12th PHYSICS

Most Important Questions

www.saiphy.com

01 Rotational Dynamics

Note: The Ⓡ marked questions are the part of reduced/non-evaluative portion for academic year 2020-21 only.

Multiple Choice Questions (1 Mark Each)

1. A diver in a swimming pool bends his head before diving. It
   (A) Increases his linear velocity
   (B) Decreases his angular velocity
   (C) Increases his moment of inertia
   (D) Decreases his moment of inertia

2. The angular momentum of a system of particles is conserved
   (A) When no external force acts upon the system
   (B) When no external torque acts upon the system
   (C) When no external impulse acts upon the system
   (D) When axis of rotation remains the same

3. A stone is tied to one end of a string. Holding the other end, the string is whirled in a horizontal plane with progressively increasing speed. It breaks at some speed because
   (A) Gravitational forces of the earth is greater than the tension in string
   (B) The required centripetal force is greater than the tension sustained by the string
   (C) The required centripetal force is lesser than the tension in the string
   (D) The centripetal force is greater than the weight of the stone

4. The moment of inertia of a circular loop of radius R, at a distance of R/2 around a rotating axis parallel to horizontal diameter of the loop is
   (A) $\frac{1}{2}MR^2$ **(B) $\frac{3}{4}MR^2$**
   (C) MR^2 (D) $2 MR^2$

Hint: $I_o = I_c + Mh^2$

$$= \frac{MR^2}{2} + M\left(\frac{R}{2}\right)^2 \quad \text{....(M.I. of circular loop along its diameter} = \frac{MR^2}{2}\text{)}$$

$$= \frac{MR^2}{2} + \frac{MR^2}{4}$$

$$= \frac{3}{4} MR^2$$

5. A 500 kg car takes a round turn of radius 50 m with a velocity of 36 km/hr. The centripetal force is
   (A) 250 N (B) 750 N
   (C) 1000 N (D) 1200 N

Hint: C.P.F. $= \frac{mv^2}{r} = \frac{500 \times 10^2}{50} = 1000$ N

6. A cyclist riding a bicycle at a speed of $14\sqrt{3}$ m/s takes a turn around a circular road of radius $20\sqrt{3}$ m without skidding. Given g = 9.8 m/s^2, what is his inclination to the vertical?
   (A) 30^0 (B) 45^0
   (C) 60^0 (D) 90^0

Hint: $\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$

$$= \tan^{-1}\left(\frac{14 \times 14 \times 3}{20 \times \sqrt{3} \times 9.8}\right)$$

$$= \tan^{-1}\left(\sqrt{3}\right)$$

$\therefore \quad \theta = 60^\circ$

7. A string of length ℓ fixed at one end carries a mass m at the other. The string makes $2/\pi$ revolutions/sec around the vertical axis through the fixed end. The tension in the string is
   (A) 2 ml (B) 4 ml
   (C) 8 ml **(D) 16 ml**

Hint: $\omega = 2\pi r = 2\pi \times \frac{2}{\pi} = 4$ rad/s

$T = ml\omega^2 = 16\ ml$

Very Short Answer (VSA) (1 Mark Each)

1. Find the radius of gyration of a uniform disc about an axis perpendicular to its plane and passing through its centre.

Ans: M.I. of a unifrom disc about an axis perpendicular to the plane and passing through its centre: $I = \frac{MR^2}{2}$

Since, $I = MK^2$

$$K = \sqrt{\frac{I}{M}} = \sqrt{\frac{MR^2}{2M}} = \frac{R}{\sqrt{2}}$$

2. Does the angle of banking depend on the mass of the vehicle?

Ans: No, angle of banking is independent of mass of the vehicle.

3. During ice ballet, while in the outer rounds, why do the dancers outstretch their arms and legs.

Ans: During ice ballet, while in the outer rounds, the dancers outstretch their arms and legs to reduce their angular speed.

4. State the principle of conservation of angular momentum.

Ans: *Angular momentum of an isolated system is conserved in the absence of an external unbalanced torque.*

5. Two bodies have their moments of inertia I and 2I respectively about their axis of rotation. If their kinetic energies of rotation are equal, then what is the ratio of their angular velocity?

Ans: Given: $(K.E.)_1 = (K.E.)_2$

$\therefore \quad \frac{1}{2}I_1\omega_1^2 = \frac{1}{2}I_2\omega_2^2$

$\therefore \quad \frac{\omega_1}{\omega_2} = \sqrt{\frac{I_2}{I_1}}$

$= \sqrt{\frac{2I}{I}}$ ....(Given: $I_1 = I$, $I_2 = 2I$)

$\frac{\omega_1}{\omega_2} = \frac{\sqrt{2}}{1}$

$\therefore \quad \omega_1 : \omega_2 = \sqrt{2} : 1$

6. A hollow sphere has radius 6.4 m. what is the minimum velocity required by a motor cyclist at bottom to complete the circle.

Ans: Minimum velocity required by a motor cyclist at bottom to complete the circle, $v_{min} = \sqrt{5rg}$

$= \sqrt{5 \times 6.4 \times 9.8}$

$= 17.7$ m/s

7. A bend in a level road has a radius of 100 m. Find the maximum speed which a car turning this bend may have without skidding, if the coefficient of friction between the tyres and road is 0.8.

Ans: Maximum speed, which a car turning the bend may have without skidding, $v_{max} = \sqrt{\mu rg}$

$= \sqrt{0.8 \times 100 \times 9.8}$

$= 28$ m/s

Short Answer I (SA1) (2 Marks Each)

1. A flywheel is revolving with a constant angular velocity. A chip of its rim breaks and flies away. What will be the effect on its angular velocity?

Ans:

i. When the chip of the rim of a flywheel revolving with a constant angular velocity breaks away, its mass will decrease.

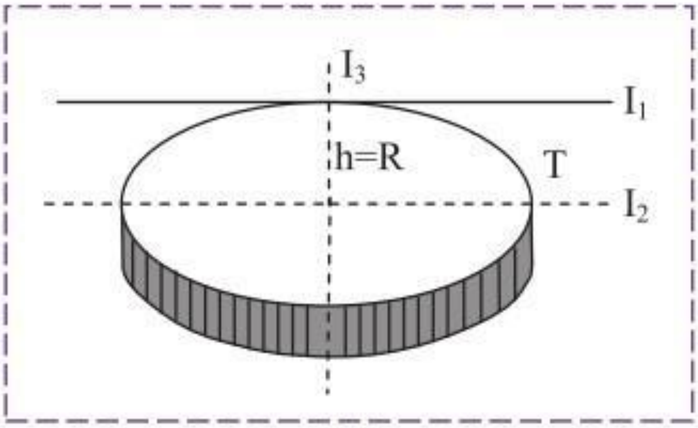
ii. Due to the decrease in its mass, the moment of inertia of the flywheel will decrease.

iii. In order to conserve angular momentum, the angular velocity of the flywheel will increase.

2. The moment of inertia of a uniform circular disc about a tangent in its own plane is $\frac{5}{4}MR^2$ where M is the mass and R is the radius of the disc. Find its moment of inertia about an axis through its centre and perpendicular to its plane.

Ans:

i. M.I. of a unifrom circular disc about a tangent in its own plane, $I_1 = \frac{5}{4}MR^2$



ii. Applying parallel axis theorem

$I_1 = I_2 + Mh^2$

$\therefore \quad I_2 = I_1 - MR^2 = \frac{5}{4}MR^2 - MR^2 = \frac{MR^2}{4}$

iii. Applying perpendicular axis theorem,

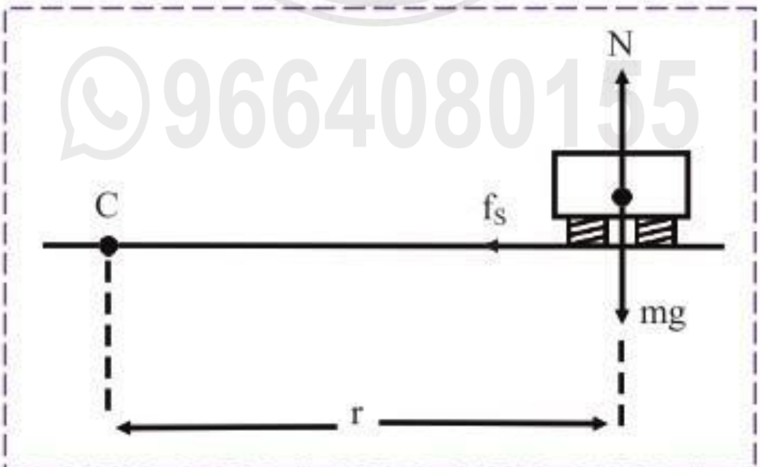
$I_3 = I_2 + I_2 = 2I_2$

$\therefore \quad I_3 = 2 \times \frac{MR^2}{4} = \frac{MR^2}{2}$

3. Derive an expression for maximum safety speed with which a vehicle should move along a curved horizontal road. State the significance of it.

Ans:

i. Consider vertical section of a car moving on a horizontal circular track having a radius 'r' with 'C' as centre of track.



ii. Forces acting on the car (considered to be a particle):

a. Weight (mg), vertically downwards,

b. Normal reaction (N), vertically upwards that balances the weight

c. Force of static friction (f_s) between road and the tyres.
Since, normal reaction balances the weight

$\therefore$ $N = mg$ ....(1)

While working in the frame of reference attached to the vehicle, the frictional force balances the centrifugal force.

$$f_s = \frac{mv^2}{r} \quad ....(2)$$

Dividing equation (2) by equation (1),

$$\therefore \frac{f_s}{N} = \frac{v^2}{rg} \quad ....(3)$$

iii. However, f_s has an upper limit $(f_s)_{max} = \mu_s N$, where μ_s is the coefficient of static friction between road and tyres of the vehicle. This imposes an upper limit to the speed v.
At the maximum possible speed,

$$\frac{(f_s)_{max}}{N} = \mu_s = \frac{v^2_{max}}{rg} \quad ....\text{[From equations (2) and (3)]}$$

$$\therefore v_{max} = \sqrt{\mu_s rg}$$

This is an expression for for maximum safety speed with which a vehicle should move along a curved horizontal road.

iv. **Significance:** The maximum safe speed of a vehicle on a curve road depends upon friction between tyres and road, radius of the curved road and acceleration due to gravity.

4. The moment of inertia of a body about a given axis is 1.2 kgm^2. Initially the body is at rest. For what duration, an angular acceleration of 25 radian/sec^2 must be applied about that axis in order to produce a rotational kinetic energy of 1500 joule?

Solution:

Given: $I = 1.2$ kgm^2, $\alpha = 25$ radian/sec^2, $\omega_0 = 0$ rad/s
$(K.E.)_{rot} = 1500$ J

To find: Time (t)

Formulae: i. $\alpha = \frac{\omega - \omega_0}{t}$ ii. $K.E. = \frac{1}{2} I\omega^2$

Calculation: From formula (i),

$$25 = \frac{\omega - 0}{t}$$

$$\therefore \omega = 25t$$

From formula (ii),

$$1500 = \frac{1}{2} \times 1.2 \times (25t)^2$$

$\therefore$ $t = \sqrt{\frac{2 \times 1500}{1.2 \times 25^2}} = \sqrt{4}$

$\therefore$ t = **2 sec.**

Ans: An angular acceleration must be applied for **2 sec**.

5. A bucket containing water is tied to one end of a rope 5 m long and it is rotated in a vertical circle about the other end. Find the number of rotations per minute in order that the water in the bucket may not spill.

Solution:

Given: r = 5 m

To find: Rotations per minute

Formulae:
i. $v = \sqrt{rg}$
ii. $v = r\omega$
iii. $n = \frac{\omega}{2\pi}$

Calculation: From formula (i),

$$v = \sqrt{5 \times 9.8} = \sqrt{49} = 7 \text{ m/s}$$

From formula (ii) and (iii),

$$n = \frac{v}{2\pi r} = \frac{7 \times 7}{2 \times 22 \times 5} = \frac{4.9}{22} \text{ r.p.s.}$$

$\therefore$ Rotations per minute $= \frac{4.9}{22} \times 60 =$ **13.37 r.p.m.**

Ans: Rotations per minute in order that the water in the bucket may not spill is **13.37 r.p.m**.

6. A body weighing 0.5 kg tied to a string is projected with a velocity of 10 m/s. The body starts whirling in a vertical circle. If the radius of the circle is 0.8 m, find the tension in the string when the body is at the top of the circle.

Solution:

Given: m = 0.5 kg, u = 10 m/s, r = 0.8 m

To find: Tension at highest point (T_H)

Formula: $T_H = \frac{m}{r}\left(u^2 - 5rg\right)$

Calculation: From formula,

$$T_H = \frac{0.5}{0.8}\left(10^2 - 5 \times 0.8 \times 9.8\right)$$

$$= \frac{0.5}{0.8}\left(100 - 39.2\right)$$

$$= \mathbf{38\ N}$$

Ans: The tension in the string when the body is at the top of the circle is **38 N**.

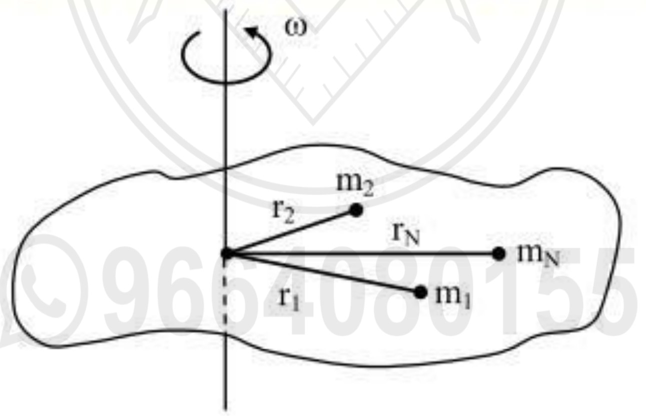
[Note:** The given question is solved in accordance with textbook method.**]

Short Answer I (SA1) (3 Marks Each)

1. Derive an expression for kinetic energy of a rotating body with uniform angular velocity.

Ans:

i. Consider a rigid object rotating with a constant angular speed ω about an axis perpendicular to the plane of paper.



