged particle produces a negative electric potential

② At infinity $r = \infty$, $V = \frac{q}{4\pi\varepsilon_0(\infty)} = 0$

③ At distance 'r', V is same and independent of direction of 'r'. Hence electrostatic potential due to a single charge is spherical symmetric.

Q.13 Define electric dipole. What is electric dipole moment. Also define axis and equator of an electric dipole.

i) Electric dipole : An electric dipole consist of two charges +q and -q separated by a finite distance 2l.



ii) Electric dipole moment (p) : The product of charge on dipole (q) and its length (2l) is called as Electric dipole moment

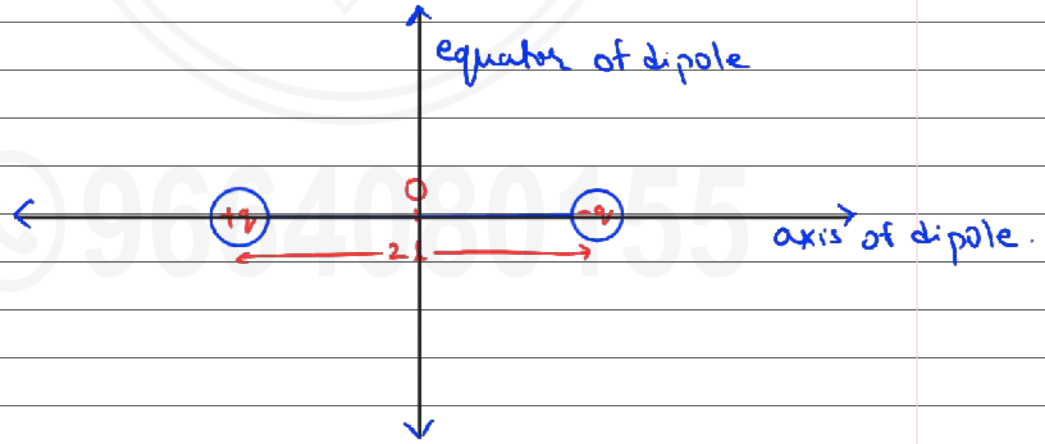
i.e $\vec{P} = q \times 2\vec{l}$

in Scalar's $P = p(2l)$

The SI unit of Electric dipole moment is (C-m). (Coloumb meter)

iii) Axis of dipole : The line joining the centre of the two charges of dipole is called the axis of dipole.

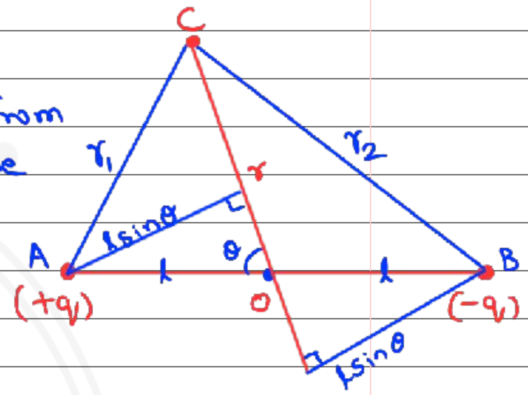
iv) Equator of dipole : The line drawn perpendicular to the axis and passing through centre of electric dipole is called equator of dipole.



Q.14 Derive an expression for electric potential due to an electric dipole. Discuss the same at axial and equatorial point.

i) Consider an electric dipole AB. Let 'O' be the centre of dipole

ii) Let 'C' be any point near the electric dipole at a distance 'r' from the centre O inclined at an angle θ with axis of the dipole.



Let r_1 and r_2 be the distances of point C from charges $(+q)$ and $(-q)$ respectively.

iii) Potential at C due to charge $+q$ at A is

$$V_1 = \frac{+q}{4\pi\varepsilon_0 r_1}$$

Potential at C due to charge $-q$ at B is

$$V_2 = \frac{-q}{4\pi\varepsilon_0 r_2}$$

iv) The potential at 'C' due to the dipole is,

$$V_c = V_1 + V_2$$

$$V_c = \frac{(+q)}{4\pi\varepsilon_0 r_1} + \frac{(-q)}{4\pi\varepsilon_0 r_2}$$

$$V_c = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_1} - \frac{1}{r_2}\right] \quad \ldots\ (1)$$

v) By geometry

$$r_1^2 = r^2 + l^2 - 2rl\cos\theta \qquad\qquad r_2^2 = r^2 + l^2 + 2rl\cos\theta$$

$$r_1^2 = r^2\left(1 + \frac{l^2}{r^2} - 2\frac{l}{r}\cos\theta\right) \qquad\qquad r_2^2 = r^2\left(1 + \frac{l^2}{r^2} + 2\frac{l}{r}\cos\theta\right)$$

For a short dipole, $2l << r$ and If $r >> l$; $\frac{l}{r}$ is small

$\therefore \frac{l^2}{r^2}$ can be neglected

$$r_1^2 = r^2\left(1 - 2\frac{l}{r}\cos\theta\right) \qquad\qquad r_2^2 = r^2\left(1 + 2\frac{l}{r}\cos\theta\right)$$

$$r_1 = r\left(1 - 2\frac{l}{r}\cos\theta\right)^{1/2} \qquad\qquad r_2 = r\left(1 + 2\frac{l}{r}\cos\theta\right)^{1/2}$$

$$\frac{1}{r_1} = \frac{1}{r}\left(1 - 2\frac{l}{r}\cos\theta\right)^{-1/2} \qquad\qquad \frac{1}{r_2} = \frac{1}{r}\left(1 + 2\frac{l}{r}\cos\theta\right)^{-1/2}$$

By using binomial expansion

$$(1+nx)^n = 1 + nx \;,\; x << 1$$

retaining only first term and neglecting higher terms.

