QUICK REVISION

12th Mathematics

All Chapterwise Formulas

For Chemistry and Physics

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MATHEMATICS BASIC FORMULAE

1. Co-ORDINATE GEOMETRY

(1) To change from cartesian coordinates to polar coordinates for x write $r\cos\theta$ and for y write $r\sin\theta$.

(2) To change from polar coordinates to cartesian coordinates, for r^2 write $x^2 + y^2$; for r cos θ write x for $r\sin\theta$. write y and for tan θ write $\frac{y}{x}$.

(3) Distance between two points (x_1, y_1) and (x_2, y_2) is :
$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(4) Distance of points (x_1, y_1) from the origin is : $\sqrt{x_1^2 + y_1^2}$

(5) Distance between (r_1, θ_1) and (r_2, θ_2) is : $\sqrt{r_1^2 + r_2^2 - 2 r_1 r_2 \cos(\theta_2 - \theta_1)}$

(6) Coordinates of the point which divides the line joining (x_1, y_1) and (x_2, y_2) internally in the ratio $m_1 : m_2$ are :

$$\left\{\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2} \middle| \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}\right\}, (m_1 - m_2 \neq 0)$$

(7) Coordinates of the point which divides the line joining (x_1, y_1) and (x_2, y_2) externally in the ratio $m_1 : m_2$ are :

$$\left\{\frac{m_1 x_2 - m_2 x_1}{m_1 - m_2} \middle| \frac{m_1 y_2 - m_2 y_2}{m_1 - m_2}\right\}, (m_1 - m_2 \neq 0)$$

(8) Coordinates of the mid-point (point which bisects) of the seg. joining (x_1, y_1) and (x_2, y_2) are : $\left[\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right]$

(9) (a) **Centriod** is the point of intersection of the medians of triangle.

(b) **In-centre** is the point of intersection of the bisectors of the angles of the triangle.

(c) **Circumcentre** is the point of intersection of the right (perpendicular) bisectors of the sides of a triangle.

(d) **Orthocentre** is the point of intersection of the altitudes (perpendicular drawn from the vertex on the opposites) of a triangle.

(10) Coordinates of the centriod of the triangle whose vertices are (x_1, y_1) (x_2, y_2); and (x_3, y_3) are : $\left\{\frac{x_1 + x_2 + x_3}{3} \middle| \frac{y_1 + y_2 + y_3}{3}\right\}$

(11) Coordinates of the in-centre of the triangle whose vertices are A (x_1, y_1); B (x_2, y_2); C (x_3, y_3) and l (BC) = a, l (CA) = b, l (AB) = c.
are $\left\{\frac{ax_1 + bx_2 + cx_3}{a+b+c} \middle| \frac{ay_1 + by_2 + cy_3}{a+b+c}\right\}$

(12) Slope of line joining two points (x_1, y_1) and (x_2, y_2) is
$m = \frac{y_2 - y_1}{x_2 - x_1}$

(13) Slope of a line is the tangent ratio of the angle which the line makes with the positive direction of the X-axis i.e. $m = \tan\theta$.

(14) Slope of the perpendicular to X-axis (parallel to Y-axis) does not exist, and the slope of line parallel to X-axis is zero.

(15) **Intercepts:** If a line cuts the X-axis at A and y-axis at B then OA is called intercept on X-axis and denoted by "a" and OB is called intercept on Y- axis and denoted by "b".

(16) $x = a$ is equation of line parallel to Y-axis and passing through (a, b) and $y = b$ is equation of the line parallel to X-axis and passing through (a, b).

(17) $x = 0$ is the equation of Y - axis and $y = 0$ is the equation of X - axis.

(18) $y = mx$ is the equation of the line through the origin and whose slope is m.

(19) $y = mx + c$ is the equation of line in **slope intercept** form.

(20) $\frac{x}{a} + \frac{y}{b} = 1$ is the equation of line in the **Double intercepts** form,
where "a" is x - intercept and "b" is y- intercept.

(21) $x\cos\alpha + y\sin\alpha = p$ is the equation of line in **normal form**, where "p" is the length of perpendicular from the origin on the line and α is the angle which the perpendicular (normal) makes with the positive direction of X-axis.

(22) $y - y_1 = m(x - x_1)$ is the slope point from of line which passes through (x_1, y_1) and whose slope is m.

(23) **Two point form:** $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$ is the equation of line which passes through the points (x_1, y_1) and (x_1, y_2).

(24) **Parametric form :** $\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta} = r$ is the equation of line which passes through the point (x_1, y_1) makes angle θ with the axis and r is the distance of any point (x, y) from (x_1, y_1).