A body of N particles

ii. For theoretical simplification, let us consider the object to be consisting of N particles of masses m_1, m_2, …..m_N at respective perpendicular distances r_1, r_2, …..r_N from the axis of rotation.

iii. As the object rotates, all these particles perform UCM with the same angular speed ω, but with different linear speeds,

$v_1 = r_1\omega$, $v_2 = r_2\omega$,…., $v_N = r_N\omega$

iv. Translational K.E. of the first particle is

$$(K.E.)_1 = \frac{1}{2}m_1v_1^2 = \frac{1}{2}m_1r_1^2\omega^2$$

Similar will be the case of all the other particles.

v. Rotational K.E. of the object, is the sum of individual translational kinetic energies.

Thus,

$$\text{Rotational K.E.} = \frac{1}{2}m_1r_1^2\omega^2 + \frac{1}{2}m_2r_2^2\omega^2 .... + \frac{1}{2}m_Nr_N^2\omega^2$$

∴ $$\text{Rotational K.E.} = \frac{1}{2}\left(m_1r_1^2 + m_2r_2^2 .... + m_Nr_N^2\right)\omega^2$$

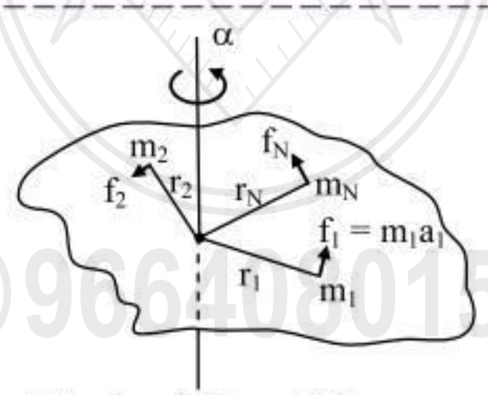
vi. But $I = \sum_{i=1}^{N} m_ir_i^2 = m_1r_1^2 + m_2r_2^2 .... + m_Nr_N^2$

∴ $$\text{Rotational K.E.} = \frac{1}{2}I\omega^2$$

2. Obtain an expression for the torque acting on a rotating body with constant angular acceleration.

Ans:

i. Consider a rigid object rotating with a constant angular acceleration 'α' about an axis perpendicular to the plane of paper.



A body of N particles

ii. Let us consider the object to be consisting of N number of particles of masses m_1, m_2,…..m_N at respective perpendicular distances r_1, r_2,…..r_N from the axis of rotation.

iii. As the object rotates, all these particles perform circular motion with same angular acceleration α, but with different linear (tangential) accelerations.

$a_1 = r_1\alpha$, $a_2 = r_2\alpha$,…., $a_N = r_N\alpha$, etc.

iv. Force experienced by the first particle is, $f_1 = m_1a_1 = m_1r_1\alpha$

As these forces are tangential, the irrespective perpendicular distances from the axis are $r_1, r_2, \ldots r_N$.

v. Thus, the torque experienced by the first particle is of magnitude $\tau_1 = f_1r_1 = m_1r_1^2\alpha$

Similarly, $\tau_2 = m_2r_2^2\alpha$, $\tau_3 = m_3r_3^2\alpha$ ….$\tau_N = m_Nr_N^2\alpha$

If the rotation is restricted to a single plane, directions of all these torques are the same, and along the axis.

vi. Magnitude of the resultant torque is then given by

$\tau = \tau_1 + \tau_2 + \ldots + \tau_N$

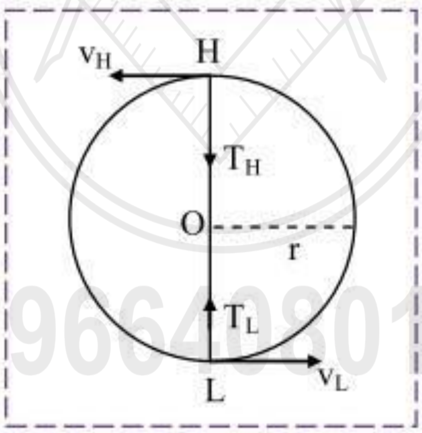
$= \left(m_1r_1^2 + m_1r_2^2 \ldots + m_Nr_N^2\right)\alpha = I\alpha$

where, $I = m_1r_1^2 + m_2r_2^2 \ldots + m_Nr_N^2$ is the moment of inertia of the object about the given axis of rotation.

3. Derive an expression for the difference in tensions at highest and lowest point for a particle performing vertical circular motion.

Ans:

i. Suppose a body of mass 'm' performs V.C.M on a circle of radius r as shown in the figure.



ii. Let,

T_L = tension at the lowest point

T_H = tension at the highest point

v_L = velocity at the lowest point

v_H = velocity at the highest point

iii. At lowest point L,

$$T_L = \frac{mv_L^2}{r} + mg \quad \ldots.(1)$$

At highest point H,

$$T_H = \frac{mv_H^2}{r} - mg \qquad \text{....(2)}$$

iv. Subtracting (1) by (2),

$$T_L - T_H = \frac{mv_L^2}{r} + mg - \left(\frac{mv_H^2}{r} - mg\right)$$

$$= \frac{m}{r}\left(v_L^2 - v_H^2\right) + 2mg$$

$\therefore$ $T_L - T_H = \frac{m}{r}\left(v_L^2 - v_H^2\right) + 2mg$ ....(3)

v. By law of conservation of energy,
(P.E + K.E) at L = (P.E + K.E) at H

$\therefore$ $0 + \frac{1}{2}mv_L^2 = mg.2r + \frac{1}{2}mv_H^2$

$\therefore$ $\frac{1}{2}m\left(v_L^2 - v_H^2\right) = mg.2r$

$\therefore$ $v_L^2 - v_H^2 = 4gr$ ....(4)

vi. From equation (3) and (4),

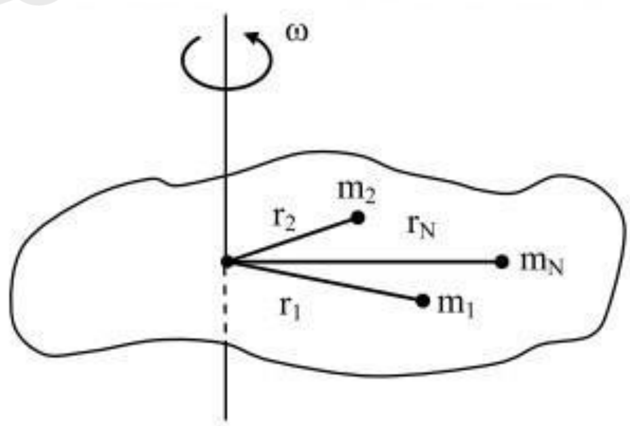
$$T_L - T_H = \frac{m}{r}(4gr) + 2mg = 4mg + 2mg$$

$\therefore$ $T_L - T_H = 6mg$

4. Obtain an expression for the angular momentum of a body rotating with uniform angular velocity.

Ans:

i. Consider a rigid object rotating with a constant angular speed 'ω' about an axis perpendicular to the plane of paper.



A body of N particles

ii. Let us consider the object to be consisting of N number of particles of masses m_1, m_2, …..m_N at respective perpendicular distances r_1, r_2, …..r_N from the axis of rotation.

iii. As the object rotates, all these particles perform UCM with same angular speed ω, but with different linear speeds

$v_1 = r_1\,\omega$, $v_2 = r_2\,\omega$, ...$v_N = r_N\,\omega$.

Directions of individual velocities $\vec{v}_1$, $\vec{v}_2$, ….. $\vec{v}_N$, are along the tangents to the irrespective tracks.

iv. Linear momentum of the first particle is of magnitude

$p_1 = m_1v_1 = m_1r_1\omega$. Its direction is along that of $\vec{v}_1$.

Its angular momentum is thus of magnitude $L_1 = p_1r_1 = m_1\,r_1^2\,\omega$

Similarly, $L_2 = m_1\,r_2^2\,\omega$, $L_3 = m_3\,r_3^2\,\omega$,…., $L_N = m_N\,r_N^2\,\omega$.

v. For a rigid body with a fixed axis of rotation, all these angular momenta are directed along the axis of rotation, and this direction can be obtained by using right hand thumb rule.

As all of them have the same direction, their magnitudes can be algebraically added.

vi. Thus, magnitude of angular momentum of the body is given by

$$L = m_1\,r_1^2\,\omega + m_1\,r_2^2\,\omega + …. + m_N\,r_N^2\,\omega$$

$$= \left(m_1r_1^2 + m_2r_2^2 + ... + m_Nr_N^2\right)\omega = I\omega$$

where, $I = m_1r_1^2 + m_2r_2^2 + ... + m_Nr_N^2$ is the moment of inertia of the body about the given axis of rotation.

5. A railway track goes around a curve having a radius of curvature of 1 km. The distance between the rails is 1 m. Find the elevation of the outer rail above the inner rail so that there is no side pressure against the rails when a train goes round the curve at 36 km/hr.

Solution:

Given: Radius of curve, r = 1 km =1000 m,
Speed of train, v = 36 km/hr =10 m/s

To find: Elevation of rails (h)

Formulae: i. $\tan\theta = \dfrac{v^2}{rg}$

ii. $h = l\sin\theta$

Calculation: From formula (i),

$$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$$

$$= \tan^{-1}\left(\frac{100}{1000\times 9.8}\right)$$

$$= \tan^{-1}(0.0102)$$

$\therefore$ $\theta = 0°35'$

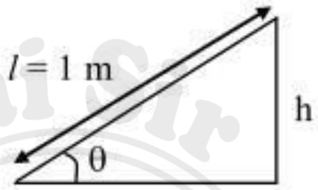
From formula (ii),

$h = l \sin\theta$

$h = 1 \times \sin(0°35')$

$= 0.0102$ m

$\therefore$ h = **1.02 cm**



Ans: The elevation of the outer rail above the inner rail is **1.02 cm**.

6. A flywheel of mass 8 kg and radius 10 cm rotating with a uniform angular speed of 5 rad / sec about its axis of rotation, is subjected to an accelerating torque of 0.01 Nm for 10 seconds. Calculate the change in its angular momentum and change in its kinetic energy.

Solution:

Given: M = 8 kg, R = 10 cm = 0.1 m,
$\omega_1 = 5$ rad/s, $\tau = 0.01$ Nm, t = 10 s

To find:
i. Change in angular momentum (ΔL)
ii. Change in K.E. (ΔK.E.)

Formulae:
i. $I = \frac{MR^2}{2}$

ii. $\tau = I\left(\frac{\omega_2 - \omega_1}{t}\right)$

iii. $\Delta L = I(\omega_2 - \omega_1)$

iv. $\Delta K.E. = \frac{1}{2}I\left(\omega_2^2 - \omega_1^2\right)$

Calculation: From formula (i),

$$I = \frac{8\times(0.1)^2}{2}$$

$= 0.04$ kgm^2

From formula (ii),

$$\omega_2 = \frac{(\tau \times t)}{I} + \omega_1$$

$$= \frac{0.01 \times 10}{0.04} + 5$$

$$= 7.5 \text{ rad/s}$$

From formula (iii),

$\Delta L = 0.04\ (7.5 - 5) = \mathbf{0.1\ kg\ m^2/s}$

From formula (iv),

$$\Delta K.E. = \frac{1}{2} \times 0.04 \times (7.5^2 - 5^2)$$

$$= \mathbf{0.625\ J}$$

Ans: The change in its angular momentum and change in its kinetic energy are **0.1 kg m²/s** and **0.625 J** respectively.

7. Two wheels of moment of inertia 4 kgm² rotate side by side at the rate of 120 rev/min and 240 rev/min respectively in the opposite directions. If now both the wheels are coupled by means of a weightless shaft so that both the wheels rotate with a common angular speed. Calculate the new speed of rotation.

Solution:

$I_1 = I_2 = I = 4$ kg m², $n_1 = 120$ r.p.m, $n_2 = 240$ r.p.m.

Initially, the angular velocities of the two wheels are $\vec{\omega}_1$ and $\vec{\omega}_2$ and, therefore the angular momentum $\vec{L}_1$ and $\vec{L}_2$ are in opposite directions.

∴ The magnitude of the total initial angular momentum, $L = -L_1 + L_2$.

∴ $L = -I\omega_1 + I\omega_2$ ….(i)

After coupling on the same shaft, the total moment of inertia is 2I.

Let, $\omega = 2\pi n$ be the common angular speed.

∴ The magnitude of the total final angular momentum $L' = 2I\omega$ ….(ii)

By the principle of conservation of angular momentum, $L = L'$

∴ Equating equation (i) and (ii), we have

$I(\omega_2 - \omega_1) = 2I\omega$

∴ $2 \times 2\pi n = 2\pi (n_2 - n_1)$

∴ $2n = n_2 - n_1$

∴ $n = \frac{n_2 - n_1}{2} = \frac{240 - 120}{2} = \mathbf{60\ r.p.m}$

Ans: The new speed of rotation of the wheels would be **60 r.p.m.**

Long Answer (LA) (4 Marks Each)

1. State and explain the theorem of parallel axes.

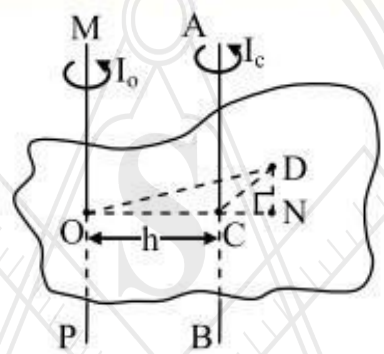
Ans: Statement: *The moment of inertia* (I_o) *of an object about any axis is the sum of its moment of inertia* (I_c) *about an axis parallel to the given axis, and passing through the centre of mass and the product of the mass of the object and the square of the distance between the two axes.*

Mathematically, $I_o = I_c + Mh^2$

Proof:

i. Consider an object of mass M. Axis MOP is any axis passing through point O.

ii. Axis ACB is passing through the centre of mass C of the object, parallel to the axis MOP, and at a distance h from it ($\therefore$ h = CO).