i.e $$\left(1 - 2\frac{l}{r}\cos\theta\right)^{-1/2} \simeq \left(1 + \frac{l}{r}\cos\theta\right)$$

$$\left(1 + 2\frac{l}{r}\cos\theta\right)^{-1/2} \simeq \left(1 - \frac{l}{r}\cos\theta\right)$$

$$\therefore \frac{1}{r_1} = \frac{1}{r}\left(1 + \frac{l}{r}\cos\theta\right)$$

$$\frac{1}{r_2} = \frac{1}{r}\left(1 - \frac{l}{r}\cos\theta\right)$$

Subsituting $\frac{1}{r_1}$ & $\frac{1}{r_2}$ in equation (1)

$$V_c = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r}\left(1+\frac{l}{r}\cos\theta\right) - \frac{1}{r}\left(1-\frac{l}{r}\cos\theta\right)\right]$$

$$V_c = \frac{q}{4\pi\varepsilon_0}\frac{1}{r}\left[\left(1+\frac{l}{r}\cos\theta\right) - \left(1-\frac{l}{r}\cos\theta\right)\right]$$

$$V_c = \frac{q}{4\pi\varepsilon_0}\frac{1}{r}\left[\cancel{1}+\frac{l}{r}\cos\theta - \cancel{1}+\frac{l}{r}\cos\theta\right]$$

$$V_c = \frac{q}{4\pi\varepsilon_0 r}\left[\frac{2l\cos\theta}{r}\right]$$

$$V_c = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2} \qquad (\because p = q \times 2l)$$

is the expression for electric potential at C.

In vectors

$$V_c = \frac{1}{4\pi\varepsilon_0}\cdot\frac{\vec{p}\cdot\hat{r}}{r^2} \qquad \left(\text{where } \hat{r} = \frac{\vec{r}}{r}\right)$$

Where $\hat{r}$ is unit vector along $\overline{OC}$.

Special cases :-

a. Potential at an axial point
$\theta = 0^\circ$ (towards $+q$) or $\theta = 180^\circ$ (toward $-q$)

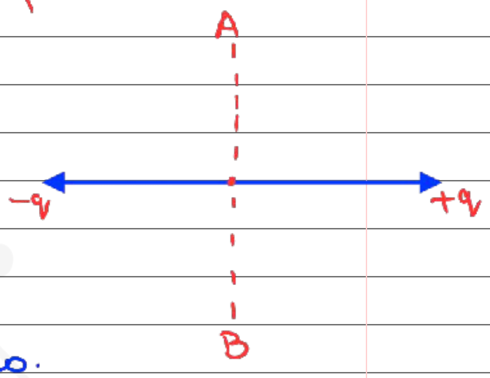
$$V_{axial} = \frac{\pm 1}{4\pi\varepsilon_0}\cdot\frac{p}{r^2}$$

This is the maximum value of the potential.

b. Potential at an equatorial point.
$\theta = 90^\circ$ and $V = 0$

Hence, the potential at any point on the equatorial line of a dipole is zero.
This is the minimum value of the magnitude of the potential of a dipole.

Q.15 A charge 'q' is moved from a point A above a dipole of dipole moment p to a point B below the dipole in equatorial plane without acceleration. Find the work done in this process.



i) Displacement of charge is along equatorial line of dipole (AB) where potential is zero throughout.

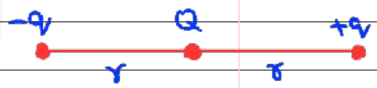
ii) As work done is

$$W = qV = q(0) = 0$$

work done in the process is zero.

Q.16 Three charges $-q$, $+Q$ and $-q$ are placed at equal distance on straight line. If the potential energy of the system of the three charges is zero, then what is the ratio of $Q:q$?

Potential energy of 3 charge - system



$$U = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}}\right)$$

Taking $r_{12} = r$, $r_{13} = 2r$, $r_{23} = r$

also $q_1 = q_3 = -q$ and $q_2 = Q$

$$U = \frac{1}{4\pi\varepsilon_0}\left(\frac{-qQ}{r} + \frac{(-q)(-q)}{2r} + \frac{-qQ}{r}\right)$$

$$U = \frac{1}{4\pi\varepsilon_0}\left(\frac{-qQ}{r} + \frac{q^2}{2r} - \frac{qQ}{r}\right)$$

Given : $U = 0$

$$\therefore \quad 0 = \frac{1}{4\pi\varepsilon_0}\left(-\frac{2qQ}{2r} + \frac{q^2}{2r}\right)$$

i.e $\quad \frac{-2qQ}{2r} + \frac{q^2}{2r} = 0$

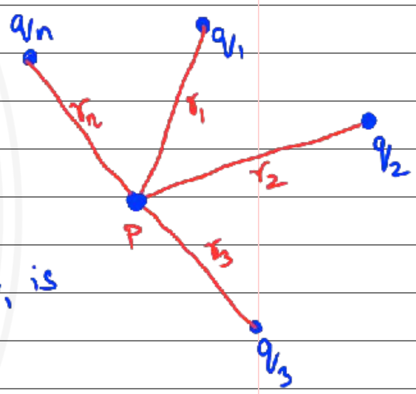
$$\therefore \quad \frac{\cancel{q}^2}{2\cancel{r}} = \frac{2\cancel{q}Q}{\cancel{r}}$$

$$\frac{q}{2} = 2Q$$

$$\therefore \quad Q = \frac{q}{4} \quad \text{or} \quad \frac{Q}{q} = \frac{1}{4}$$

Hence $Q:q = 1:4$

Q.17 Derive an expression for electrostatic potential due to system of charges.



i) Consider a system of charges $q_1, q_2, \ldots\ldots q_n$ at distances $r_1, r_2 \ldots r_n$ respectively from point P as shown in figure.

ii) The potential V_1 at P due to the charge q_1 is

$$V_1 = \frac{1}{4\pi\varepsilon_0} \frac{q_1}{r_1}$$

Similarly,

$$V_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_2}{r_2}, \quad V_3 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_3}{r_3}, \ldots\ldots, V_n = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_n}{r_n}$$

By the superposition principle, the potential V at point P due to the system charges is the algebraic sum of the potential due to the individual charges.

i.e $\quad V = V_1 + V_2 + V_3 + \cdots\cdots + V_n$

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \cdots + \frac{q_n}{r_n}\right) = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{r_i}$$

iii) for a continuous charge distribution, summation should be replaced by integration.

Q.18 Define equipotential surface. state and explain its properties.

i) An equipotential surface is a surface with the same electric potential at every point.