(25) Every first degree equation in x and y always represents a straight line $ax + by + c = 0$ is the general equation of line whose,

(a) Slope $= -\frac{a}{b} = \left[\frac{\text{coefficient of } x}{\text{confficient of } y}\right]$

(b) X - intercept $= -\frac{c}{a}$

(c) Y - intercept $= -\frac{c}{b}$

(26) Length of the perpendicular from (x_1, y_1) on the line $ax + by + c =$ is :

$$\left[\frac{ax_1 + by_1 + c}{\sqrt{a^2+b^2}}\right]$$

(27) To find the coordinates of point of intersection of two curves or two line, solve their equation simultaneously.

(28) The equation of any line through the point of intersection of two given lines is

(L.H.S of one line) + K (L.H.S. of 2nd line) = 0

(Right Hand side of both lines being zero)

2. TRIGONOMETRY

(1) $\sin^2\theta + \cos^2\theta = 1$; $\sin^2\theta = 1 - \cos^2\theta$, $\cos^2\theta = 1 - \sin^2\theta$,

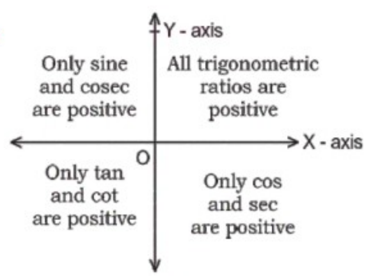
(2) $\tan\theta = \frac{\sin\theta}{\cos\theta}$; $\cos\theta = \frac{\cos\theta}{\sin\theta}$; $\sec\theta = \frac{1}{\cos\theta}$;

$\text{cosec}\,\theta = \frac{1}{\sin\theta}$; $\cot\theta = \frac{1}{\tan\theta}$

(3) $1 + \tan^2\theta = \sec^2\theta$; $\tan^2\theta = \sec^2\theta - 1$; $\sec^2\theta - \tan^2\theta = 1$.

(4) $1 + \cot^2\theta = \text{cosec}^2\theta$; $\cot^2\theta = \text{cosec}^2\theta - 1$; $\text{cosec}^2\theta - \cot^2\theta = 1$.

(5) **Sign conventions:**



(6)

Angle / Ratio	0^0 0^c	30^0 $\left(\frac{\pi}{6}\right)^c$	45^0 $\left(\frac{\pi}{4}\right)^c$	60^0 $\left(\frac{\pi}{3}\right)^c$	90^0 $\left(\frac{\pi}{2}\right)^c$	120^0 $\left(\frac{2\pi}{3}\right)$	135^0 $\left(\frac{3\pi}{4}\right)^c$	150^0 $\left(\frac{5\pi}{6}\right)^c$	180^0 π^c
sin	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	0
cos	1	$\sqrt{\frac{3}{2}}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$\frac{-1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1
tan	0	$\frac{2}{\sqrt{3}}$	1	$\sqrt{3}$	∞	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0

(7) $\sin(-\theta) = \sin\theta$; $\cos(-\theta) = \cos\theta$; $\tan(-\theta) = -\tan\theta$.

(8)

$\sin(90-\theta) = \cos\theta$	$\sin(90+\theta) = \cot\theta$	$\sin(180-\theta) = \sin\theta$
$\cot(90-\theta) = \sin\theta$	$\cos(90+\theta) = -\sin\theta$	$\cos(180-\theta) = -\cos\theta$
$\tan(90-\theta) = \cot\theta$	$\tan(90+\theta) = -\cot\theta$	$\tan(180-\theta) = -\tan\theta$
$\cot(90-\theta) = \tan\theta$	$\cot(90+\theta) = -\tan\theta$	$\cot(180-\theta) = \cot\theta$
$\sec(90-\theta) = \text{cosec}\,\theta$	$\text{set}(90+\theta) = -\text{cosec}\,\theta$	$\sec(180-\theta) = -\sec\theta$
$\text{cosec}(90-\theta) = \sec\theta$	$\text{cosec}(90+\theta) = \sec\theta$	$\text{cosec}(180-\theta) = \text{cosec}\,\theta$

(9) $\sin(A+B) = \sin A\cos B + \cos A\sin B$

$\sin(A-B) = \cos A\sin B - \sin A\cos B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\cos(A-B) = \cos A\cos B + \sin A\sin B$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$

$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$

(10) $\tan\left[\frac{\pi}{4} - A\right] = \frac{1-\tan A}{1+\tan A}$; $\tan\left[\frac{\pi}{4} + A\right] = \frac{1+\tan A}{1-\tan A}$

(11) $\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

$\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

$\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

$\cos C - \cos D = 2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{D-C}{2}\right)$

(12) $2\sin A\cos B = \sin(A+B) + \sin(A-B)$

$2\cos A\sin B = \sin(A+B) - \sin(A-B)$

$2\cos A\cos B = \cos(A+B) + \cos(A+B)$

$2\sin A\sin B = \cos(A-B) - \cos(A+B)$

(13) $\cos(A+B)\cos(A-B) = \cos^2 A - \sin^2 B$

$\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B$

(14) $\sin 2\theta = 2\sin\theta\cos\theta = \frac{2\tan\theta}{1+\tan^2\theta}$

(15) $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$

(16) $1 + \cos 2\theta = 2\cos^2\theta$; $1 - \cos 2\theta = 2\sin^2\theta$.