Theorem of parallel axes

iii. Consider a mass element 'dm' located at point D. Perpendicular on OC (produced) from point D is DN.

iv. Moment of inertia of the object about the axis ACB is $I_c = \int (DC)^2\, dm$, and about the axis MOP it is $I_o = \int (DO)^2\, dm$.

v. $I_o = \int (DO)^2\, dm = \int \left[(DN)^2 + (NO)^2\right] dm$

$= \int \left[(DN)^2 + (NC)^2 + 2 \cdot NC \cdot CO + (CO)^2\right] dm$

$= \int \left[(DC)^2 + 2NC \cdot h + h^2\right] dm$

....(using Pythagoras theorem in ΔDNC)

$= \int (DC)^2\, dm + 2h \int NC \cdot dm + h^2 \int dm$

Now, $\int (DC)^2\, dm = I_C$ and $\int dm = M$

vi. NC is the distance of a point from the centre of mass. Any mass distribution is symmetric about the centre of mass. Thus, from the definition of the centre of mass, $\int NC \cdot dm = 0$

$\therefore$ $I_o = I_c + Mh^2$

This is the mathematical form of the theorem of parallel axes.

2. What is a conical pendulum? Obtain an expression for its time period.

Ans: *A tiny mass (assumed to be a point object and called a bob) connected to a long, flexible, massless, inextensible string, and suspended to a rigid support revolves in such a way that the string moves along the surface of a right circular cone of vertical axis and the point object performs a uniform horizontal circular motion. Such a system is called a* ***conical pendulum.***

Expression for its time period:

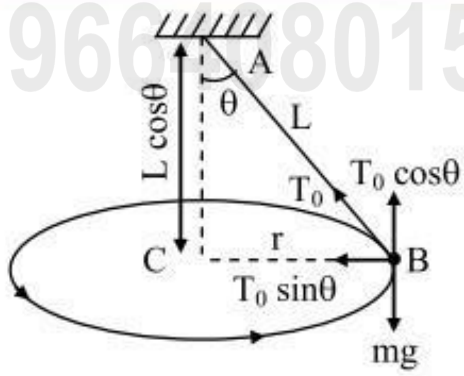
i. Consider the vertical section of a conical pendulum having bob (point mass) of mass m and string of length 'L'.

Here, θ is the angle made by the string with the vertical, at any position (semi-vertical angle of the cone)

ii. In a given position B, the forces acting on the bob are

a. its weight 'mg' directed vertically downwards

b. the force 'T_0' due to the tension in the string, directed along the string, towards the support A.



In an inertial frame

iii. As the motion of the bob is a horizontal circular motion, the resultant force must be horizontal and directed towards the centre C of the circular motion.

For this, tension (T_0) in the string is resolved into

a. $T_0 \cos\theta$: vertical component

b. $T_0 \sin\theta$: horizontal component

iv. The vertical component ($T_0 \cos\theta$) balances the weight 'mg'.

∴ $mg = T_0 \cos\theta$ ....(1)

The horizontal component $T_0 \sin\theta$ then becomes the resultant force which is centripetal.

$mr\omega^2 = T_0 \sin\theta$ ....(2)

Dividing equation (2) by equation (1),

$\omega^2 = \frac{g\sin\theta}{r\cos\theta}$ ....(3)

v. From the figure,

$\sin\theta = \frac{r}{L}$

∴ $r = L\sin\theta$ ....(4)

From equation (3) and (4),

∴ $\omega^2 = \frac{g\sin\theta}{L.\sin\theta.\cos\theta}$

$\omega = \sqrt{\frac{g}{L\cos\theta}}$

vi. If T is the period of revolution of the bob, then

$\omega = \frac{2\pi}{T} = \sqrt{\frac{g}{L\cos\theta}}$

∴ Period, $T = 2\pi\sqrt{\frac{L\cos\theta}{g}}$

3. Obtain an expression for maximum safety speed with which a vehicle can be safely driven along a curved banked road.

OR

Show that the angle of banking is independent of mass of vehicle.

Ans:

i. The vertical section of a vehicle on a curved road (considering friction) of radius 'r' banked at an angle 'θ' with the horizontal is shown in the figure.

If the vehicle is running exactly at the optimum speed , then the forces acting on the vehicle are

a. weight mg acting vertically downwards

b. normal reaction N acting perpendicular to the road.

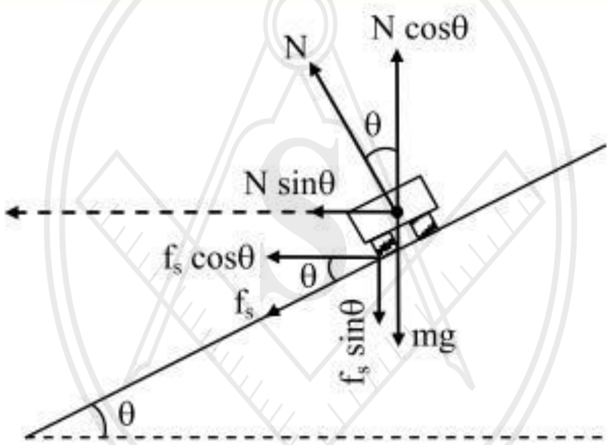
ii. But in practice, vehicles never travel exactly with this speed.

iii. Hence, for speeds other than this, the component of force of static friction between road and the tyres helps us, up to a certain limit.

iv. For maximum possible speed,

The component N sinθ is less than the centrifugal force $\frac{mv^2}{r}$.

$\therefore \quad \frac{mv^2}{r} > N \sin\theta$



Banked road: upper speed limit

v. In this case, the direction of force of static friction (f_s) between road and the tyres is directed along the inclination of the road, downwards.

vi. The horizontal component ($f_s \cos\theta$) is parallel to Nsinθ.

These two forces take care of the necessary centripetal force (or balance the centrifugal force).

$\therefore \quad \frac{mv^2}{r} = N \sin\theta + f_s \cos\theta \qquad \ldots(1)$

vii. The vertical component, N cosθ balances the component $f_s \sin\theta$ and weight 'mg'.

$\therefore \quad N \cos\theta = f_s \sin\theta + mg$

$\therefore \quad mg = N \cos\theta - f_s \sin\theta \qquad \ldots.(2)$

viii. For maximum possible speed, f_s is maximum and equal to $\mu_s N$.
From equations (1) and (2),

$$v_{max} = \sqrt{rg\left(\frac{\tan\theta + \mu_s}{1 - \mu_s \tan\theta}\right)} \quad ....(3)$$

This is an expression for maximum safety speed with which a vehicle can be safely driven along a curved banked road (considering friction).

ix. If $\mu_s = 0$, then equation (3) becomes,

$$v_{max} = \sqrt{rg\left[\frac{0 + \tan\theta}{1 - 0\tan\theta}\right]}$$

$\therefore \quad v_{max} = \sqrt{rg\tan\theta} \quad ....(4)$

This is an expression for maximum safety speed with which a vehicle can be safely driven along a curved banked road (neglecting friction).

x. From equation (3) and equation (4) we can write,

$$\left(\frac{\tan\theta + \mu_s}{1 - \mu_s \tan\theta}\right) = \frac{v_{max}^2}{rg} \quad ....(5)$$

and $\tan\theta = \frac{v_{max}^2}{rg}$

$$\therefore \quad \theta = \tan^{-1}\left(\frac{v_{max}^2}{rg}\right) \quad ....(6)$$

From equation (5) and equation (6), angle of banking is independent of mass of vehicle.

02 Mechanical Properties of Fluids

Multiple Choice Questions (1 Mark Each)

1. Insect moves over surface of water because of
   (A) Elasticity (B) **Surface tension**
   (C) Friction (D) Viscosity

2. The water droplets are spherical in free fall due to
   (A) gravity (B) intermolecular attraction
   (C) **Surface tension** (D) Viscosity

3. Surface tension of a liquid at critical temperature is
   (A) Infinity
   (B) **Zero**
   (C) Same as any other temperature
   (D) Cannot be determined

4. Unit of coefficient of viscosity is
   (A) Ns/m (B) Ns^2/m
   (C) Ns^2/m^2 (D) $\mathbf{Ns/m^2}$

5. Two capillary tubes of radii 0.6 cm and 0.3 cm are dipped in the same liquid. The ratio of heights through which the liquid will rise in the tubes is
   (A) 2:1 (B) **1:2**
   (C) 4:1 (D) 1:4

Hint: $h = \frac{2T}{r\rho g} \Rightarrow h \propto \frac{1}{r}$

$\therefore \quad \frac{h_1}{h_2} = \frac{r_2}{r_1}$

$\therefore \quad \frac{h_1}{h_2} = \frac{0.3}{0.6} = 1:2$

6. The energy stored in a soap bubble of diameter 6 cm and T = 0.04 N/m is nearly

(A) 0.9×10^{-3} J (B) 0.4×10^{-3} J
(C) 0.7×10^{-3} J (D) 0.5×10^{-3} J

Hint: Given: d = 6 cm

$\therefore$ r = 3 cm = 3×10^{-2} m

$E = T\,(dA)$

$= 0.04 \times 2 \times 4 \times 3.142 \times (3 \times 10^{-2})^2$

$= 0.9 \times 10^{-3}$ J

7. Two stones with radii 1:2 fall from a great height through atmosphere. Their terminal velocities are in the ratio

(A) 2:1 **(B) 1:4**
(C) 4:1 (D) 1:2

Hint: $v = \frac{2}{9}\frac{r^2 g(\rho - \sigma)}{\eta}$

$\therefore \quad v \propto r^2$

$\therefore \quad \frac{v_1}{v_2} = \frac{r_1^2}{r_2^2}$

$\therefore \quad \frac{v_1}{v_2} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

Very Short Answer (VSA) (1 Mark Each)

1. What is surface film?

Ans: *A layer of surface of a liquid whose thickness is equal to the range of intermolecular force is called **surface film**.*

2. What are cohesive forces?

Ans: *The force of attraction between the molecules of the same substance is called **cohesive force**.*

3. What will be the shape of liquid meniscus for obtuse angle of contact?

Ans: The shape of liquid meniscus for obtuse angle of contact will be convex.

4. What is the net weight of a body when it falls with terminal velocity through a viscous medium?

Ans: The net weight of a body when it falls with terminal velocity through a viscous medium is zero.

5. A square metal plate of area 100 cm^2 moves parallel to another plate with a velocity of 10 cm/s, both plates immersed in water. If the viscous force is 200 dyne and viscosity of water is 0.01 poise, what is the distance between them?

Ans: $F = \eta A \frac{dv}{dx}$

$\therefore \quad dx = \eta A \frac{dv}{F} = 0.01 \times 100 \times \frac{10}{200} = 0.05 \text{ cm}$

6. The relative velocity between two parallel layers of water is 8 cm/s and perpendicular distance between them is 0.1 cm. Calculate the velocity gradient.

Ans: Velocity gradient, $v_g = \frac{dv}{dx} = \frac{8}{0.1} = 80/s$

7. Water rises to a height of 20 mm in a capillary tube. If the radius made 1/3 rd of its previous value, to what height will the water now rise in the tube?

Ans: As, $\frac{h_1}{h_2} = \frac{r_2}{r_1}$

$\therefore \quad h_2 = \frac{h_1 r_1}{r_2} = 20 \times \frac{r_1}{(r_1/3)} = 60 \text{ mm}$

Short Answer I (SA1) (2 Marks Each)

1. State properties of an ideal fluid.

Ans: An ideal fluid has the following properties:

i. It is incompressible i.e., its density is constant.

ii. Its flow is irrotational i.e., its flow is smooth with no turbulences in the flow.

iii. It is non-viscous i.e., there is no internal friction in the flow and hence the fluid has no viscosity.

iv. Its flow is steady i.e., its velocity at each point is constant in time.

2. Compare streamline flow and turbulent flow.

Ans:

	Streamline flow	Turbulent flow
i.	The smooth flow of a fluid, with velocity smaller than certain critical velocity (limiting value of velocity) is called streamline flow or laminar flow of a fluid.	The irregular and unsteady flow of a fluid when its velocity increases beyond critical velocity is called turbulent flow.
ii.	In a streamline flow, velocity of a fluid at a given point is always constant.	In a turbulent flow, the velocity of a fluid at any point does not remain constant.
iii.	Two streamlines can never intersect, i.e., they are always parallel and hence can never form eddies.	In a turbulent flow, at some points, the fluid may have rotational motion which gives rise to eddies.
iv.	Streamline flow over a plane surface can be assumed to be divided into a number of plane layers. In a flow of liquid through a pipe of uniform cross-sectional area, all the streamlines will be parallel to the axis of the tube.	A flow tube loses its order and particles move in random direction.

3. Define surface tension and angle of contact.

Ans: ***Surface tension*** *is defined as the tangential force acting per unit length on both sides of an imaginary line drawn on the free surface of liquid.*

The angle of contact *(θ) between a liquid and a solid surface is defined as the angle between the tangents drawn to the free surface of the liquid and surface of the solid at the point of contact, measured within the liquid.*

4. Calculate the rise of water inside a clean glass capillary tube of radius 0.1 mm, when immersed in water of surface tension 7×10^{-2} N/m. The angle of contact between water and glass is zero, density of water is $1000 kg/m^3$, $g = 9.8 m/s^2$

Ans: ***Solution:***

Given: $r = 0.1 \text{ mm} = 10^{-4}$ m, $T = 7 \times 10^{-2}$ N/m, $\theta = 0^\circ$,

$\rho = 1000 \text{ kg/m}^3$, $g = 9.8 \text{ m/s}^2$

To find: Height of capillary rise (h)

Formula: $h = \frac{2T\cos\theta}{r\rho g}$

Calculation: From formula,

$$h = \frac{2 \times (7 \times 10^{-2}) \times \cos 0^\circ}{10^{-4} \times 10^{3} \times 9.8}$$

$$= \frac{14 \times 10^{-1}}{9.8}$$

$$= \frac{1}{7}$$

$$= \mathbf{0.1429\ m}$$

Ans: The rise of water inside the glass capillary is of **0.1429 m.**

5. A rain drop of radius 0.3 mm falls through air with a terminal velocity of 1 m/s. The viscosity of air is 18 × 10^{-6} Ns/m^2. Find the viscous force on the rain drop.