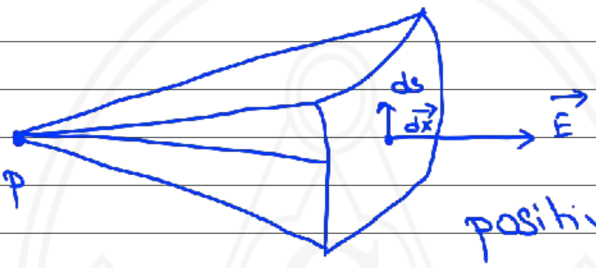
ii) Equipotential surfaces can be drawn through any region in which there is an electric field.

iii) No work is required to move a test charge along an equipotential surface.

i.e $\quad W_{QP} = V_P - V_Q = 0$

Since $V_P = V_Q$

iv) Electric field intensity $\vec{E}$ is always normal to equipotential surface.



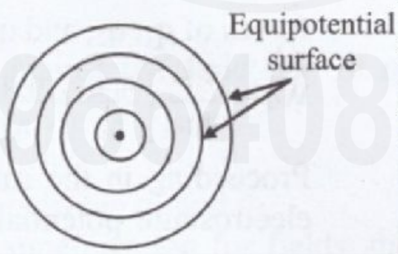
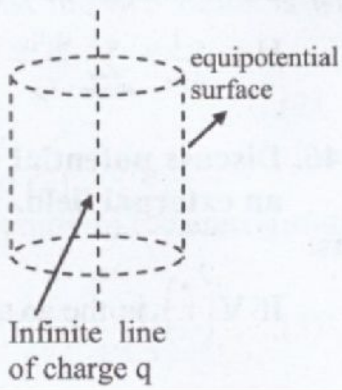
dx is the small distance over the equipotential surface through which unit positive charge is carried.

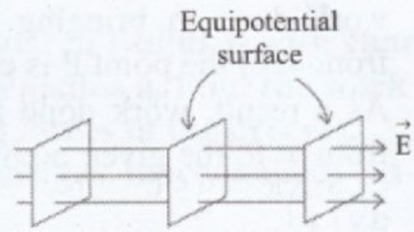
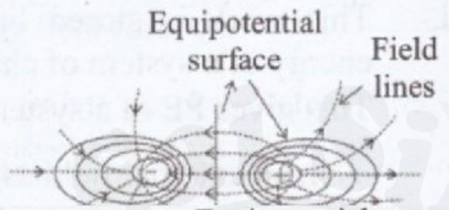
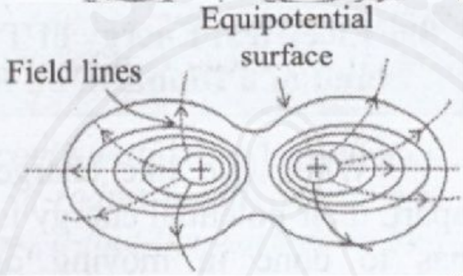
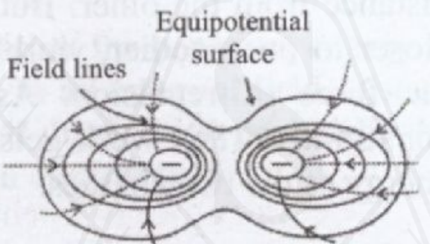
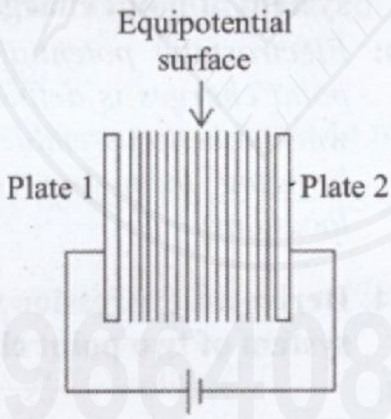
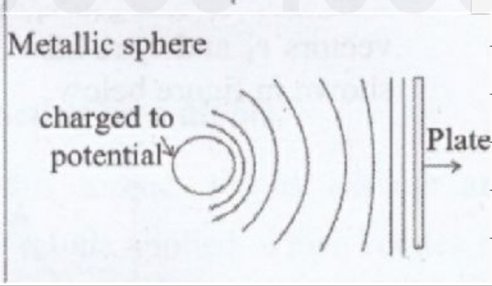
i.e $\quad dW = \vec{E}\cdot\vec{dx} = E\cdot dx\cos\theta = E\cdot dx\cos 90^\circ = 0$

v) No two equipotential surfaces can intersect each other.

vi) Like the lines of forces, the equipotential surface gives a visual picture of both the direction and the magnitude of electric field in a region of space.

Q.19 Draw figures of equipotential surfaces for following charge distributions.

Point charge	
Line charge	

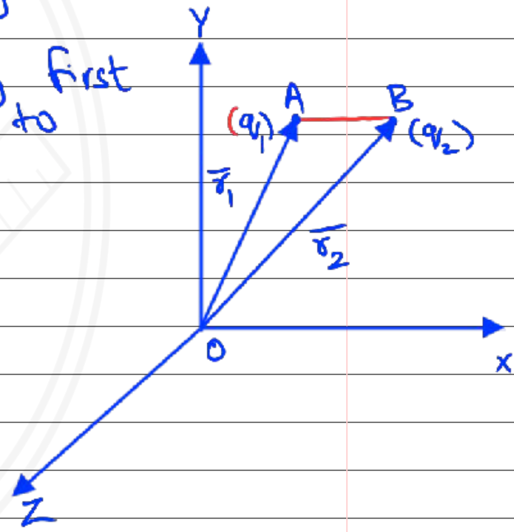
Uniform electric field	
Electric dipole	
Two identical positive charges	
Two identical negative charges	
Between 2 plane metallic sheets connected to a cell.	
When one of the sheets is replaced by a charged metallic sphere.	

Q.20 Define electrostatics potential energy of a system of point charges. Derive an expression for potential energy of a system of two point charges.

Electrostatic potential energy of a system of point charges is defined as the total amount of work done to assemble the system of charges by bringing them infinity to their present locations.