(17) $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$;

(18) $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$; $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$;

$\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$

(19) $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

(20) $\cos A = \frac{b^2+c^2-a^2}{2bc}$; $\cos B = \frac{c^2+a^2-b^2}{2ca}$; $\cos C = \frac{a^2+b^2-c^2}{2ab}$

(21) $a = b\cos C + c\cos B$; $b = c\cos A + a\cos C$; $c = a\cos B + b\cos A$

(22) Area of triangle $= \frac{1}{2}bc\sin A = \frac{1}{2}ca\sin B = \frac{1}{2}ab\sin C$

(23) $1 \pm \sin A = (\cos\frac{A}{2} \pm \sin\frac{A}{2})^2$

(24) $\sec A \pm \tan A = \tan\left(\frac{\pi}{4} \pm \frac{A}{2}\right)$

(25) $\operatorname{cosec} A - \cot A = \tan\frac{A}{2}$

(26) $\operatorname{cosec} A + \cot A = \cot\frac{A}{2}$

3. PAIR OF LINES

(1) A homogeneous equation is that equation in which sum of the powers of x and y is the the same in each term.

(2) If m_1 and m_2 be the slopes of the lines represented by

$ax^2 + 2hxy + by^2 = 0$, then

$m_1 + m_2 + - \frac{2h}{b} = -\left(\frac{\text{coefficien of } xy}{\text{coefficient of } y^2}\right)$

and $m_1 + m_2 = \frac{a}{b} = \left(\frac{\text{coefficien of } x^2}{\text{coefficient of } y^2}\right)$

(3) If θ be the acute angle between the lines represented by

$ax^2 + 2hxy + by^2 = 0$, then,

$\tan\theta = \left|\frac{2\sqrt{h^2-ab}}{a+b}\right|$

These lines will be co-incident (parallel) if $h^2 = ab$ and perpendicular if $a + b = 0$.

(4) The condition that the general equation of the second degree viz

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ may represent a pair of straight line is $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

i.e. $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$.

(5) $ax^2 + 2hxy + by^2 = 0$ and $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ are pairs of parallel lines.

(6) The point of intersection of lines $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ is obtained by solving the equation $ax + hy + g = 0$ and $hx + by + f = 0$

(7) Joint equation of two lines can be obtained by multiplying the two equations of lines and equating to zero ($uv = 0$, where $u = 0$, $v = 0$).

(8) If the origin is changed to (h,k) and the axis remain parallel to the original axis then for x and y put $x' + h$ and $y' + k$ respectively.

4. CIRCLE

(1) $x^2 + y^2 = a^2$ is the equation of circle whose center is (0,0) and radius is a.

(2) $(x - h)^2 + (y - k)^2 = a^2$ is the equation of a circle whose centre is (h, k) and radius is a.

(3) $x^2 + y^2 + 2gx + 2fy + c = 0$ is a general equation of circle, its centre is $(-g, -f)$ and radius is $\sqrt{g^2+f^2-c}$

(4) **Diameter form:** $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$ is the equation of a circle whose, (x_1, y_1) and $x_2, y_2)$ are ends of diameter.

(5) Conditions for an equation to represent a circle are:

(a) Equation of the circle is of the second degree in x and y.

(b) The coefficient of x^2 and y^2 must be equal.

(c) There is no xy term in the equation (coefficient of xy must be zero).

(6) To find the equation of the tangent at (x_1, y_1) on any curve rule is :

In the given equation of the curve for x^2 put xx_1; for y^2 put yy_1; for $2x$ put $x + x_1$ and for $2y$ put $y + y_2$

(7) For the equation of tangent from a point outside the circle or given slope or parallel to a given line or perpendicular to a given line use $y = mx + c$ or $y - y_1 = m(x - x_1)$.

(8) For the circle $x^2 + y^2 = a^2$

(a) Equation of tangle at (x_1, y_1) is $xx_1 + yy_1 = a^2$

(b) Equation of tangent at $(a\cos\theta, a\sin\theta)$ is $x\cos\theta + y\sin\theta = a$.

(c) Tangent in terms of slope m is

$$y = mx \pm a\sqrt{m^2+1}$$

(9) For the circle $x^2 + y^2 + 2gx + 2fy + c = 0$

(a) Equation of tangent at = (x_1, y_1) is

$$xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$$

(b) Length of tangent from (x_1, y_1) is

$$\sqrt{x_1^2+y_1^2+2gx_1+2fy_1+c}$$

(10) For the point P (x, y), x is abscissa of P and y is ordinate of P.