Solution:

Given: $r = 0.3\ mm = 3 \times 10^{-4}\ m$, $v = 1\ m/s$, $\eta = 18 \times 10^{-6}\ Ns/m^2$

To find: Viscous force (F)

Formula: $F = 6\pi\eta rv$

Calculation: From formula,

$$F = 6 \times 3.142 \times 18 \times 10^{-6} \times 3 \times 10^{-4} \times 1$$

$$= 324 \times 3.142 \times 10^{-10}$$

$$= \text{antilog}\{\log 324 + \log 3.142\} \times 10^{-10}$$

$$= \text{antilog}\{2.5105 + 0.4972\} \times 10^{-10}$$

$$= \text{antilog}\{3.0077\} \times 10^{-10}$$

$$= 1.017 \times 10^{3} \times 10^{-10}$$

$$= \mathbf{1.017 \times 10^{-7}\ N}$$

Ans: The viscous force is **1.017 × 10^{-7} N**

6. Two soap bubbles have radius in the ratio 2 : 3. Compare the works done in blowing these bubbles.

Solution:

Given: $r_1 : r_2 = 2 : 3$

To find: $W_1 : W_2$

Formula: $W = 2TdA$

Calculation: From formula,

Work done to blow both bubbles,

$W_1 = 2T\left(4\pi r_1^2\right)$ and $W_2 = 2T\left(4\pi r_2^2\right)$

$\therefore \quad \dfrac{W_1}{W_2} = \left(\dfrac{r_1}{r_2}\right)^2 = \left(\dfrac{2}{3}\right)^2$

$\therefore \quad W_1 : W_2 = \mathbf{4 : 9}$

Ans: The ratio of work done to blow the bubbles is **4 : 9.**

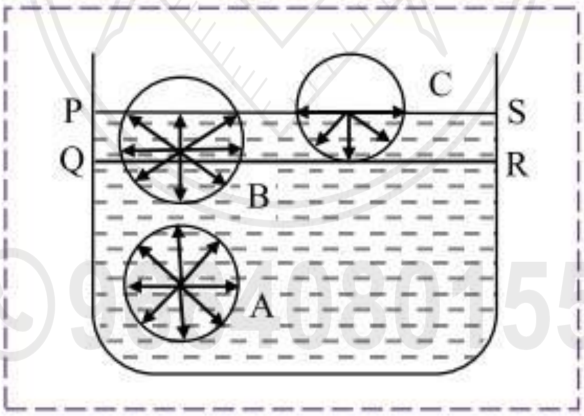
Short Answer II (SA1) (3 Marks Each)

1. Explain the phenomena of surface tension on the basis of molecular theory.

Ans: Molecular theory of surface tension:

i. Let PQRS = Surface film of liquid in a container containing liquid.
PS is the free surface of liquid and QR is the inner layer parallel to PS at distance equal to the range of molecular force.

ii. Now consider three molecules A, B and C in a liquid in a vessel such that the molecule A is well inside the liquid, the molecule B within surface film and molecule C is on the surface of liquid as shown in the figure.



iii. The sphere of influence of the molecule A is entirely inside the liquid. As a result, molecule A is acted upon by equal cohesive forces in all directions. Thus, the net cohesive force acting on molecule A is zero.

iv. For molecule B, a large part of its sphere of influence is inside the liquid and a smaller part is outside the surface (in air). The adhesive force acting on molecule B due to air molecules above it and within its sphere of influence is weak compared to the strong downward cohesive force acting on the molecule. As a result, the molecule B gets attracted inside the liquid.

v. For molecule C, half of the sphere of influence is in air and half is in liquid. As density of air is much less than that of liquid, number of air molecules within the sphere of influence of the molecule C above the free surface of the liquid is much less than the numbers of liquid molecules within the sphere of influence that lies within the liquid. Thus, the adhesive force due to the air molecules acting on molecule C is weak compared to the cohesive force acting on the molecule. As a result, the molecule C also gets attracted inside the liquid.

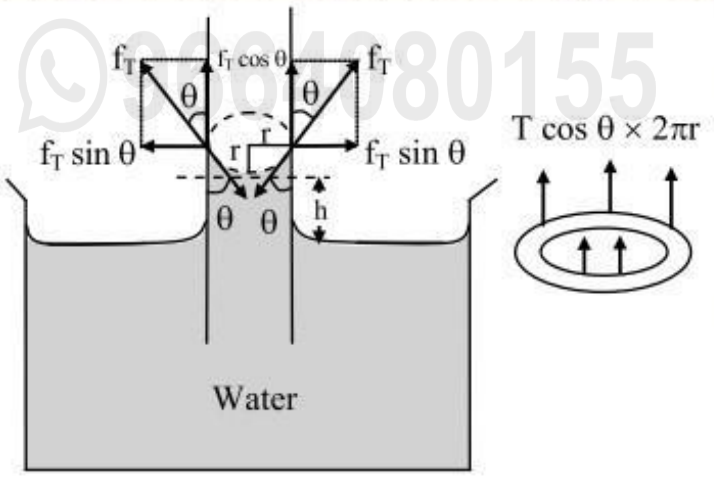
vi. Thus, all molecules in the surface film are acted upon by an unbalanced net cohesive force directed into the liquid. Therefore, the molecules in the surface film are pulled inside the liquid. This minimizes the total number of molecules in the surface film. As a result, the surface film remains under tension. The surface film of a liquid behaves like a stretched elastic membrane. This tension is known as surface tension and the force due to it acts tangential to the free surface of a liquid.

2. Obtain an expression for the capillary rise or fall using forces method.

Ans: Expression for capillary rise or fall using forces method:

i. When glass capillary tube is dipped into a liquid, then the liquid rises in the capillary against gravity.
Hence, the weight of the liquid column must be equal and opposite to the component of force due to surface tension at the point of contact.

ii. The length of liquid in contact inside the capillary is the circumference $2\pi r$.
Let, r = radius of capillary tube
h = height of liquid level in the tube
T = surface tension of liquid
ρ = density of liquid
g = acceleration due to gravity



Rise of liquid in capillary tube

iii. The force of magnitude f_T acts tangentially on unit length of liquid surface which is in contact with wall of capillary tube and is given as,
$f_T = T \times 2\pi r$
This force can be resolved into two components:

a. $f_T \cos\theta$-vertically upward and
b. $f_T \sin\theta$-along horizontal

iv. Vertical component is effective. Horizontal component is not responsible for capillary rise.

v. Vertical component of force acting on liquid column
$(f_T)_v$ = force per unit length × circumference
$= T \cos\theta \times 2\pi r$

vi. Upward force balances weight of liquid in the capillary.
$W = mg = V\rho g = \pi r^2 h\rho g$
where, V = volume of liquid rise in the tube (ignoring the liquid in the concave meniscus.)
m = mass of the liquid in the capillary rise.
This must be equal and opposite to the vertical component of the force due to surface tension.

vii. If liquid in meniscus is neglected, then for equilibrium.,
$2\pi r\, T \cos\theta = \pi r^2 h \rho g$

$\therefore \quad h = \dfrac{2T\cos\theta}{r\rho g} \quad ....(1)$

This is the required expression for rise or fall of liquid in capillary tube.

3. State Stoke's law and give two factors affecting angle of contact.

Ans:

Statement: *The viscous force acting on a small sphere falling through a medium is directly proportional to the radius* (r) *of the sphere, its velocity* (v) *through fluid and coefficient of viscosity* (η) *of the fluid.*

Two factors affecting angle of contact:

i. The nature of the liquid and the solid in contact.
ii. Impurities present in the liquid change the angle of contact.

4. Twenty seven droplets of water, each of radius 0.1 mm coalesce into a single drop. Find the change in surface energy. Surface tension of water is 0.072 N/m.

Solution:

Given: $r = 0.1 \text{ mm} = 10^{-4} \text{ m}, n = 27, T = 0.072 \text{ N/m}$

To find: Change in surface energy (W)

Formula: W = TdA

Calculation: Volume of a single drop = $\frac{4}{3}\pi R^3$ and

Volume of a single droplet = $\frac{4}{3}\pi r^3$

$\therefore$ We have, $\frac{4}{3}\pi R^3 = n \times \frac{4}{3}\pi r^3$ or $R^3 = nr^3$

$\therefore$ $R = \sqrt[3]{n}\, r = \sqrt[3]{27} \times 10^{-4} = 3 \times 10^{-4}\ m^2$

From formula,

$W = T(n \times 4\pi r^2 - 4\pi R^2)$

$= 4\pi T(nr^2 - R^2)$

$= 4 \times 3.142 \times 0.072 \times [27 \times (10^{-4})^2 - (3 \times 10^{-4})^2]$

$= 3.142 \times 0.288 \times 18 \times 10^{-8}$

$= \text{antilog}\ \{\log(3.142) + \log(0.288) + \log(18)\}$

$= \text{antilog}\ \{0.4972 + \bar{1}.4594 + 1.2553\} \times 10^{-8}$

$= \text{antilog}\ \{1.2119\} \times 10^{-8}$

$= 16.29 \times 10^{-8}$

$= \mathbf{1.629 \times 10^{-7}\ J}$

Ans: Change in the surface energy is **1.629×10^{-7} J.**

5. A U-tube is made up of capillaries of bore 1 mm and 2 mm respectively. The tube is held vertically and partially filled with a liquid of surface tension 49 dyne/cm and zero angle of contact. Calculate the density of liquid, if the difference in the levels of the meniscus is 1.25 cm. Take g = 980 cm/s²

Solution:

Let 'r_1' be the radius of one bore and 'r_2' be the radius of second bore of the U-tube. Then, if 'h_1' and 'h_2' are the height of water on two sides, then

$$h_1 = \frac{2T\cos\theta}{r_1 \rho g} \quad \text{....(i)}$$

and $$h_2 = \frac{2T\cos\theta}{r_2 \rho g} \quad \text{....(ii)}$$

Subtracting equation (ii) from equation (i),

$$h_1 - h_2 = \frac{2T\cos\theta}{r_1 \rho g} - \frac{2T\cos\theta}{r_2 \rho g} = \frac{2T\cos\theta}{\rho g}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$\therefore \quad \rho = \frac{2T\cos\theta}{(h_1 - h_2)g}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$

Given: $T = 49$ dyne/cm, $\theta = 0°$,

$h_1 - h_2 = 1.25$ cm, $g = 980$ cm/s^2

$r_1 = \frac{1}{2}$ mm $= 0.5$ mm $= 5 \times 10^{-2}$ cm

$r_2 = \frac{2}{2}$ mm $= 1$ mm $= 10^{-1}$ cm

$\therefore \quad \rho = \frac{2 \times 49 \times \cos 0°}{1.25 \times 980} \times \left[\frac{1}{5\times10^{-2}} - \frac{1}{10^{-1}}\right]$

$= \frac{1}{12.5}[20 - 10]$

$= \frac{10}{12.5} = \frac{1}{1.25}$

Using reciprocal table,

$\rho = \mathbf{0.8\ g/cm^3}$

Ans: The density of liquid is **0.8 g/cm^3**.

6. **A rectangular wire frame of size 2 cm × 2 cm, is dipped in a soap solution and taken out. A soap film is formed, it the size of the film is changed to 3 cm × 3 cm, Calculate the work done in the process. The surface tension of soap film is 3×10^{-2} N/m.**

Solution:

Given: $A_1 = 2$ cm × 2 cm $= 4 \times 10^{-4}$ m^2,

$A_2 = 3$ cm × 3 cm $= 9 \times 10^{-4}$ m^2,

$T = 3 \times 10^{-2}$ N/m

To find: Work done (W)

Formula: W = (T dA)

Calculation: As rectangular film has two surfaces,

$dA = 2(A_2 - A_1)$

From formula,

$W = 3 \times 10^{-2} \times 2 \times (9 \times 10^{-4} - 4 \times 10^{-4})$

$= 6 \times 10^{-2} \times 5 \times 10^{-4}$

$\therefore \quad W = 30 \times 10^{-6} = \mathbf{3 \times 10^{-5}\ J}$

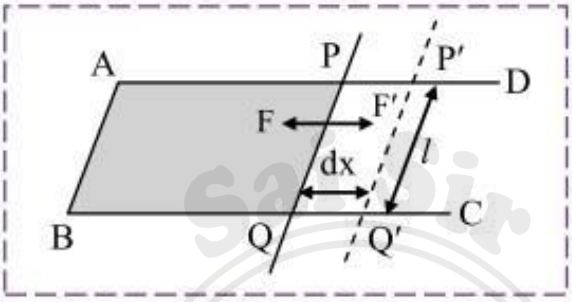
Ans: The work done in increasing film is **3×10^{-5} J.**

Long Answer (LA) (4 Marks Each)

1. Derive the relation between surface energy and surface tension.

Ans: Relation between surface tension and surface energy:

i. Let ABCD be a rectangular frame of wire, fitted with a movable arm PQ.



ii. The frame held in horizontal position is dipped into soap solution and taken out so that a soap film APQB is formed. Due to surface tension of soap solution, a force 'F' will act on each arm of the frame. Under the action of this force, the movable arm PQ moves towards AB.

iii. Magnitude of force due to surface tension is,

$F = 2Tl$. ....[$\because T = F/l$]

(A factor of 2 appears because soap film has two surfaces which are in contact with wire.)

iv. Let the wire PQ be pulled outwards through a small distance 'dx' to the position P′Q′, by applying an external force F′ isothermally, which is equal and opposite to F. Work done by this force, $dW = F'dx = 2Tl dx$.

v. But, $2l dx = dA$ = increase in area of two surfaces of film.

$\therefore$ $dW = T\,dA$

vi. This work done in stretching the film is stored in the area dA in the form of potential energy (surface energy).

$\therefore$ Surface energy, $E = T\,dA$

$\therefore$ $\frac{E}{dA} = T$

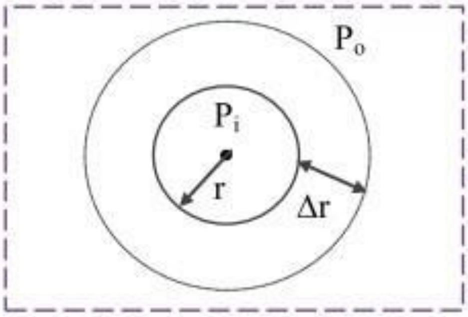
Hence,

surface tension = surface energy per unit area.

vii Thus, surface tension is equal to the mechanical work done per unit surface area of the liquid, which is also called as surface energy.