Expression for potential energy of a system of two point charges.

i) Consider two charges q_1 and q_2 with position vector r_1 and r_2 relative to some origin (O) as shown



ii) No work is done while bring first charge (q_1) to position A due to absence of electric field.

i.e $W_1 = 0$

Now charge 'q_1' produces a potential in space given by

$$V_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1}{r_1} \quad \ldots (1)$$

iii) Work done to bring charge q_2 from infinite to B (at $\bar{r}_2$) is

$$W_2 = \left(\begin{array}{c}\text{Potential at B due} \\ \text{charge } q_1\end{array}\right) \times (\text{charge } q_2)$$

$$W_2 = \left(\frac{q_1}{4\pi\varepsilon_0 r_{12}}\right) \times q_2$$

....(Where $AB = r_{12}$)

iv) This work done in bringing the two charges to their respective locations is stored as the potential energy of the configuration of two charges.

$$U = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r_{12}}$$

This is the required expression and can be generalised for a system of any number of point charges.

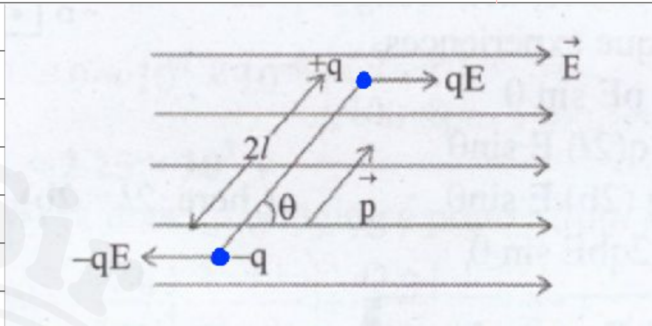
i.e $$U = \frac{1}{4\pi\varepsilon_0} \sum_{\text{all pairs}} \frac{q_j q_k}{r_{jk}}$$

Note :-

Potential Energy (P.E) = Work done (W) = Change in P.E

= charge (q) X potential (V)

Q.21 obtain an expression for potential energy of an electric dipole in an external field. Discuss potential energy if dipole is (i) perpendicular and (ii) parallel to electric field.



Couple acting on a dipole

i) Consider a dipole with charges $-q$ and $+q$ separated by a finite distance '$2l$', placed in a uniform electric field $\vec{E}$.

ii) It experiences a torque $\vec{\tau}$ which tends to rotate the dipole. This torque (τ) is given by

$$\vec{\tau} = \vec{p} \times \vec{E} = pE\sin\theta \quad \ldots (1)$$

iii) To neutralize this torque, let us assume an external torque $\vec{\tau}_{ext}$ be applied, which rotates dipole in the plane of the paper from angle θ_0 to angle θ, without angular acceleration and at an infinitesimal angular speed.

iv) Work done by the external torque

$$W = \int_{\theta_0}^{\theta} \tau_{ext} \cdot d\theta = \int_{\theta_0}^{\theta} pE\sin\theta \, d\theta$$

$$W = pE\left[-\cos\theta\right]_{\theta_0}^{\theta}$$

$$W = pE\left[-\cos\theta - (-\cos\theta_0)\right]$$

$$W = pE\left[-\cos\theta + \cos\theta_0\right]$$

$$W = pE\left[\cos\theta_0 - \cos\theta\right]$$

This work done is stored as the potential energy of the system in the position when the dipole makes an angle θ with the electric field.

v) Thus, potential energy of electric dipole in external electric field is

$$U(\theta) - U(\theta_0) = pE(\cos\theta_0 - \cos\theta)$$

Special case :- Choosing $U(\theta_0) = 0$ we get,

case a :- If initially the dipole is perpendicular to the field $\vec{E}$.

i.e $\theta_0 = \frac{\pi}{2}$ then

$$U(\theta) = pE\left(\cos\frac{\pi}{2} - \cos\theta\right) = -pE\cos\theta$$

$$\therefore \quad U(\theta) = -\vec{p}\cdot\vec{E}$$

Case b :- If initially the dipole is parallel to the field $\vec{E}$ then $\theta_0 = 0$

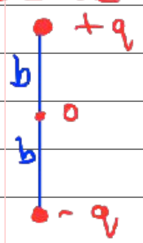
$$U(\theta) = pE(\cos 0 - \cos\theta) = pE(1 - \cos\theta)$$

Q.22 A dipole with its charges, $-q$ and $+q$ located at the points $(0,-b,0)$ and $(0,+b,0)$ is present in a uniform electric field E. The equipotential surface of this field are planes parallel to the YZ planes.

1. What is the direction of the electric field E?

As the equipotential surfaces of uniform electric field are plane parallel to YZ planes, direction of $\vec{E}$ must be perpendicular to YZ planes i.e along X-axis

2. How much torque would the dipole experience in this field?



Torque experiences, $\tau = pE\sin\theta$

$\tau = q(2l)E\sin\theta$

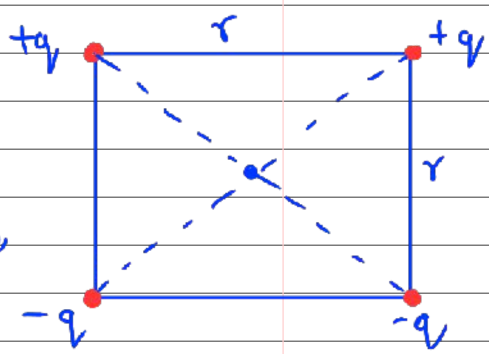
$\tau = q(2b)E\sin\theta$

$\tau = 2qbE\sin\theta$

Q.23 Calculate the electrostatic potential energy of the System of Charges shown in the figure

$d(AB) = d(BC) = d(CD) = d(AD) = r$

$d(AC) = d(BD) = \sqrt{2}\cdot r$



Assuming potential at ∞ as zero, potential energy of system of charges

$$U = \frac{1}{4\pi\varepsilon_0}\sum_{j,k}\frac{q_j q_k}{r_{jk}}$$

$$U = \frac{1}{4\pi\varepsilon_0}\left[\frac{qq}{d(AB)} + \frac{q(-q)}{d(AD)} + \frac{q(-q)}{d(AC)} + \frac{q(-q)}{d(BC)} + \frac{q(-q)}{d(BD)} + \frac{-q(-q)}{d(CD)}\right]$$

$$U = \frac{1}{4\pi\varepsilon_0}\left[\cancel{\frac{q^2}{r}} - \cancel{\frac{q^2}{r}} - \frac{q^2}{\sqrt{2}r} - \cancel{\frac{q^2}{r}} - \frac{q^2}{\sqrt{2}r} + \cancel{\frac{q^2}{r}}\right]$$

$$U = \frac{1}{4\pi\varepsilon_0}\left[\frac{-2q^2}{\sqrt{2}\,r}\right]$$

$$U = \frac{1}{4\pi\varepsilon_0}\left[\frac{-\sqrt{2}\cdot\cancel{\sqrt{2}}\,q^2}{\cancel{\sqrt{2}}\,r}\right]$$

$$U = \frac{1}{4\pi\varepsilon_0}\left[\frac{-\sqrt{2}\,q^2}{r}\right] = \frac{-\sqrt{2}\,q^2}{4\pi\varepsilon_0 r}$$