5. PARABOLA

(1) Distance of any point P on the parabola from the focus S is always equal to perpendicular distance of P from the directrix i.e. SP = PM.

(2) Parametric equation of parabola $y^2 = 4ax$ is $x = at^2$, $y = 2at$

Coordinates of any point (t) is $(at^2, 2at)$

(3) Different types of standard parabola :

Parabola	Focus	Directrix	Latus recturm	Axis of parabola (axis of symmetry)
$y^2 = 4ax$	$(a, 0)$	$x = -a$	$4a$	$y = 0$
$y^2 = -4ax$	$(-a, 0)$	$x = a$	$4a$	$y = 0$
$x^2 = 4by$	$(0, b)$	$y = -b$	$4b$	$x = 0$
$x^2 = -4by$	$(0, -b)$	$y = b$	$4b$	$x = 0$

(4) For the parabola $y^2 = 4ax$

(a) Equation of tangent at (x_1, y_1) is $yy_1 = 2a(x + x_1)$.

(b) Parametric equation of tangent at $(at_1^2, 2at)$ is $yt_1 = x + at_1^2$

(c) Tangent in terms of slope m is $y = mx + \frac{a}{m}$ and its point of contact is $(a/m^2, 2a/m)$

(d) If P (t_1) and Q (t_2) are the ends of a focal chord then $t_2t_1 = -1$

(e) Focal distance of a point P (x_1, y_1) is $x_1 + a$.

6. ELLIPSE

(1) Distance of any point on an ellipse from the focus = e (perpendicular distance of the point from the corresponding diectrix) i.e. SP = e PM

(2) Different types of ellipse :

Ellipse	Focus	Directrix	Latus Rectum	Equation of axis	Ends of L.R
$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ $(a > b)$	$(\pm ae, 0)$	$x = \pm\frac{a}{e}$	$\frac{2b^2}{a}$	Major axis $y = 0$ Minor axis, $x = 0$	$\left(ae, \frac{b^2}{a}\right)$ $\left(ae, \frac{-b^2}{a}\right)$

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ (a<b)	(0, ± be)	$y = \pm\frac{b}{e}$	$\frac{2a^2}{b}$	Major axis $x = 0$ Minor axis $y = 0$	$\left(\frac{-a^2}{b}, be\right)$ $\left(\frac{-a^2}{b}, be\right)$

(3) Parametric equation of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$ $(a > b)$ is
$x = a\cos\theta$ and $y = b\sin\theta$.

(4) For the ellipse $\frac{x^2}{y^2}+\frac{y^2}{b^2} = 1$, $a > b$, $b^2 = a^2(1 - e^2)$
and $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$, $a < b$, $a^2 = b^2(1 - e^2)$

(5) For the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$ $(a > b)$

(a) Equation of tangent at (x_1, y_1) is
$\frac{xx_1}{a^2}+\frac{yy_1}{b^2} = 1$.

(b) Equation of tangent in terms of its slope m is
$y = mx \pm \sqrt{a^2m^2+b^2}$

(c) Tangent at $(a\cos\theta, b\sin\theta)$ is
$\frac{x\cos\theta}{a}+\frac{y\sin\theta}{b} = 1$

(6) Focal distance of a point P (x_1, y_1) is SP = $|a - ex_1|$
and SP = $|ex_1 + a|$

7. HYPERBOLA

(1) Distance of a point on the hyperbola from the focus = e (perpendicular distance of the point from the corresponding directrix) i.e. SP = ePM

(2) Different types of Hyperbola;

Hyperbola	Focus	Directrix	L.R	End of L.R	Eqn of axis
$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$	(± ae, 0)	$\sqrt{x_1^2+y_1^2}$.	$\frac{2b^2}{a}$	$(ae, \frac{b^2}{a})$ $(ae, \frac{-b^2}{a})$	Transverse axis $y = 0$, Conjugate axis, $x = 0$
$\frac{y^2}{b^2}-\frac{x^2}{a^2}=1$	(0, ± be)	$Y = \pm\frac{b}{e}$	$\frac{2a^2}{b}$	$(\frac{a^2}{b}, be)$ $(-\frac{a^2}{b}, be)$	Transverse axis $x = 0$ Conjugate axis $y = 0$

(3) For the hyperbola $\frac{y^2}{a^2}-\frac{y^2}{b^2} = 1$, $b^2 = a^2(e^2 - 1)$ and for
$\frac{y^2}{b^2}-\frac{x^2}{a^2} = 1$, $a^2 = b^2(e^2 - 1)$.