2. Obtain Laplace's law of spherical membrane.

Ans: Derivation for Laplace's law for spherical membrane:



i. Free surface of drops or bubbles are spherical in shape.
Let,
P_i = inside pressure of a drop or air bubble
P_o = outside pressure of bubble
r = radius of drop or bubble.

ii. As drop is spherical, $P_i > P_o$

∴ excess pressure inside drop = $P_i - P_o$

iii. Let the radius of drop increases from r to r + Δr so that inside pressure remains constant.

iv. Initial area of drop $A_1 = 4\pi r^2$,
Final surface area of drop $A_2 = 4\pi (r + \Delta r)^2$
Increase in surface area,
$\Delta A = A_2 - A_1 = 4\pi[(r + \Delta r)^2 - r^2]$
$= 4\pi[r^2 + 2r\Delta r + \Delta r^2 - r^2]$
$= 8\pi r\Delta r + 4\pi\Delta r^2$

v. As Δr is very small, the term containing Δr^2 can be neglected.

∴ $dA = 8\pi r\Delta r$

vi. Work done by force of surface tension,
$dW = TdA = (8\pi r\Delta r)T$ ….(1)
This work done is also equal to the product of the force F which causes increase in the area of the bubble and the displacement Δr which is the increase in the radius of the bubble.

∴ $dW = F\Delta r$
The excess force is given by,
(Excess pressure) × (Surface area)

∴ $F = (P_i - P_o) \times 4\pi r^2$

∴ $dF = (P_i - P_o)A$
$dW = F\Delta r = (P_i - P_o) A\Delta r$

From equation (1),

$(P_i - P_o) A\Delta r = (8\pi r\Delta r) T$

$\therefore \quad P_i - P_o = \dfrac{8\pi r\Delta rT}{4\pi r^2\Delta r}$ ....($\because A = 4\pi r^2$)

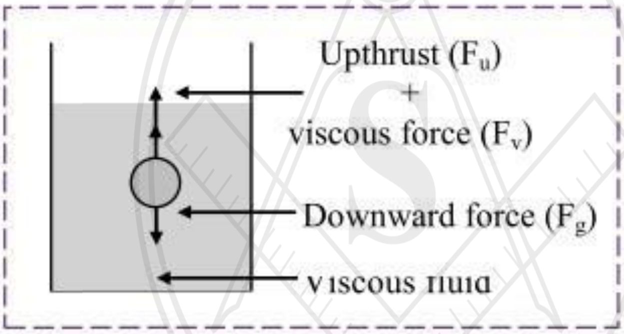
$\therefore \quad P_i - P_o = \dfrac{2T}{r}$ ....(2)

Equation (2) represents Laplace's law of spherical membrane.

3. Derive an expression for terminal velocity of the sphere falling under gravity through a viscous medium.

Ans:

i. Consider a sphere of radius (r) and density (ρ) falling under gravity through a liquid of density (σ) and coefficient of viscosity (η) as shown in figure.



ii. Forces acting on the sphere during downward motion are,

a. Viscous force = $F_v = 6\pi\eta rv$ (directed upwards)

b. Weight of the sphere, (F_g)

$mg = \dfrac{4}{3}\pi r^3\rho g$ (directed downwards)

c. Upward thrust as Buoyant force (F_u)

$F_u = \dfrac{4}{3}\pi r^3\sigma g$ (directed upwards)

iii. As the downward velocity increases, the viscous force increases. A stage is reached, when sphere attains terminal velocity.

iv. When the sphere attains the terminal velocity, the total downward force acting on the sphere is balanced by the total upward force acting on the sphere.

$\therefore$ Total downward force = Total upward force

∴ Weight of sphere (mg) = Viscous Force + Buoyant force due to medium

∴ $\frac{4}{3}\pi r^3 \rho g = 6\pi\eta r v + \frac{4}{3}\pi r^3 \sigma g$

∴ $6\pi\eta r v = \left(\frac{4}{3}\pi r^3 \rho g\right) - \left(\frac{4}{3}\pi r^3 \sigma g\right)$

∴ $6\pi\eta r v = \left(\frac{4}{3}\right)\pi r^3 g\,(\rho - \sigma)$

∴ $v = \left(\frac{4}{3}\right)\pi r^3 g\,(\rho - \sigma) \times \frac{1}{6\pi\eta r}$

∴ $v = \frac{2}{9}\ \frac{r^2 g(\rho - \sigma)}{\eta}$ ....(1)

This is the expression for terminal velocity of the sphere falling under gravity through a viscous medium.

03 Kinetic Theory of gases and Radiation

Multiple Choice Questions (1 Mark Each)

1. The average energy per molecule is proportional to
   (A) the pressure of the gas
   (B) the volume of the gas
   (C) the absolute temperature of the gas
   (D) the mass of the gas

2. The number of degrees of freedom, for the vibrational motion of a polyatomic molecule depends on the
   (A) geometric structure of the molecule
   (B) mass of the molecule
   (C) energy of the molecule
   (D) absolute temperature of the molecule

3. The power radiated by a perfect blackbody depends only on its
   (A) material
   (B) nature of surface
   (C) colour
   (D) temperature

4. If the absolute temperature of a body is doubled, the power radiated will increase by a factor of
   (A) 2
   (B) 4
   (C) 8
   (D) 16

5. Calculate the value of λ_{max} for radiation from a body having surface temperature 3000 K.(b = 2.897×10^{-3} m K)
   (A) 9935 Å
   (B) 9656 Å
   (C) 9421 Å
   (D) 9178 Å

Hint: $\lambda_{max} = \frac{b}{T} = \frac{2.897 \times 10^{-3}}{3000} = 9656$ Å

6. The molar specific heat of a gas at constant volume is 12307.69 J kg^{-1} K^{-1}. If the ratio of the two specific heats is 1.65, calculate the difference between the two molar specific heats of gas.
   (A) 7999 J kg^{-1} K^{-1}
   (B) 7245 J kg^{-1} K^{-1}
   (C) 6890 J kg^{-1} K^{-1}
   (D) 4067 J kg^{-1} K^{-1}

Hint: Given: $C_v = 12307.69\ J\ kg^{-1}\ K^{-1}$

$$\frac{C_p}{C_v} = 1.65$$

Using property of dividendo,

$$\frac{C_p - C_v}{C_v} = \frac{1.65 - 1}{1}$$

$$C_p - C_v = 0.65 \times C_v$$
$$= 0.65 \times 12307.69$$
$$\approx 7999\ J\ kg^{-1}\ K^{-1}$$

7. Calculate the energy radiated in one minute by a blackbody of surface area 200 cm^2 at 127°C ($\sigma = 5.7 \times 10^{-8}\ J\ m^{-2}\ s^{-1}\ K^{-4}$)

(A) 1367.04 J (B) 1698.04 J
(C) 1751.04 J (D) 1856.04 J

Hint: $Q = \sigma A t T^4$
$= 5.7 \times 10^{-8} \times 200 \times 10^{-4} \times 60 \times (400)^4$
$= 1751.04\ J$

Very Short Answer (VSA) (1 Mark Each)

1. **Under which condition laws of Boyle, Charles, and Gay-Lussac are valid?**

Ans: The laws of Boyle, Charles, and Gay-Lussac are strictly valid for real gases, only if the pressure of the gas is not too high and the temperature is not close to the liquefaction temperature of the gas.

2. **On what, the values of absorption coefficient, reflection coefficient and transmission coefficient depend, in addition to the material of the object on which the radiation is incident?**

Ans: The values of absorption coefficient, reflection coefficient and transmission coefficient depend on the wavelength of the incident radiation, in addition to the material of the object on which the radiation is incident.

3. **Why the temperature of all bodies remains constant at room temperature?**

Ans: All bodies radiate as well as absorb radiation at room temperature, but their rate of emission and rate of absorption are same, hence their temperature remains constant.

4. Above what temperature all bodies radiate electromagnetic radiation?

Ans: All bodies at a temperature above 0 K radiate electromagnetic radiation.

5. If the density of nitrogen is 1.25 kg/m^3 at a pressure of 10^5 Pa, find the root mean square velocity of nitrogen molecules.

Ans: $v_{rms} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3 \times 10^5}{1.25}} = 489.89$ m/s

*[**Note:** Since density value given in question is of nitrogen, question is modified to obtain RMS velocity of nitrogen molecules.]*

6. Find kinetic energy of 3 litre of a gas at S.T.P, given standard pressure is 1.013 × 10^5 N/m^2.

Ans: K.E. $= \frac{3}{2}PV = \frac{3}{2} \times 1.013 \times 10^5 \times 3 \times 10^{-3} = 455.85$ J

7. Determine the pressure of nitrogen at 0°C if the density of nitrogen at N.T.P. is 1.25 kg/m^3 and R.M.S. speed of the molecules at N.T.P. is 489 m/s.

Ans: $P = \frac{1}{3}\frac{mN}{V}v_{rms}^2 = \frac{1}{3}\rho v_{rms}^2 = \frac{1}{3} \times 1.25 \times 489^2 = 99633.75$ Nm

Short Answer I (SA1) (2 Marks Each)

1. State factors on which the amount of heat radiated by a body depends.

Ans: Amount of heat radiated by a body depends on:

i. The absolute temperature of the body (T)

ii. The nature of the body – the material, nature of surface - polished or not, etc.

iii. Surface area of the body (A)

iv. Time duration for which body emits radiation (t)

2. Show that for monoatomic gas the ratio of the two specific heats is 5 : 3.

Ans:

i. For a monatomic gas enclosed in a container, held at a constant temperature T and containing N_A atoms, each atom has only 3 translational degrees of freedom.

ii. Therefore, average energy per atom is $\frac{3}{2}k_BT$ and the total internal energy per mole is, $E = \frac{3}{2}N_Ak_BT$

iii. Molar specific heat at constant volume

$$C_V = \frac{dE}{dT} = \frac{3}{2}N_Ak_B = \frac{3}{2}R$$

iv. Using Mayer's relation, $C_P = R + C_V$

$\therefore$ $C_P = \frac{5}{2}R$

$\therefore$ $\frac{C_P}{C_V} = \frac{5}{3}$

3. Show that for diatomic gas the ratio of the two specific heats is 7 : 5.

Ans:

i. For a gas consisting of diatomic molecules such as O_2, N_2, CO, HCl, enclosed in a container held at a constant temperature T, if treated as a rigid rotator, each molecule will have 3 translational and 2 rotational degrees of freedom.

ii. According to the law of equipartition of energy, the internal energy of one mole of gas is,

$$E = \frac{3}{2}N_Ak_BT + \frac{2}{2}N_Ak_BT$$

$$= \frac{5}{2}N_Ak_BT$$

iii. The molar specific heat at constant volume will be,

$$C_V = \frac{5}{2}N_Ak_B$$

$$= \frac{5}{2}R$$

iv. Using Mayer's relation,

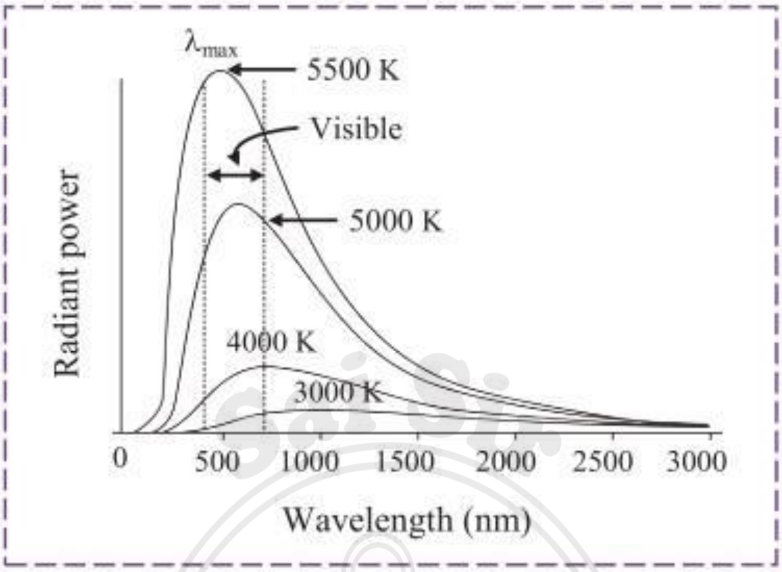
$C_P = R + C_v$

$\therefore$ $C_P = \frac{7}{2}R$

$\therefore$ $\frac{C_P}{C_V} = \frac{7}{5}$

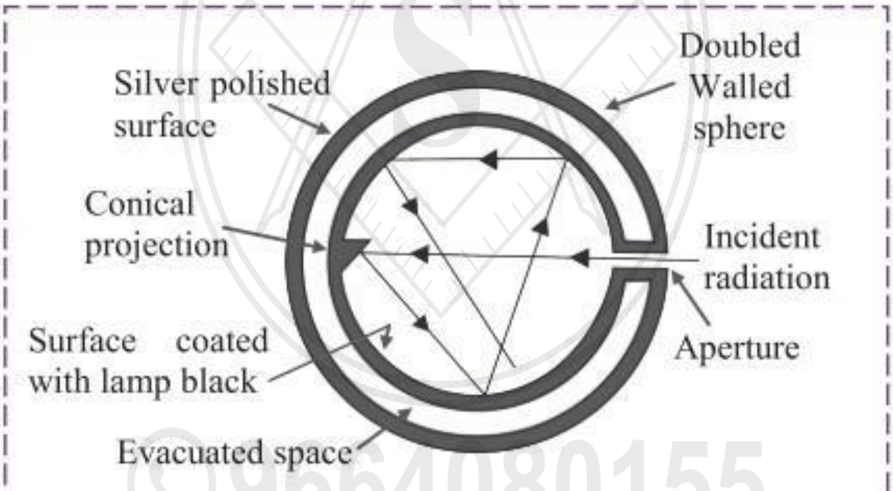
4. Show the graphical representation of radiant power of a black body per unit range of wavelength as a function of wavelength.