Q.24 Distinguish between conductors and insulators

Sr. No.	Conductors	Insulators
i.	Conductors are materials or substances which allow electricity to flow through them.	Insulators are materials or substances which resist electricity to flow through them.
ii.	They contain a large number of free charge carriers (free electrons). For example, in a metal the outer (valence) electrons are loosely bound to the nucleus and are thus free for conductivity, when an external electric field is applied.	They do not free charges carriers.
iii.	A conductor can carry any distribution of external electric charges on its surface or in interior and electric field in interior can be zero.	An insulator can carry any distribution of external electric charges on its surface or in its interior and the electric field in the interior can have non zero values.
iv.	Examples: Metals, humans, earth and animal bodies	Examples: Wood, glass, ebonite

Q.25 Explain why insulating material can be considered as a collection of molecules that are not easily ionized.

i) In insulators, the electrons are tightly bound to the nucleus and are thus not available for conductivity

ii) There are no free charges since all the charges are bound to the nucleus.

iii) This makes removing charge to form an ion of a substance extremely difficult.

iv) Hence, an insulating material can be considered as a collection of molecules that are not easily ionized.

Q.26 list properties of conductor in electrostatic conditions

i) In the interior of a conductor, net electrostatic field is zero.

ii) Potential is constant within and on the surface of a conductor.

iii) In static situation, the interior of a conductor can have no charge.

iv) Electric field just outside a charged conductor is perpendicular to the surface of the conductor at every point.

v) surface charge density of a conductor could be different at different points.

Q.27 Explain electrostatic shielding with example.

i. To protect a delicate instrument from the disturbing effects of other charged bodies near it, it is placed inside a hollow conductor where E = 0. This is called electrostatic shielding.

ii. Thin metal foils are used in making the shields.

Examples:

a. During lightning and thunder storm it is always advisable to stay inside the car than near a tree in open ground, since the car acts as a shield.

b. **Faraday Cages:** It is an enclosure which is used to block the external electric fields in conductive materials.

c. **Electro-magnetic shielding:** MRI scanning rooms are built in such a manner that they prevent the mixing of the external radio frequency signals with the MRI machine.

Q.28 The safest way to protect yourself from lightening is to be inside a car. Justify.

The body of the car is metallic. It provides electrostatic shielding to the person in the car because electric field inside the car is zero.

The discharging due to lightning passes to the ground through the metallic body of the car thereby keeping person sitting inside safe.

Q.29 Define free charges and bound charges

Free charges: In metallic conductors, the electrons in the outermost shells of the atoms are loosely bound to the nucleus and hence can easily get detached and move freely inside the metal. When an external electric field is applied, they drift in a direction opposite to the direction of the applied electric field. These charges are called free charges.

Bound charges: The nucleus, which consist of the positive ions and the electrons of the inner shells, remain held in their fixed positions. These immobile charges are called bound charges.

Q.30 Explain concept of electric polarisation.

i. Certain substances when are placed in an external field, their positive and negative charges get displaced in opposite directions and the molecules develop a net dipole moment. This is called polarization of the material.

ii. The dipole moment per unit volume is called polarization and is denoted by $\vec{P}$. For linear isotropic dielectrics $\vec{P} = \chi_e \vec{E}$.

Where, χ_e is a constant called electric susceptibility of the dielectric medium.

iii. **Examples:** Dielectrics substances show electric polarisation.

Q.31 What is electric susceptibility of dielectric medium?

i. *A quantity that describes electrical behaviour of a dielectric is called as electric susceptibility of dielectric medium.*

ii. It is denoted by χ_e.

iii. It is constant for a dielectric but has different values for different dielectrics.

iv. For vacuum $\chi_e = 0$.

Q.32 Explain concept of dielectrics.

i. *Dielectrics are non-conducting substances which cannot transmit electric charge through them.*

Examples:

Glass, wax, water, wood, mica, rubber, stone, plastic, etc.

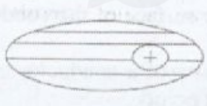
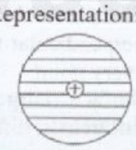
ii. Dielectric substances do not contain any free electrons in them, so they have no charge carriers.

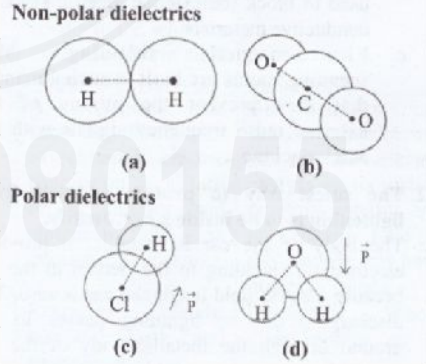
iii. Dielectrics can be polarised through small localised displacement of charges.

iv. Dielectrics are insulates which can be used to store electrical energy.

v. Dielectrics can be classified as polar dielectrics and non-polar dielectrics.

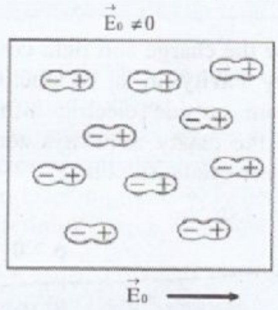
Q.33 Distinguish between polar and non-polar dielectrics.