(4) Parametric equations of hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2} = 1$ are
$x = a\sec\theta$, $y = b\tan\theta$

(5) For the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2} = 1$

(a) Equation of tangent at (x_1, y_1) are $\frac{xx_1}{a^2}-\frac{yy_1}{b^2} = 1$

(b) Equation of tangent in terms of its slope m is
$y = mx \pm \sqrt{a^2m^2-b^2}$

(c) Equation of tangent at $(a\sec\theta, b\tan\theta)$ is
$\frac{x\sec\theta}{a}-\frac{y\tan\theta}{b} = 1$

(d) Focal distance of P (x_1, y_1) is SP = $|ex_1 - a|$ and S'P = $|ex_1 + a|$

8. SOLID GEOMERY

(1) Distance between (x_1, y_1, z_1) and (x_2, y_2, z_2) is $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

(2) Distance of $(x_1, y_1\ z_1)$ from origin $\sqrt{x_1^2+y_1^2+z_1^2}$

(3) Coordinates of point which divides the line joining (x_1, y_1, z_1) and (x_2, y_2, z_2) internally in the ratio m:n are

$$\left[\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}, \frac{mz_2+nz_1}{m+n}\right], m+n \neq 0$$

(4) Coordinates of point which divides the joint of (x_1, y_1, z_1) and (x_2, y_2, z_2) externally in the ratio m:n are

$$\left[\frac{mx_2-nx_1}{m-n}, \frac{my_2-ny_1}{m-n}, \frac{mz_2-nz_1}{m-n}\right], m-n \neq 0$$

(5) Coordinates of mid point of join of (x_1, y_1, z_1) and (x_2, y_2, z_2) are

$$\left|\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right|$$

(6) Coordinates of centriod of triangle whose vertices are (x_1, y_1, z_1) (x_2, y_2, z_2) and (x_2, y_3, z_3) are

$$\left[\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right]$$

(7) Direction cosines of X - axis are 1, 0, 0

(8) Direction cosines of Y - axis are 0, 1, 0

(9) Direction cosines of Z - axis are 0, 0, 1

(10) If OP = r and direction cosines of OP are l, m, n then the coordinates of P are $(l\,r, mr, nr)$

(11) If l, m, n are direction cosines of a line then $l^2 + m^2 + n^2 = 1$

(12) If l, m, n are direction cosines and, a, b, c are direction ratios of a line

then $l = \frac{a}{\pm\sqrt{a^2+b^2+c^2}}$, $m = \frac{b}{\pm\sqrt{a^2+b^2+c^2}}$, $n = \frac{c}{\pm\sqrt{a^2+b^2+c^2}}$

(13) If $l, m, n,$ are direction cosines of a line then a unit vector along the line is $l\,\bar{i} + m\bar{j} + n\,\bar{k}$

(14) If a, b, c are direction ratio of line then a vector along the line is $a\bar{i} + b\bar{j} + c\bar{k}$

9. VECTORS

(1) $\bar{a} \cdot \bar{b} = ab\cos\theta = a_1a_2 + b_1b_2 + c_1c_2$

(2) Projection of $\bar{a}$ on $\bar{b} = \frac{\bar{a}\cdot\bar{b}}{|\bar{b}|}$ and projection of $\bar{b}$ on $\bar{a} = \frac{\bar{a}\cdot\bar{b}}{|\bar{a}|}$

(3) $\bar{a}\times\bar{b} = ab\sin\hat{\theta}_n = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$

$\bar{a}\times\bar{b} = -(\bar{b}\times\bar{a})$

(4) $\bar{a}\cdot\bar{b}\times\bar{c} = \left[\bar{a}\ \bar{b}\ \bar{c}\right] = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

(5) Vector area of Δ ABC is

$\frac{1}{2}(\overline{AB}\times\overline{AC}) = \frac{1}{2}(\bar{a}\times\bar{b}+\bar{b}\times\bar{c}+\bar{c}\times\bar{a})$

and area of $\Delta ABC = \frac{1}{2}\left|\overline{AB}\times\overline{AC}\right|$

(6) Volume of parallelepiped : $\left[\bar{a}\ \bar{b}\ \bar{c}\right] = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \left|\overline{AB}\ \ \overline{AC}\ \ \overline{AD}\right|$

(7) Volume of Terahedra ABCD is = $\frac{1}{2}\left|\overline{AB}\ \ \overline{AC}\ \ \overline{AD}\right|$

(8) Work done by force $\overline{F}$ in moving a particle from A to B = $\overline{AB}\cdot\overline{F}$

(9) Moment of force $\vec{F}$ acting at A about a point B is $\overline{M} = \overrightarrow{BA} \times \vec{F}$

10. PROBABILTY

(1) Probability of an event A is $P(A) = \frac{n(A)}{n(S)}$ $0 \le P(A) \le 1$.