Ans:



5. Draw neat labelled diagram of Ferry's black body.

Ans:



Ferry's blackbody

6. Compare the rate of radiation of metal body at 727°C and 227°C.

Solution:

Given: $T_1 = 727\ °C = 727 + 273 = 1000\ K$,
$T_2 = 227\ °C = 227 + 273 = 500\ K$

To find: Ratio of radiation $\left[\left(\frac{dQ}{dt}\right)_1 \Big/ \left(\frac{dQ}{dt}\right)_2\right]$

Formula: $\frac{dQ}{dt} = \sigma A e T^4$

Calculation: From formula,

$$\left(\frac{dQ}{dt}\right)_1 = \sigma Ae\, T_1^4 \quad ....(1)$$

$$\left(\frac{dQ}{dt}\right)_2 = \sigma Ae\, T_2^4 \quad ....(2)$$

Dividing equation (1) by (2),

$$\left(\frac{dQ}{dt}\right)_1 \Big/ \left(\frac{dQ}{dt}\right)_2 = \frac{\sigma AeT_1^4}{\sigma AeT_2^4} = \left(\frac{T_1}{T_2}\right)^4$$

$$= \frac{(1000)^4}{(500)^4} = \mathbf{16}$$

Ans: The rate of radiation of metal sphere at 727 °C and 227 °C is **16 : 1**.

7. 1000 calories of radiant heat is incident on a body. If the body absorbs 400 calories of heat, find the coefficient of emission of the body.

Solution:

Given: $Q = 1000$ cal, $Q_a = 400$ cal

To find: Coefficient of emission (e)

Formula: i. $a = \frac{Q_a}{Q}$ ii. $a = e$

Calculation: From formula (i),

$$a = \frac{400}{1000} = 0.4$$

From Formula (ii),

$e = a = \mathbf{0.4}$

Ans: The coefficient of emission of the body is **0.4**.

8. A metal cube of length 4 cm radiates heat at the rate of 10 J/s. Find its emissive power at given temperature.

Solution:

Given: $l = 4$ cm, $A = 6l^2 = 6 \times 16\ cm^2 = 96 \times 10^{-4}\ m^2$

$$\frac{Q}{t} = 10\ J/s$$

To find: Emissive power (E)

Formula: $E = \frac{Q}{At}$

Calculation: From formula,

$$E = \frac{10}{96 \times 10^{-4}}$$

$$= \mathbf{1041.66\ J/m^2s}$$

Ans: The emissive power at given temperature is **$1041.66\ J/m^2s$**.

Short Answer II (SA1) (3 Marks Each)

1. Show that the root mean square speed of the molecules of gas is directly proportional to the square root of the absolute temperature of the gas.

Ans:

i. Average value of the pressure of the gas is,

$$P = \frac{1}{3}\frac{N}{V}m\overline{v^2}$$

ii. Thus, the mean square velocity of molecule will be,

$$\overline{v^2} = \frac{3PV}{Nm}$$

iii. Using ideal gas equation,

$$PV = nRT$$

$$\overline{v^2} = \frac{3nRT}{Nm}$$

iv. But, $n = \frac{N}{N_A}$

$\therefore$ $$\overline{v^2} = \frac{3NRT}{N_A Nm}$$

v. Also, $mN_A = M_0$ (Molar mass of the gas)

$\therefore$ $$\overline{v^2} = \frac{3RT}{M_0}$$

$\therefore$ $$\sqrt{\overline{v^2}} = v_{rms} = \sqrt{\frac{3RT}{M_0}}$$

vi. As R and M_0 in the above equation are constant,

$\therefore$ $v_{rms} \propto \sqrt{T}$

2. Show that the average energy of the molecules of gas is directly proportional to the absolute temperature of gas.

Ans:

i. From kinetic theory of gases pressure of the gas is given as,

$$P = \frac{1}{3}\frac{N}{V}m\overline{v^2}$$

$$\therefore \quad PV = \frac{1}{3}Nm\overline{v^2}$$

$$\therefore \quad PV = \frac{2}{3}N\left(\frac{1}{2}m\overline{v^2}\right) \quad ....(1)$$

ii. The quantity $\frac{1}{2}m\overline{v^2}$ represents the average translational kinetic energy of an ideal gas molecule.

iii. Therefore, the total energy of the gas is,

$$E = N\left(\frac{1}{2}m\overline{v^2}\right)$$

iv. Substituting in equation (1),

$$PV = \frac{2}{3}E$$

v. From ideal gas equation,

$$PV = Nk_BT = \frac{2}{3}E \quad ....(2)$$

$$\therefore \quad E = \frac{3}{2}Nk_BT \quad ....(3)$$

vi. From equation (3), it can be concluded that the average energy of the molecules of gas is directly proportional to absolute temperature of the gas.

3. Calculate the ratio of two specific heats of polyatomic gas molecule.

Ans:

i. Gases which have molecules containing more than two atoms are termed as polyatomic gases.

ii. Each molecule of the polyatomic gas has 3 translational degrees of freedom. Only linear molecules have 2 degrees of freedom for rotation. All other polyatomic molecules have 3 degrees of freedom for rotation.

iii. The number of degrees of freedom (f), for the vibrational motion of a polyatomic molecule depends on the geometric structure of the molecule i.e., the arrangement of atoms in a molecule.

iv. Each such degree of freedom contributes average energy $2 \times \frac{1}{2}k_BT$ from kinetic energy and potential energy terms.

v. Therefore for 1 mole of a polyatomic gas, the internal energy is

$$E = \frac{3}{2}N_Ak_BT + \frac{3}{2}N_Ak_BT + f \times \frac{2}{2}N_Ak_BT$$
$$= (3 + f)\ N_Ak_BT$$

vi. The molar specific heats at constant volume and constant pressure are given as

$C_V = (3 + f)\ R$ and $C_P = (4 + f)\ R$

$$\therefore \quad \gamma = \frac{C_P}{C_V} = \frac{4+f}{3+f}$$

4. Explain the construction and working of Ferry's black body.

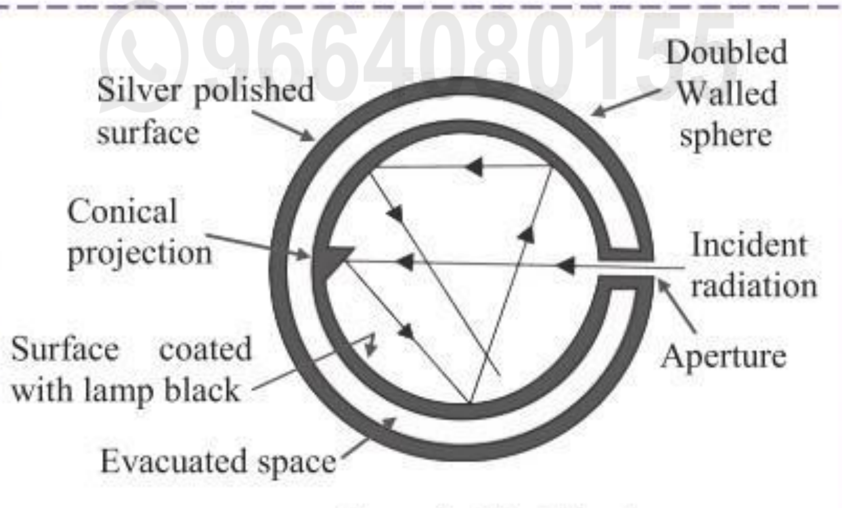
Ans:

i. Ferry's perfectly blackbody consists of a double walled hollow sphere having tiny hole or aperture, through which radiant heat can enter.

ii. The space between the walls is evacuated and outer surface of the sphere is silvered.

iii. The inner surface of sphere is coated with lampblack.

iv. There is a conical projection on the inner surface of sphere opposite the aperture. The projection ensures that a ray travelling along the axis of the aperture is not incident normally on the surface and is therefore not reflected back along the same path.



Ferry's blackbody

v. A heat ray entering the sphere through the aperture suffers multiple reflections and is almost completely absorbed inside.

vi. Thus, the aperture behaves like a perfect blackbody.

vii. The effective area of perfectly blackbody is equal to the area of the aperture.

5. Compare the rates of emission of heat by a blackbody maintained at 627°C and at 127°C, if the blackbodies are surrounded by an enclosure at 27°C. What would be the ratio of their rates of loss of heat?

Solution:

Given: $T_0 = 27\ °C = 27 + 273 = 300\ K$

$T_1 = 627\ °C = 627 + 273 = 900\ K$

$T_2 = 127\ °C = 127 + 273 = 400\ K$

To find: Ratio of rate of loss of heat ($R_1 : R_2$)

Formula: $R = \frac{dQ}{dt} = e\sigma A\,(T^4 - T_0^4)$

Calculation: From formula,

$$R_1 = \left(\frac{dQ}{dt}\right)_1 = e\sigma A(\,T_1^4 - T_0^4) \quad \text{....(1)}$$

$$R_2 = \left(\frac{dQ}{dt}\right)_2 = e\sigma A(\,T_2^4 - T_0^4) \quad \text{....(2)}$$

Dividing equation (1) by equation (2),

$$\frac{R_1}{R_2} = \frac{T_1^4 - T_0^4}{T_2^2 - T_0^4} = \frac{900^4 - 300^4}{400^4 - 300^4}$$

$$= \frac{9^4 - 3^4}{4^4 - 3^4}$$

$$= \frac{6480}{175}$$

= antilog {log (6480) – log (175)}

= antilog {3.8116 – 2.2430}

= antilog {1.5686}

= **37.03**

Ans: The ratio of the rate of energy radiated is **37.03 : 1.**

[Note: *Answer calculated above is in accordance with textual method of calculation.****]***

6. Determine the molecular kinetic energy (i) per mole (ii) per gram (iii) per molecule of nitrogen molecules at 227°C, $R = 8.310$ J $mole^{-1}$ K^{-1}, No = 6.03×10^{26} molecules K $mole^{-1}$. Molecular weight of nitrogen = 28.

Solution:

Given: $T = 227\ °C = 227 + 273 = 500$ K

$M = 28\ g = 28 \times 10^{-3}$ kg

$R = 8.31\ Jmol^{-1}\ K^{-1}$

Avogadro's number, $N_A = 6.03 \times 10^{26}$ molecules $Kmol^{-1}$

To find:
i. KE / mole
ii. KE / g
iii. KE / molecule

Formulae:
i. $\text{KE / mole} = \frac{3}{2}RT$

ii. $\text{KE / gram} = \frac{3}{2}\frac{RT}{M_0}$

iii. $\text{KE / molecule} = \frac{3}{2}\frac{RT}{N_A}$

Calculation: From formula (i),

$$\text{KE / mole} = \frac{3}{2} \times 8.31 \times 500$$

$$= \mathbf{6.232 \times 10^3\ J/mol}$$

From formula (ii),

$$\text{KE / gram} = \frac{3}{2} \times \frac{8.31 \times 500}{28}$$

$$= \mathbf{0.2225 \times 10^3\ J/g}$$

From formula (iii),

$$\text{KE / molecule} = \frac{3}{2} \times \frac{8.31 \times 500}{6.03 \times 10^{26}}$$

$$= \mathbf{1.0335 \times 10^{-23}\ J}$$

Ans: i. KE / mol is **6.232×10^3 J/mole.**

ii. KE / g is **0.2225×10^3 J/g.**

iii. KE / molecule is **1.0335×10^{-23} J.**

*[**Note:** Answers calculated above are in accordance with textual methods of calculation.]*

7. The velocity of three molecules, are 2 km s^{-1}, 4 km s^{-1}, 6 km s^{-1}. Find (i) mean square velocity (ii) root mean square velocity.

Solution:

Given: $v_1 = 2$ km/s, $v_2 = 4$ km/s, $v_3 = 6$ km/s,

To find: i. Mean square velocity
ii. Root mean square velocity

Formulae: i. $\overline{v^2} = \frac{v_1^2 + v_2^2 + v_3^2}{3}$ ii. $v_{rms} = \sqrt{\frac{v_1^2 + v_2^2 + v_3^2}{3}}$

Calculation: From formula (i),

$$\overline{v^2} = \frac{2^2 + 4^2 + 6^2}{3} = \mathbf{18.66\ km^2/s^2}$$

From formula (ii),

$$v_{rms} = \sqrt{\frac{2^2 + 4^2 + 6^2}{3}} = \sqrt{18.66} = \mathbf{4.319\ km/s}$$

Ans: i. Mean square velocity is **18.66 km^2/s^2**.
ii. Root mean square velocity **4.319 km/s**.

[Note: *Mean square velocity is expressed in considering the units used for velocity in the question.****]***

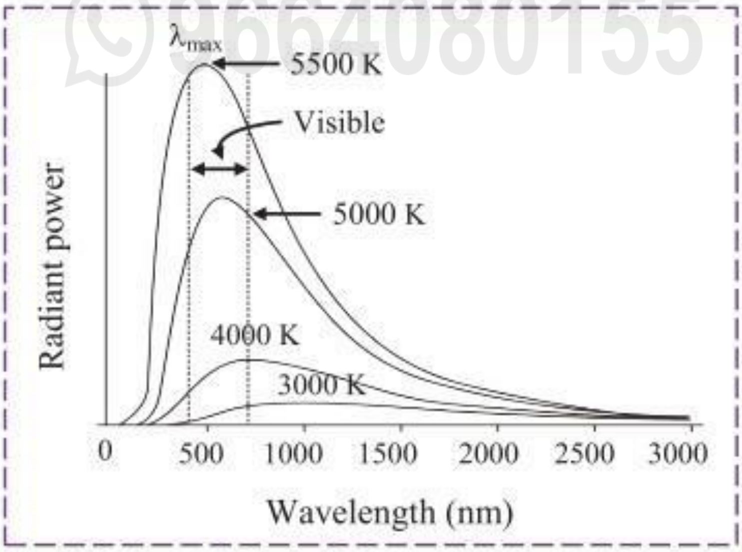
Long Answer (LA) (4 Marks Each)

1. Explain spectral distribution of a blackbody radiation.

Ans:

i. The rate of emission per unit area or power per unit area of a surface is defined as a function of the wavelength λ of the emitted radiation.

ii. Scientists studied the energy distribution of blackbody radiation as a function of wavelength.



iii. By keeping the source of radiation (such as a cavity radiator) at different temperatures they measured the radiant power corresponding to different wavelengths. The measurements were represented graphically in the form of curves showing variation of radiant power per unit area as a function of wavelength λ at different constant temperatures as shown in figure.