Sr. No.	Polar dielectric	Non-polar dielectric
i.	A dielectric molecule in which the centre of mass of positive charges (protons) does not coincide with the centre of mass of negative charges (electrons), because of the asymmetric shape of the molecules is called polar dielectric.	A dielectric in which the centre of mass of the positive charges coincides with the centre of mass of the negative charges is called a non-polar dielectric.
ii.	Representation: 	Representation: 
iii.	They have permanent dipole moments of the order of 10^{-30} Cm. They act as tiny electric dipoles, as the charges are separated by a small distance.	These have symmetrical shapes and have zero dipole moment in the normal state.
iv.	Examples: HCl, water, alcohol, NH_3	Examples: H_2, N_2, O_2, CO_2, benzene, methane



Q.32 Explain polarization of a non-polar dielectric in an external electric field.

i. In the presence of an external electric field E_0, the centres of the positive charge in each molecule of a non-polar dielectric is pulled in the direction of E_0, while the centres of the negative charges are displaced in the opposite direction. Thus, the two centres are separated and the molecule gets distorted.

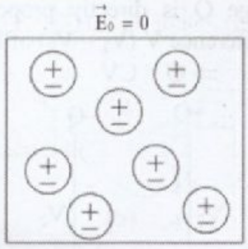


In presence of an external field

ii. The displacement of the charges stops when the force exerted on them by the external field is balanced by the restoring force between the charges in the molecule.

iii. Each molecule becomes a tiny dipole having a dipole moment.

iv. The induced dipole moments of different molecules add up giving a net dipole moment to the dielectric in the presence of the external field.



The non polar dielectric in absence of electric field

Q.33 Explain polarization of a polar dielectric in an external electric field.

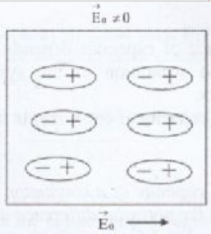
i. The molecules of a polar dielectric have tiny permanent dipole moments. Due to thermal agitation in the material in the absence of any external electric field, these dipole moments are randomly oriented. Hence the total dipole moment is zero.

ii. When an external electric field is applied the dipole moments of different molecules tend to align with the field. As a result, the dielectric develops a net dipole moment in the direction of the external field. Hence the dielectric is polarized.

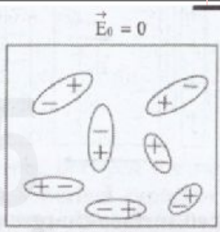
iii. The extent of polarization depends on the relative values of the two opposing energies:

a. The applied external electric field which tends to align the dipole with the field.

b. Thermal energy tending to randomise the alignment of the dipole.



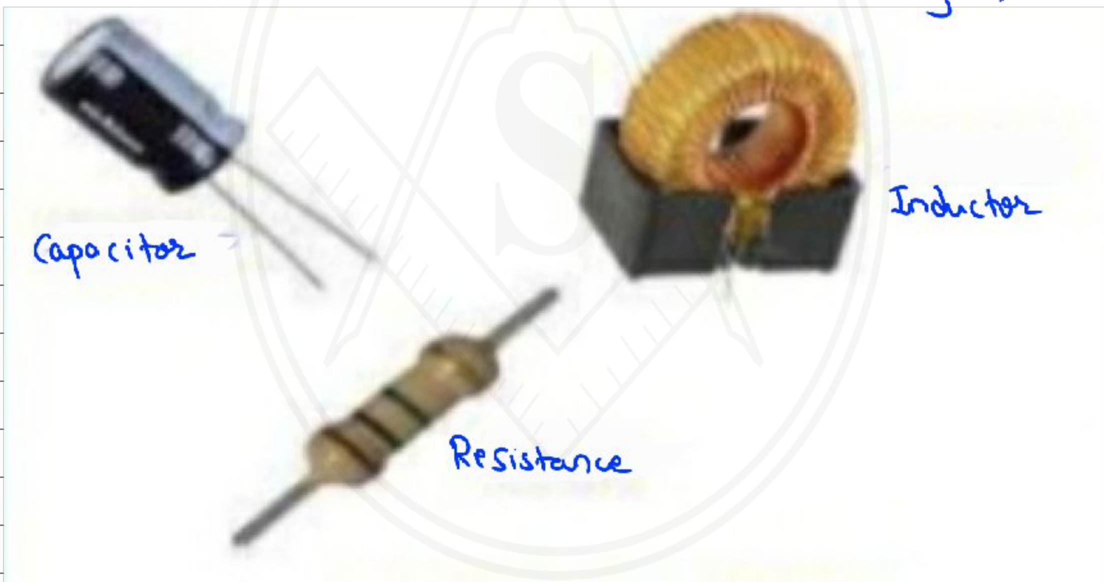
In presence of a strong external field



The polar dielectric in absence of electric field

- ❑ Resistor: the ability to resist the flow of electric current through it.
- ❑ Capacitor: the ability to oppose the change of voltage across it.
- ❑ Inductor: the ability to oppose the change of current flowing through it.

(ohm Ω) (farad F) (henry H)



Q.34 What is capacitor?

Capacitor is a system consisting of two conductors having equal and opposite charges separated by an insulator or dielectric

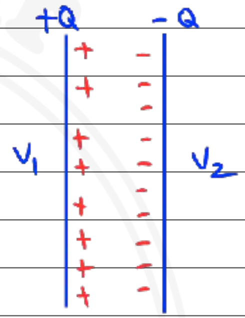
Q.35 Define capacity of a conductor. State and define the SI unit of capacity of conductor. Write its dimensions.

i) The ability of a conductor to store the electric charge is called capacity of conductor.

ii) When the charges on the conductors are increased, potential difference between them also gets increased. Thus, charge Q is directly proportional to the potential difference 'V' i.e $(V_2 - V_1)$ of the conductors.

i.e $Q \propto V$

$\Rightarrow$ $Q = CV$



where, C = Constant of proportionality or capacity of conductor or capacitance

$$\therefore C = \frac{Q}{V}$$

iii) Capacitance of capacitor depends on the size, shape and separation of the system of two conductors.

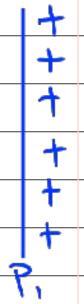
iv) SI unit of capacity of conductor is farad (F)

$$\therefore 1F = \frac{1C}{1V}$$

Thus, the capacity of a conductor is said to be 1 farad if the potential difference across it rises by 1 volt, when 1C charge is given to it.