(2) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

If A and B are mutually exclusive then

$P(A \cap B = 0$ and $P(A \cup B) = P(A) + P(B)$.

(3) $P(A) = 1 - P(A')$

(4) $P(A \cap B) = P(A) . P(B/A) = P(B) . P(A/B)$

11. DIFFERENTIAL CALCULAS

(1) $f'(x) \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$; where $f(x)$ is derivative of function $f(x)$ wth respectve to x $f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

(2) $\frac{d}{dx}(a) = 0$, where a is constant; $\frac{d}{dx}(x) = 1$,

$\frac{d}{dx}(ax) = a$, $\frac{d}{dx}(\frac{1}{x}) = \frac{-1}{x^2}$; $\frac{d}{dx}(\frac{1}{u}) = \frac{-1}{u^2} \times \frac{du}{dx}$

$\frac{d}{dx}(\frac{1}{u^n}) = \frac{-n}{u^{n+1}} , \frac{du}{dx}$

$\frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}$; $\frac{d}{dx}\sqrt{u} = \frac{1}{2\sqrt{u}} \times \frac{d}{dx}u$, where $u = f(x)$

(3) $\frac{d}{dx}[x^n] = n[x]^{n-1}$; $\frac{d}{dx}[u^n] = nu^{n-1}\frac{du}{dx}$; $\frac{dy^n}{dx} = ny^{n-1}\frac{dy}{dx}$

(4) $\frac{d}{dx}\log x = \frac{1}{x}$; $\frac{d}{dx}(\log u) = \frac{1}{u} \times \frac{du}{dx}$

$\frac{d}{dx}\log_a x \times \frac{1}{x \log a}$; $\frac{d}{dx}\log_a u = \frac{1}{u \log a} \times \frac{du}{dx}$

(5) $\frac{d}{dx}[a^x] = a^x \log a$; $\frac{d}{dx}[a^u] = a^u \log a \times \frac{du}{dx}$

(6) $\frac{d}{dx}[e^x] = e^x$; $\frac{d}{dx}[e^u] = e^u \times \frac{du}{dx}$

(7) $\frac{d}{dx}[\sin x] = \cos x$; $\frac{d}{dx}[\sin u] = \cos u \times \frac{du}{dx}$

(8) $\frac{d}{dx}[\cos x] = -\sin x$; $\frac{d}{dx}[\cos u] = -\sin u \times \frac{du}{dx}$

(9) $\frac{d}{dx}\tan x = \sec^2 x$; $\frac{d}{dx}\tan u = \sec^2 u \times \frac{du}{dx}$

(10) $\frac{d}{dx}\cot x = -\text{cosec}^2 x$; $\frac{d}{dx}\cot u = -\text{cosec}^2 u \times \frac{du}{dx}$

(11) $\frac{d}{dx}\sec x = \sec x \tan x$; $\frac{d}{dx}\sec u = \sec u \times \tan u \times \frac{du}{dx}$

(12) $\frac{d}{dx}\text{cosec } x = -\text{cosec } x \cot x$;

$\frac{d}{dx}\text{cosec } u = -\text{cosec } u \times \cot u \times \frac{du}{dx}$

(13) $\frac{d}{dx}\sin^2 x = 2 \sin x \times \frac{d}{dx}(\sin x) = 2 \sin x \cos x = \sin 2x$

$\frac{d}{dx}\sin^n x = n \sin^{n-1} x \times \frac{d}{dx}\sin x = n \sin^{n-1} x \cos x$.

(14) $\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}} \times \frac{d}{dx}(\sin^{-1} u) = \frac{1}{\sqrt{1-u^2}} \times \frac{du}{dx}$

(15) $\frac{d}{dx}\cos^{-1} x = \frac{-1}{\sqrt{1-x^2}}$; $\frac{d}{dx}(\cos^{-1} u) = \frac{1}{\sqrt{1-u^2}} \times \frac{du}{dx}$

(16) $\frac{d}{dx}\tan^{-1} x = \frac{-1}{1+x^2}$; $\frac{d}{dx}(\tan^{-1} u) = \frac{1}{\sqrt{1+u^2}} \times \frac{du}{dx}$

(17) $\frac{d}{dx}\cot^{-1} x = \frac{-1}{1+x^2}$; $\frac{d}{dx}\cot^{-1} u = \frac{-1}{1+u^2} \times \frac{du}{dx}$

(18) $\frac{d}{dx}\sec^{-1} x = \frac{1}{x\sqrt{x^2-1}}$; $\frac{d}{dx}\sec^{-1} u = \frac{1}{u\sqrt{u^2-1}} \times \frac{du}{dx}$