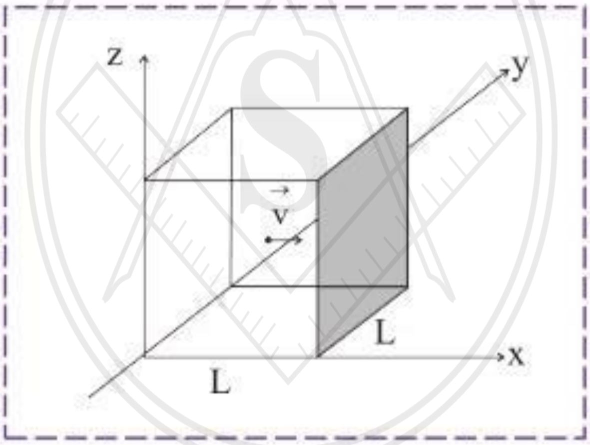
2. Derive expression for average pressure of an ideal gas.

Ans:

i. Let there be n moles of an ideal gas enclosed in a cubical box of volume V $(= L^3)$ with sides of the box parallel to the coordinate axes, as shown in figure. The walls of the box are kept at a constant temperature T.

ii. The gas molecules are in continuous random motion, colliding with each other and hitting the walls of the box and bouncing back.

iii. As per one of the assumptions, we neglect intermolecular collisions and consider only elastic collisions with the walls.



iv. A typical molecule moving with the velocity $\vec{v}$, about to collide elastically with the shaded wall of the cube parallel to yz-plane.

v. During elastic collision, the component v_x of the velocity will get reversed, keeping v_y and v_z components unaltered.

vi. Hence the change in momentum of the particle is only in the x component of the momentum, Δp_x is given by,

Δp_x = final momentum – initial momentum

$= (-mv_x) - (mv_x) = -2\,mv_x$

vii. Thus, the momentum transferred to the wall during collision is $+2mv_x$. The re-bounced molecule then goes to the opposite wall and collides with it.

viii. After colliding with the shaded wall, the molecule travels to the opposite wall and travels back towards the shaded wall again.

ix. This means that the molecule travels a distance of 2L in between two collisions.

x. As L is the length of the cubical box, the time for the molecule to travel back and forth to the shaded wall is $\Delta t = \frac{2L}{v_x}$.

xi. Average force exerted on the shaded wall by molecule 1 is given as,
Average force = Average rate of change of momentum

$\therefore$ $F_{avg} = \frac{2mv_{x_1}}{2L/v_{x_1}} = \frac{mv_{x_1}^2}{L}$ ....(1)

where v_{x_1} is the x component of the velocity of molecule 1.

xii. Considering other molecules 2, 3, 4 ... with the respective x components of velocities $v_{x_2}, v_{x_3}, v_{x_4}$ ...., the total average force on the wall is,

$F_{avg} = \frac{m}{L}\left(v_{x_1}^2 + v_{x_2}^2 + v_{x_3}^2 + ...\right)$ ....[From (1)]

$\therefore$ The average pressure

$$P = \frac{\text{Average force}}{\text{Area of shaded wall}}$$

$$= \frac{m\left(v_{x_1}^2 + v_{x_2}^2 + ...\right)}{L \times L^2}$$

xiii. The average of the square of the x component of the velocities is given by,

$$\overline{v_x^2} = \frac{v_{x_1}^2 + v_{x_2}^2 + v_{x_3}^2 + ... + v_N^2}{N}$$

$\therefore$ $P = \frac{mN\overline{v_x^2}}{V}$

where $\overline{v_x^2}$ is the average over all possible values of v_x.

xiv. Now, $\overline{v^2} = \overline{v_x^2} + \overline{v_y^2} + \overline{v_z^2}$

By symmetry, $\overline{v_x^2} = \overline{v_y^2} = \overline{v_z^2} = \frac{1}{3}\overline{v^2}$ since the molecules have no preferred direction to move.

Therefore, average pressure, $P = \frac{1}{3}\frac{N}{V}m\overline{v^2}$

3. Derive Mayer's relation.

Ans:

i. Consider one mole of an ideal gas that is enclosed in a cylinder by light, frictionless airtight piston.

ii. Let P, V and T be the pressure, volume and temperature respectively of the gas.

iii. If the gas is heated so that its temperature rises by dT, but the volume remains constant, then the amount of heat supplied to the gas (dQ_1) is used to increase the internal energy of the gas (dE). Since volume of the gas is constant, no work is done in moving the piston.

∴ $dQ_1 = dE = C_V\, dT$ ….(1)

where C_V is the molar specific heat of the gas at constant volume.

iv. On the other hand, if the gas is heated to the same temperature, at constant pressure, volume of the gas increases by an amount say dV. The amount of heat supplied to the gas is used to increase the internal energy of the gas as well as to move the piston backwards to allow expansion of gas. The work done to move the piston dW = PdV.

∴ $dQ_2 = dE + dW = C_P dT$ ….(2)

Where, C_P is the molar specific heat of the gas at constant pressure.

v. From equations (1) and (2),

∴ $C_P\, dT = C_V\, dT + dW$

∴ $(C_P - C_V)\, dT = PdV$ ….(3)

vi. For one mole of gas,

$PV = RT$

∴ $P\, dV = R\, dT$, since pressure is constant.

Substituting equation (3), we get

$(C_P - C_V)\, dT = R\, dT$

∴ $C_P - C_V = R$

This is known as Mayer's relation between C_P and C_V.

vii. Also $C_P = M_0S_P$ and $C_V = M_0S_V$, where M_0 is the molar mass of the gas and S_P and S_V are respective principal specific heats. Thus,

$M_0S_P - M_0S_V = R/J$

Where, J is mechanical equivalent of heat.

∴ $S_P - S_V = \dfrac{R}{M_0 J}$

04 Thermodynamics

Note: The Ⓡ *marked questions are the part of reduced/non-evaluative portion for academic year 2020-21 only.*

Multiple Choice Questions (1 Mark Each)

1. Which of the following is correct?
   When the energy is transferred to a system from its environment,
   (A) system gains energy.
   (B) system loses energy.
   (C) system releases energy.
   (D) system does not exchange energy.

2. Which of the following system freely allows exchange of energy and matter with its environment?
   (A) Closed (B) Isolated
   (C) Open (D) Partially closed

3. Two systems at same temperature are said to be in
   (A) chemical equilibrium **(B) thermal equilibrium**
   (C) mechanical equilibrium (D) electrical equilibrium

Ⓡ 4. For work done to be reversible, the process should be
   (A) cyclic (B) isobaric
   (C) isochoric **(D) adiabatic**

5. A gas in a closed container is heated with 10 J of energy. Causing the lid of the container to rise 2 m with 3 N of force. What is the total change in energy of the system?
   (A) 10 J **(B) 4 J**
   (C) –4 J (D) –10 J

Hint: Given that, Q = 10 J

Work done (W) = Force × displacement

= 3 × 2

= 6 J

∴ From first law of thermodynamics,

$\Delta U = |Q| - |W| = 10 - 6 = 4$ J

6. The second law of thermodynamics deals with transfer of
   (A) work done (B) energy
   (C) momentum **(D) heat**

7. Heating a gas in a constant volume container is an example of which process?
   (A) isochoric (B) adiabatic
   (C) isobaric (D) cyclic

*[**Note:** While heating a gas in a constant volume container, $\Delta V = 0$, therefore, by definition the process is isochoric.]*

Very Short Answer (VSA) (1 Mark Each)

1. When are the two objects said to be in thermal equilibrium?

Ans: When two objects are at the same temperature, they are in thermal equilibrium.

2. The science of measuring temperatures is called as?

Ans: The science of measuring temperatures is called thermometry.

3. State zeroth law of thermodynamics.

Ans: **Statement:** *If two systems are each in thermal equilibrium with a third system, they are also in thermal equilibrium with each other.*

4. What is energy associated with the random, disordered motion of the molecules of a system called as?

Ans: Energy associated with the random, disordered motion of the molecules of a system called as internal energy.

5. A group of objects that can form a unit which may have ability to exchange energy with its surrounding is called what?

Ans: A group of objects that can form a unit which may have ability to exchange energy with its surrounding is called as a thermodynamic system.

6. On what basis a thermodynamic system can be classified?

Ans: Thermodynamic systems can be classified on the basis of the possible transfer of heat and matter to environment.

7. What is a thermodynamic process?

Ans: A process by which two or more of state variables of a system can be changed is called a thermodynamic process.

8. Define heat.

Ans: ***Heat*** *is the form of energy that transfers due to temperature difference between the system and surroundings.*

9. What is the internal energy of the system, when the amount of heat Q is added to the system and the system does not do any work during the process?

Ans: The internal energy of the system is Q.

10. When does a system lose energy to its surrounding and its internal energy decreases?

Ans: When the system does some work to increase its volume, and no heat is added to it while expanding, the system loses energy to its surrounding and its internal energy decreases.

11. State first law of thermodynamics.

Ans: **Statement:** *When the amount of heat Q is added to a system, its internal energy is increased by an amount ΔU and the remaining is lost in the form of work done W on the surrounding.*

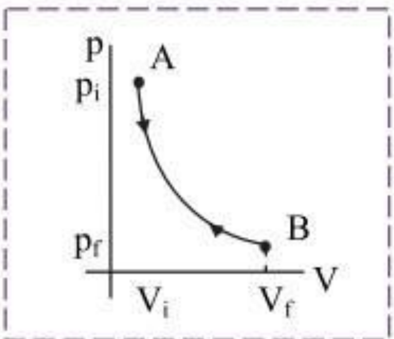
12. A system releases 100 kJ of heat while 80 kJ of work is done on the system. Calculate the change in internal energy.

Ans: $\Delta U = Q - W = 100 - 80 = 20$ kJ

Short Answer I (SA1) (2 Marks Each)

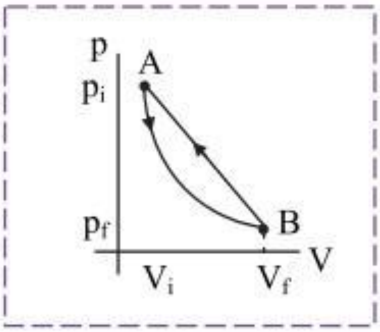
1. Draw p-V diagram of reversible process.

Ans: **p-V diagram of reversible process:**



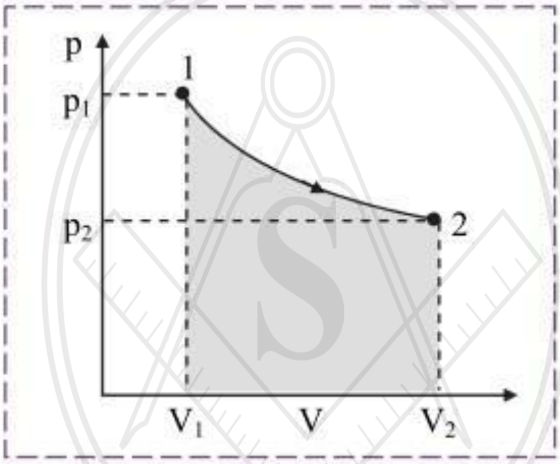
2. Draw p-V diagram of irreversible process.

Ans: p-V diagram of irreversible process:



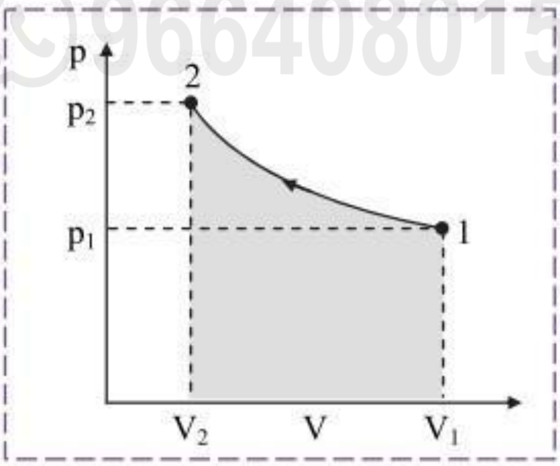
3. Draw p-V diagram showing positive work with varying pressure.

Ans: p-V diagram showing positive work with varying pressure:



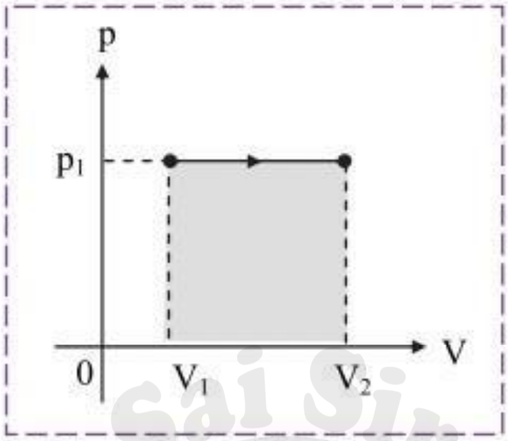
4. Draw p-V diagram showing negative work with varying pressure.