Dimensions : $[M^{-1} L^{-2} T^{4} A^{2}]$

Q.36 Explain principle of a capacitor.



i) Consider a metal plate P_1 having area A with some positive charge $+Q$ be given to the plate.

ii) Let its potential be V. Its capacity is given by

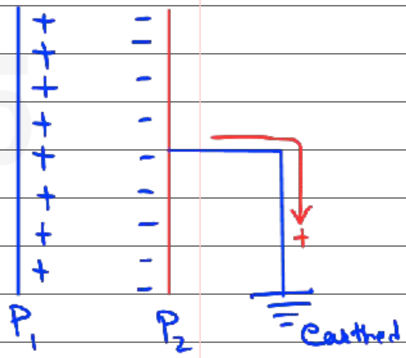
$$C_1 = \frac{Q}{V}$$

iii) Now consider another insulated metal plate P_2 held near the plate P_1. By induction a negative charge is produced on the nearer face and an equal positive charge develops on the farther face of P_2 as shown in figure.

iv) The induced negative charge lowers the potential of plate P_1, while the induced positive charge raises its potential.

v) As the induced negative charge is closer to P_1 it is more effective and thus there is a net reduction in potential of plate P_1.

vi) If the outer surface of P_2 is connected to earth the induced positive charges on P_2 being free flows to earth. The induced negative charge on P_2 stays on it, as it is bound to positive charge of P_1. This greatly reduces the potential of P_2 as show in figure.



vii) If V_1 is the potential on plate P_2 due to charge $(-Q)$ then the net potential of the system will now be $(V - V_1)$

Hence the new capacity $C_2 = \frac{Q}{V - V_1}$

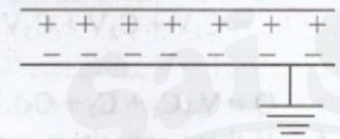
i.e $C_2 > C_1$, thus capacity of metal plate P_1 is increased by placing an identical earth connected metal plate P_2 near it.

Q.37 How is capacitor represented symbolically? State various types of capacitors. State formula for capacitance in each case

Symbol: 

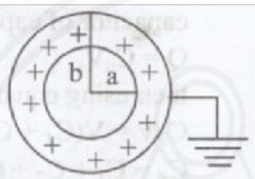
a. Parallel plate capacitor:

$$C = \frac{k\varepsilon_0 A}{d}$$



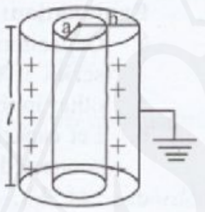
b. Spherical capacitor:

$$C = 4\pi\varepsilon_0\left(\frac{ab}{b-a}\right)$$



c. Cylindrical capacitor:

$$C = \frac{2\pi\varepsilon_0 l}{\log_e\left(\frac{b}{a}\right)} = \frac{2\pi\varepsilon_0 l}{2.303\log\left(\frac{b}{a}\right)}$$



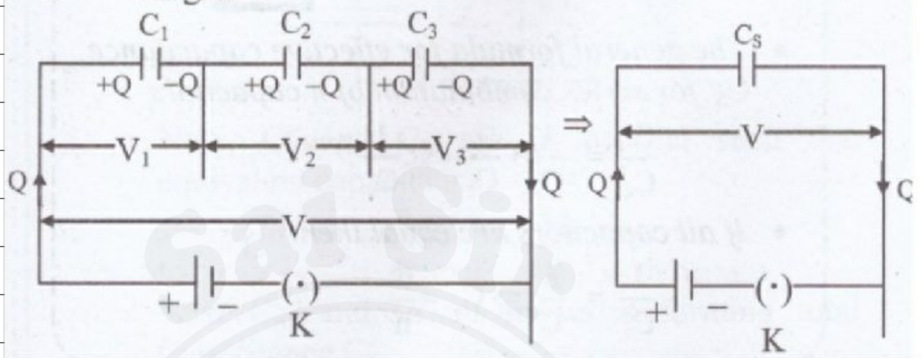
Q.38 If the difference between the radii of the two spheres of a spherical capacitor is increased, state whether the capacitance will increase or decrease.

For a spherical capacitor $C = 4\pi\varepsilon_0\left(\frac{ab}{b-b}\right)$

If $(b-a)$ increases, due to inverse relation with capacitance (C), capacitance of capacitor will decrease.

Q.39 Derive an expression for the effective capacitance of three parallel plate capacitors connected in series.

i. Diagram:



i) Capacitors are said to be connected in series, if they are connected one after the other.

ii) Let C_1, C_2 and C_3 be three capacitors connected in series.

iii) Let V_1, V_2 and V_3 be corresponding potential difference accross each capacitor respectively.

iv) Let 'V' be the applied potential difference, and let 'Q' be the amount of charge received by each capacitor respectively such that,

$$V_1 = \frac{Q}{C_1}, \quad V_2 = \frac{Q}{C_2}, \quad V_3 = \frac{Q}{C_3}$$

v) Let C_s be the equivalent series capacitance, Such that

$$V = \frac{Q}{C_s}$$

Since $\quad V = V_1 + V_2 + V_3$

$$\frac{Q}{C_s} = \frac{Q}{C_1} + \frac{Q_1}{C_2} + \frac{Q}{C_3}$$

$$\frac{\cancel{Q}}{C_s} = \cancel{Q}\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

If there are 'n' number of capacitors connected in series then,

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots \frac{1}{C_n}$$

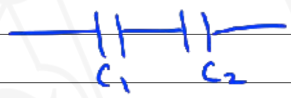
Note:- If 'n' capacitor of equal value are connected in series then

$$\frac{1}{C_s} = \frac{n}{C}$$

i.e $C_s = \frac{C}{n}$

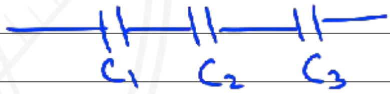
If two capacitors are connected in series then