(19) $\frac{d}{dx}\text{cosec}^{-1} x = \frac{-1}{x\sqrt{x^2-1}}$; $\frac{d}{dx}\text{cosec}^{-1} u = \frac{-1}{u\sqrt{u^2-1}} \times \frac{du}{dx}$

(20) $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

$\frac{d}{dx}(uvw) = vw\frac{du}{dx} + uw\frac{dv}{dx} + uv\frac{dw}{dx}$

(21) $\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^{2}}, v \neq 0.$

(22) $\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}$

(23) $f(x+h)=f(x)+hf'(x)$

(24) Error in y is $\delta y=\frac{dy}{dx}\times\delta x$,

Relative error in y is $\frac{\delta y}{y}$ and percentage error in $y=\frac{\delta y}{y}\times 100$

(25) Velocity $v=\frac{ds}{dt}$, acceleration $a=\frac{dv}{dt}=\frac{d}{dt}\left(\frac{ds}{dt}\right)=\frac{d^2s}{dt^2}$

12. INTEGRAL CALCULAS

(1) $\int(u+v+w+\ldots)\,dx=\int u\,dx+\int v\,dx+\int w\,dx+\ldots$

(2) $\int a\,f(x)=a\int f(x)\,dx$, where 'a' is a constant.

(3) $\int x^{n}\,dx=\frac{x^{n}+1}{n+}+c,(n\neq-1);\int(ax+b)^{n}=\frac{1}{a}\,\frac{(ax+b)^{n+1}}{n+1}+c$

(4) $\int[f(x)]^{n}f'(x)\,dx=\frac{[f(x)^{n}+1]}{n+1}+c, \quad (n\neq-1)$

(5) $\int\frac{1}{x}dx=\log x+c;\int\frac{1}{ax+b}dx=\frac{1}{a}\log[ax+b]+c;$

$\int\frac{f'(x)}{f(x)}dx=\log|f(x)|+c;$

The integral of a function in which the numerator is the differential coefficient of the denominator is log (Denominator).

(6) $\int\sqrt{x}\,dx=\frac{2}{3}x^{3/2}+c;\quad\int\sqrt{ax+b}\,dx=\frac{2}{3a}(ax+b)^{3/2}+c$

(7) $\int a^{x}\,dx=\frac{a^{x}}{\log a}+c;\ \int a^{bx+c}\,dx=\frac{1}{b}\,\frac{a^{bx+c}}{\log a}+c$

(8) $\int e^{x}\,dx\ e^{x}+c;\ \int e^{ax+b}\,dx=\frac{1}{a}e^{ax+b}+c.$

(9) $\int\sin x\,dx=-\cos x+c;\int\sin(ax+b)\,dx=\frac{-1}{a}\cos(ax+b)+c$

(10) $\int\cos x\,dx=\sin x+c;\int\cos(ax+b)\,dx=\frac{1}{a}\sin(ax+b)+c$

(11) $\int\tan x\,dx=\log\sec x+c;\int\tan(ax+b)\,dx=\frac{1}{a}\log\sec(ax+b)+c$

(12) $\int\cot x\,dx=\log\sin x+c;\int\cot(ax+b)\,dx=\frac{1}{a}\log\sin(ax+b)+c$

(13) $\int\sec x\,dx=\log|\sec x+\tan x|+c$

$=\log\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)+c$

$\int\sec(ax+b)\,dx=\frac{1}{a}\log|\sec(ax+b)+\tan(ax+b)|+c$

$=\frac{1}{a}\log\tan\left|\frac{ax+b}{2}+\frac{\pi}{4}\right|c$

(14) $\int\operatorname{cosec}x\,dx=\log|\operatorname{cosec}x-\cot x|$

$=\log\tan\left(\frac{x}{2}\right)+c$

$\int\operatorname{cosec}(ax+b)\,dx=\frac{1}{a}\log|\operatorname{cosec}(ac+b)-\cot(ax+b)|+c$

$=\frac{1}{a}\log\tan\left|\frac{ax+b}{2}\right|+c$

(15) $\int\sec^{2}x\,dx=\tan x+c;$

$\int\sec^{2}(ax+b)\,dx=\frac{1}{a}\tan(ax+b)+c$

(16) $\int\operatorname{cosec}^{2}x\,dx=-\cot x+c;\int\operatorname{cosec}^{2}(ax+b)\,dx=\frac{1}{a}\cot(ax+b)+c$

(17) $\int\sec x\tan x\,dx=\sec x+c;\int\sec(ax+b)\tan(ax+b)\,dx=\frac{1}{a}\sec(ax+b)+c$

(18) $\int\operatorname{cosec}x\cot x\,dx=-\operatorname{cosec}x+c;$

$\int\operatorname{cosec}(ax+b)\cot(ax+b)\,dx=\frac{1}{a}\operatorname{cosec}(ax+b)+c$

(19) $\int\frac{dx}{\sqrt{1+x^{2}}}=\sin^{-1}x+c=-\cos^{-1}x+c$

(20) $\int \frac{dx}{1+x^2} = \tan^{-1} x + c = -\cot^{-1} x + c$

(21) $\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + c;$

$\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1} x + c = -\text{cosec}^{-1}\, x$

13. NINE IMPORTANT RESULTS

(1) $\int \frac{dx}{\sqrt{a^2+x^2}} = \sin^{-1}\frac{x}{a} + c = -\cos^{-1}\left(\frac{x}{a}\right) + c$

(2) $\int \frac{dx}{\sqrt{x^2+a^2}} = \log[x+\sqrt{x^2+a^2}] + c$

(3) $\int \frac{dx}{\sqrt{x^2-a^2}} = \log\left[x+\sqrt{x^2-a^2}\right] + c$

(4) $\int \sqrt{a^2-x^2}\, dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + c$

(5) $\int \sqrt{x^2+a^2}\, dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right| + c$

(6) $\int \sqrt{x^2-a^2}\, dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a}{2}\log\left[x+\sqrt{x^2-a^2}\right] + c$

(7) $\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + c$

(8) $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$

(9) $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\log\left|\frac{x-a}{x-a}\right| + c$

14. INTEGRATION BY SUBSTITUTION

If the integrand contain		Proper substitution to be used
(1)	$\sqrt{a^2-x^2}$	$x = a\sin\theta$
(2)	$\sqrt{x^2+a^2}$	$x = a\tan\theta$
(3)	$\sqrt{x^2-a^2}$	$x = a\sec\theta$
(4)	$e^{f(x)}$	$f(x) = t$
(5)	Any odd power of sin x	$\cos x = t$
(6)	Any odd power of cos x	$\sin x = t$
(7)	Odd powers of both sin x and cos x	Put that function = t which is of the higher power
(8)	Any even inverse function	Inverse function = t
(9)	Any power of sec x	$\tan x = t$
(10)	Any even power of cosec x	$\cot x = t$
(11)	Function of e^x	$e^x = t$
(12)	$\frac{1}{a+b\sin x}, \frac{1}{a+b\cos x}, \frac{1}{a+b\cos x+c\sin x}$	$\tan\frac{x}{2} = t$ then $dx = \frac{2dt}{1+t^2}$; $\sin x = \frac{2t}{1+t^2}; \cos x = \frac{1+t^2}{1+t^2}$
(13)	$\frac{1}{a+b\sin 2x}, \frac{1}{a+b\cos 2x}$	$\tan x = t$, then $dx = \frac{dt}{1+t^2}$; $\sin 2t = \frac{2t}{1+t^2}$; $\cos 2x = \frac{1-t^2}{1+t^2}$
(14)	$\frac{1}{a^2\sin^2 x + b^2\cos^2 x}$	Divide numerator and demoninator by $\cos^2 x$ and put $\tan x = t$
(15)	$\frac{1}{x(px^m+q)}$	$x^m = t$

(16)	Expression containing fractional power of x or $(ax+b)$	x or $ax+b=t^k$ where k is the L.C.M. of the denominators of the fractional indices.

15. INTEGRATION BY PARTS

(1) Integral of the product of two function

= Fist function × Integral of 2nd –

$\int$[Differential coefficient of 1st × Integral of 2nd]dx

i.e. $\int [I \times II]\,dx = I \times \int II\,dx - \int \left[\frac{d}{dx} I \times \int II\,dx\right] dx$

Note:

(1) The choice of first and second function should be according to the order of the letters of the word LIATE. Where L = Logarithmic; I = inverse; A = Algebric; T = Trignometric; E = Exponential function

(2) If the integrand is product of same type of function take that function as second which is orally integrable.

(3) If there is only one function whose integral is not known multiply it by one and take one as the 2nd function.

16. DEFINITE INTEGRALS

(1) $\int_a^b f(x)\,dx = [g(x)]_a^b = g(b) - g(a)$, where $\int f(x)\,dx = g(x)$

(2) $\int_a^b f(x)\,dx = \int_a^b f(t)\,dt = \int_a^b f(m)\,dm$

(3) $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

(4) $\int_a^b f(x) = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx,\ a < c < b.$

(5) $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx;\ \int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

(6) $\int_{-a}^a f(x)\,dx = 2\int_0^a f(x)\,dx$ if f is even $\int_{-a}^a f(x)\,dx = 0$ if f is odd.

(7) $\int_0^{2a} f(x)\,dx = \int_0^a f(x)\,dx + \int_0^a f(2a-x)\,dx$

If $f(2a-x) = f(x)$ then $\int_0^{2a} f(x)\,dx = 2\int_0^a f(x)\,dx$

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