$$C_s = \frac{C_1 C_2}{C_1 + C_2}$$



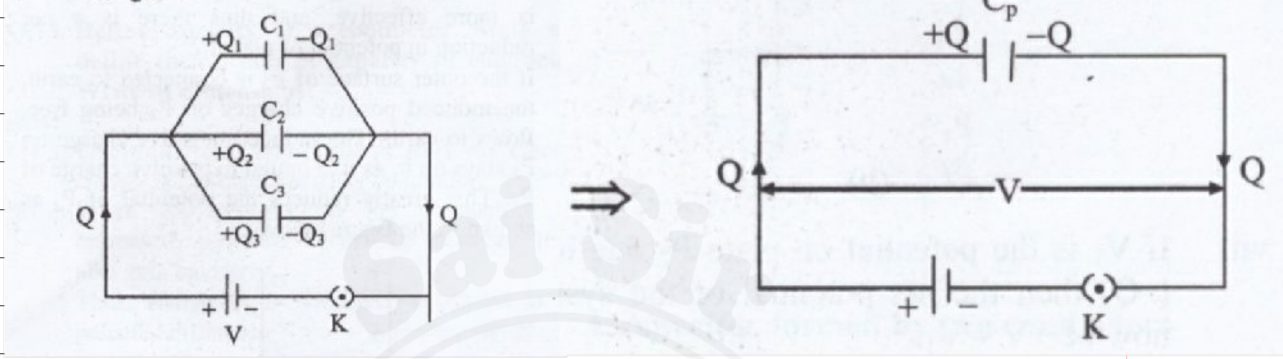
If three capacitors are connected in series then

$$C_s = \frac{C_1 C_2 C_3}{C_1 C_2 + C_2 C_3 + C_3 C_1}$$



Q.40 Derive an expression for effective capacitance of three capacitors connected in parallel.

i. **Diagram:**



i) Capacitors are said to be connected in parallel, if they are connected between two common points or junctions.

ii) Consider three capacitors of capacitances C_1, C_2 and C_3 connected in parallel.
Let Q_1, Q_2 and Q_3 be the charge deposited on the capacitors as shown in the figure.

iii) If 'V' is the applied potential across the combination.

iv) Since different current flows through differen branch so the charges on each capacitor is given as

$$Q_1 = C_1 V \ ; \ Q_2 = C_2 V \quad \text{and} \quad Q_3 = C_3 V$$

v) If C_p is the equivalent capacitor of the combination then

$$Q = C_p V$$

According to the principle of conservation of charge

$$Q = Q_1 + Q_2 + Q_3$$
$$C_p V = C_1 V + C_2 V + C_3 V$$
$$C_p \cancel{V} = \cancel{V}(C_1 + C_2 + C_3)$$

$$C_p = C_1 + C_2 + C_3$$

If there are 'n' capacitor connected in parallel combination then

$$C_p = C_1 + C_2 + C_3 + \cdots\cdots + C_n$$

If all capacitors are equal then

$$C_p = nC$$

Q.41 Give the difference between the combination of capacitors

Capacitors in series:

a. Potential difference across each capacitor is different.

b. Charge on each capacitor is same.

c. This arrangement stands high voltage.

d. Series combination is used when a high voltage is to be divided on several capacitors. Capacitor with minimum capacitance has the maximum potential difference between the plates.

e. This arrangement cannot store large number of charges.

Capacitors in parallel:

a. Potential difference across each capacitor is same.

b. Charge on each capacitor is different.

c. This arrangement stands low voltage.

d. Capacitors are combined in parallel when we require a large capacitance at small potentials.

e. This arrangement can store large number of charges.

Q.42 Give the effect of dielectrics on the capacity of parallel plate capacitor.

i) Consider two parallel metal plates P_1 and P_2 each having area 'A' separated by a small distance 'd' with air between the plates.

ii) The capacity of parallel plate capacitor with air as dielectric is given by

$$C_{air} = \frac{A\varepsilon_0}{d} \quad \ldots (1)$$

iii) Let a dielectric material of dielectric constant 'K' be introduced in the space between two parallel plates, such that the dielectric completely fills the space between them.

iv) The capacity of parallel plate capacitor with dielectric of dielectric constant 'K' is given by

$$C_d = \frac{KA\varepsilon_0}{d} \quad \ldots (2)$$

Dividing eq (2) by (1)

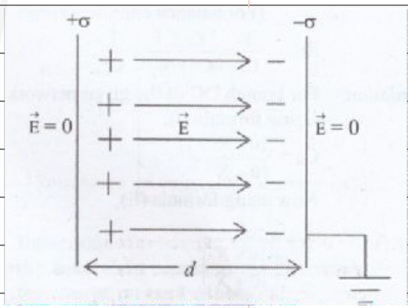
$$\frac{C_d}{C_{air}} = \frac{\frac{KA\varepsilon_0}{d}}{\frac{A\varepsilon_0}{d}} = K$$

$$\therefore C_d = KC_{air}$$

Thus the capacity of parallel plate capacitor in a medium filled with dielectric is 'K' times its capacity in air.

Q.43 Obtain an expression for capacitance of a parallel plate capacitor without a dielectric.

i) A parallel plate capacitor consists of two thin conducting plates each of area A, held parallel to each other, at a suitable distance 'd' apart.



ii) The plates are separated by an insulating medium like paper, air, mica, glass etc.
One of the plates is insulated and the other is earthed as shown in figure

iii) When a charge +Q is given to the insulated plate, then a charge −Q is induced on the inner face of earthed plate and +Q is induced on its farther face.
But as this face is earthed the charge +Q being free, flows to earth.

iv) In the outer regions the electric fields due to the two charged plates cancel out. Making net field zero.

$$E_{outer} = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = 0$$

v) In the inner regions between the two capacitor plates the electric fields due to the two charged plates adds up. The net field is thus

$$E_{outer} = E = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0} \quad \cdots (1)$$

$$\left(\text{as } \sigma = \frac{Q}{A}\right)$$

The direction of E is from positive to negative

vi) Let V be the potential difference between the two plates. Then electric field between the plates is given by

$$E = \frac{V}{d}$$

$$\text{i.e. } V = Ed$$

$$\therefore V = \left(\frac{Q}{A\varepsilon_0}\right) d \quad \cdots (ii)$$

Thus capacitance of the parallel plat capacitor is given by

$$C = \frac{Q}{V}$$

$$C = \frac{Q}{\left(\frac{Q}{A\varepsilon_0}\right) d} = \frac{A\varepsilon_0}{d}$$

This is the required expression.

If there are 'n' parallel plates then there will be (n-1) capacitors, hence

$$C = (n-1)\frac{A\varepsilon_0}{d}$$