Ans: p-V diagram showing negative work with varying pressure:



5. Draw p-V diagram showing positive work at constant pressure.

Ans: p-V diagram showing positive work at constant pressure:



6. 3 mole of a gas at temperature 400 K expands isothermally from initial volume of 4 litre to final volume of 8 litre. Find the work done by the gas. (R = 8.31 J mol^{-1} K^{-1})

Solution:

Given: n = 3 mol, T = 400 K, V_i = 4 L, V_f = 8 L, R = 8.31 J mol^{-1} K^{-1}

To find: Work done by gas (W)

Formula: $W_{isothermal} = nRT \, ln \frac{V_f}{V_i}$

Calculation: From formula (i),

$$W = 3 \times 8.31 \times 400 \times ln\left(\frac{8}{4}\right)$$

Now, $ln\ x = 2.303 \times \log(x)$

$$\therefore \quad ln\left(\frac{8}{4}\right) = ln\ (2) = 2.303 \log (2)$$

$$\therefore \quad W = 1200 \times 8.31 \times 2.303 \times 0.3010$$

$= \text{antilog}\{\log(1200) + \log(8.31) + \log(2.303) + \log(0.3010)\}$

$= \text{antilog}\{3.0792 + 0.9196 + 0.3623 + (\bar{1}.4786)\}$

$= \text{antilog}\{3.8397\}$

$= 6.913 \times 10^3$ J

= **6.913 kJ**

Ans: Work done by gas is **6.913 kJ.**

7. An ideal gas of volume 2 L is adiabatically compressed to $(1/10)^{th}$ of its initial volume. Its initial pressure is 1.01×10^5 Pa, calculate the final pressure. (Given $\gamma = 1.4$)

Solution:

Given: $V_i = 2$ L, $V_f = \frac{V_i}{10} \Rightarrow \frac{Vi}{V_f} = 10$, $P_i = 1.01 \times 10^5$ Pa, $\gamma = 1.4$

To find: Final pressure (p_f)

Formula: For adiabatic process: $p_f V_f^\gamma = p_i V_i^\gamma$

Calculation: From formula,

$$p_f = p_i \times \left(\frac{V_i}{V_f}\right)^\gamma$$
$$= 1.01 \times 10^5 \times (10)^{1.4}$$
$$= \text{antilog}\{\log(1.01) + 1.4 \times \log(10)\} \times 10^5$$
$$= \text{antilog}\{0.0043 + (1.4 \times 1)\} \times 10^5$$
$$= \text{antilog}\{1.4043\} \times 10^5$$
$$= 2.537 \times 10^6 = \mathbf{25.37 \times 10^5\ Pa}$$

Ans: The final pressure (p_f) is $\mathbf{25.37 \times 10^5}$ **Pa**

8. Explain the cyclic process.

Ans:

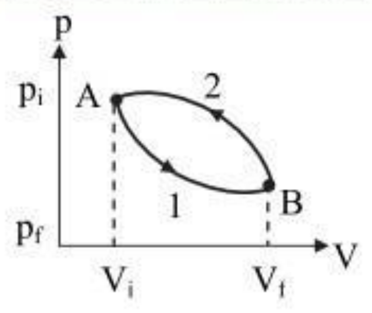
i. A thermodynamic process that returns a system to its initial state is a cyclic process.

ii. In this process, the initial and the final state is the same.

iii. For a cyclic process, the total change in the internal energy of a system is zero. ($\Delta U = 0$).

iv. According to the first law of thermodynamics, we have, for a cyclic process, Q = W

v. The given figure shows the p-V diagram of a cyclic process which is a closed loop.

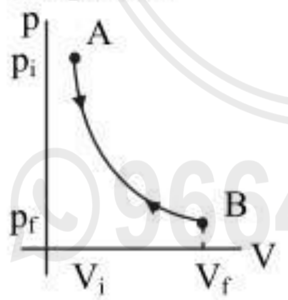
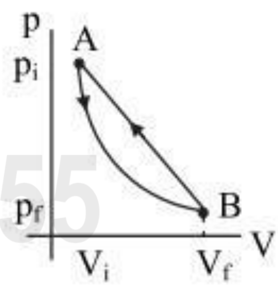


p-V diagram of cyclic process

vi. Working of all heat engines is a cyclic process.

9. Differentiate between reversible and irreversible process.

Ans:

Sr. No.	Reversible process	Irreversible process
i.	A reversible process is a change that can be retraced in reverse (opposite) direction.	An irreversible process is a change that cannot be retraced in reverse (opposite) direction.
ii.	The path of a reversible process is the same in the forward and the reverse direction.	The path of an irreversible process is not the same in the forward and the reverse direction.
iii.	Reversible changes are very slow and there is no loss of any energy in the process.	There is a permanent loss of energy from the system due to friction or other dissipative forces in an irreversible process.
iv.	The system comes back to its initial state after it is taken along the reverse path.	The change of state depends upon the path taken to change the state during an irreversible process.
v.	Reversible processes are ideal processes.	Irreversible processes are real processes.
vi.	**p-V diagram:** 	**p-V diagram:** 

[Any four points]

10. State the assumptions made for thermodynamic processes.

Ans: Assumptions made for studying various thermodynamic processes:

i. Majority of the thermodynamic processes are reversible. That is, they are quasi-static in nature. They are extremely slow and the system undergoes infinitesimal change at every stage except the adiabatic processes. The system is, therefore, in thermodynamic equilibrium during all the change.

ii. The system involved in all the processes is an ideal gas enclosed in a cylinder having a movable, frictionless, and massless piston.

iii. The ideal gas equation is applicable to the system.

Short Answer II (SA1) (3 Marks Each)

1. Classify and explain thermodynamic system.

Ans: Thermodynamic systems can be classified as open, closed and isolated systems on the basis of the possible transfer of heat and matter to environment.

i. An **open system** is a system that freely allows exchange of energy and matter with its environment.

ii. A **closed system** does not allow the exchange of matter but allows energy to be transferred.

iii. An **isolated system** is completely sealed (isolated from its environment). Matter as well as heat cannot be exchanged with its environment.

2. Explain given cases related to energy transfer between the system and surrounding –

i. energy transferred (Q) > 0

ii. energy transferred (Q) < 0

iii. energy transferred (Q) = 0

Ans: Consider a system with temperature T_S is kept in an environment of temperature T_E. Let Q be the energy transferred between the system and the environment.

i. When the temperature of the system is less than that of environment ($T_S < T_E$), the energy flows into the system as shown in figure (a).

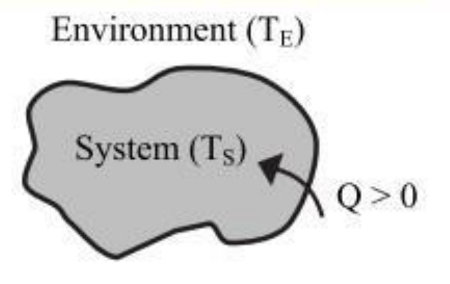


Figure (a): $T_S < T_E$

As a result, the system gains energy and Q is positive.

ii. When $T_S > T_E$, the system loses energy i.e., the energy flows from system into the environment as shown in figure (b).

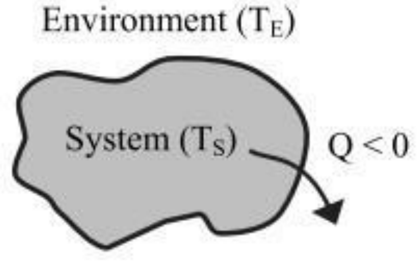


Figure (b): $T_S > T_E$

In this case, Q is negative.

iii. For $T_S = T_E$, as shown in figure (c), the system and the environment are in thermal equilibrium and there is no transfer of energy i.e., Q = 0.

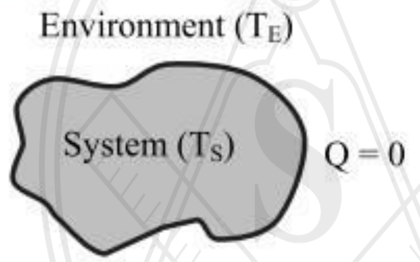


Figure (c): $T_S = T_E$

3. Explain the different ways through which internal energy of the system can be changed.

Ans:

i. Internal energy of a system can be changed by changing temperature of the system:

a. If the system is placed in environment which is at temperature lower than the system, i.e., $T_S > T_E$, the energy is transferred from the system to the environment causing decrease in the internal energy of the system.

b. If the system is placed in environment which is at temperature higher than the system, i.e., $T_E > T_S$, the energy is transferred from the environment to the system causing increase in the internal energy of the system.

ii. Internal energy of a system can be changed by doing some work:

a. When some work is done on the system by the environment, the system gains the energy and its temperature increases causing increase in internal energy.

b. When some work is done by the system on the environment, the system loses the energy and its temperature decreases causing decrease in internal energy.

4. Write a note on thermodynamic equilibrium.

Ans:

i. A system is said to be in thermodynamic equilibrium when the following three conditions of equilibrium are satisfied simultaneously: mechanical equilibrium, chemical equilibrium, and thermal equilibrium.

ii. Mechanical equilibrium:

a. For a system to be in mechanical equilibrium, there should not be any unbalanced forces acting within the system and between the system and its surrounding.

b. Also, the pressure in the system should be same throughout the system and should not change with time.

iii. Chemical equilibrium:

a. For a system to be in chemical equilibrium there should be no chemical reactions going on within the system.

b. There is no transfer of matter from one part of the system to the other due to diffusion.

c. A system to be in chemical equilibrium its chemical composition has to be same throughout and should not change with time.

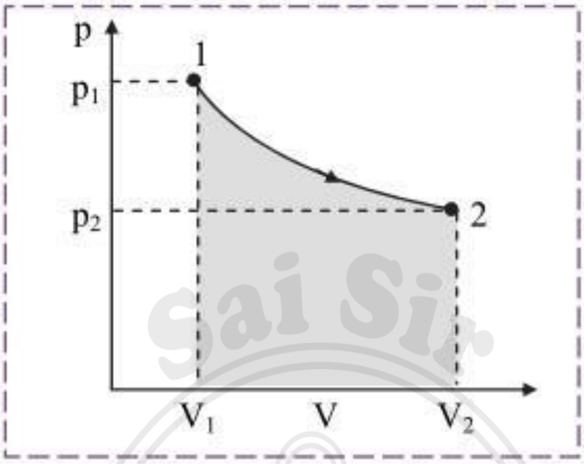
iv. Thermal equilibrium:

For a system to be in thermal equilibrium, the temperature of the system should be uniform throughout and it should not change with time. A system when in thermal equilibrium is described in terms of state variables.

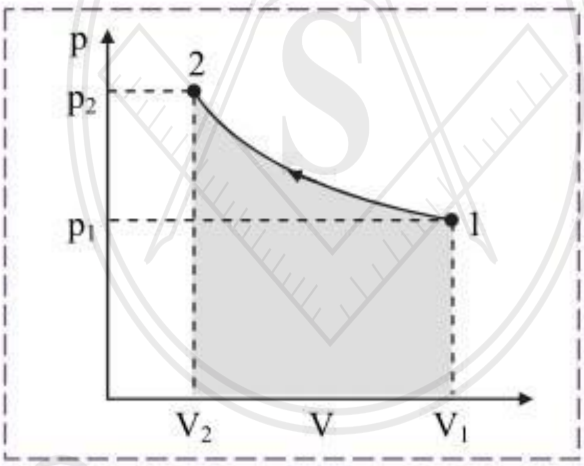
5. Explain graphically (i) positive work with varying pressure, (ii) negative work with varying pressure and (iii) positive work at constant pressure.

Ans:

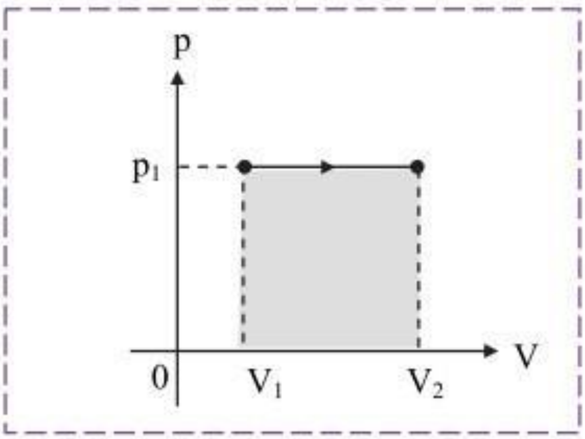
i. p-V diagram showing positive work with varying pressure:



ii. p-V diagram showing negative work with varying pressure:



iii. p-V diagram showing positive work at constant pressure:



6. Write a note on free expansion.

Ans:

i. Free expansions are adiabatic expansions thus there is no exchange of heat between a system and its environment.

ii. Also, there is no work done on the system or by the system. Q = W = 0, and according to the first law of thermodynamics, $\Delta U = 0$.

iii. For example, when a balloon is ruptured suddenly, or a tyre is suddenly punctured, the air inside rushes out rapidly but there is no displacement of a piston or any other surface.

iv. Free expansion is different than other thermodynamic processes because it is an uncontrolled change. It is an instantaneous change and the system is not in thermodynamic equilibrium.

v. A free expansion cannot be plotted on a p-V diagram. Only its initial and the final state can be plotted.

7. One gram of water (1 cm^3) becomes 1671 cm^3 of steam at a pressure of 1 atm. The latent heat of vaporization at this pressure is 2256 J/g. Calculate the external work and the increase in internal energy.

Solution:

Given: $m = 1$ g, $L_{vap} = 2256$ J/g,
$p = 1.01 \times 10^5$ Pa, T = 100 °C = 373 K,
$V_{steam} = 1671$ cm^3, $V_{liq} = 1$ cm^3

$\therefore$ $\Delta V = (V_{steam} - V_{liq}) = (1671 - 1) = 1670 \text{ cm}^3 = 1670 \times 10^{-6} \text{ m}^3$

To find: i. External work done by the system (W)
ii. Increase in internal energy (ΔU)

Formulae: i. $Q = mL$
ii. $W = p\Delta V$
iii. $\Delta U = Q - W$

Calculation: From formula (i),

$Q = 1 \times 2256 = 2256$ J

From formula (ii),

$W = (1.01 \times 10^5) \times 1670 \times 10^{-6}$

$= 1687 \times 10^{-1}$ J

$\approx$ **169 J**

From formula (iii),

$\Delta U = 2256 - 169 =$ **2087 J**

Ans: i. External work done by the system is **169 J.**

iii. Increase in the internal energy is **2087 J.**

8. Calculate the fall in temperature of helium initially at 15 °C when it is suddenly expanded to 8 times its original volume ($\gamma = 5/3$).

Solution:

Given: $T_i = 15\ °C = 15 + 273 = 288\ K$

$\gamma = \frac{5}{3}$, $V_f = 8\ V_i$

To find: Fall in temperature (ΔT)

Formulae: $T_f V_f^{\gamma-1} = T_i V_i^{\gamma-1}$

Calculation: From formula,

$$T_f = T_i\left(\frac{V_i}{V_f}\right)^{\gamma-1}$$

$$= 288\left(\frac{1}{8}\right)^{\frac{5}{3}-1}$$

$$= 288\left(\frac{1}{8}\right)^{\frac{2}{3}}$$

$$= 288 \times \frac{1}{4}$$

$$= 72\ K$$

$$= 72 - 273$$

$$= -201\ °C$$

$\therefore$ $\Delta T = T_f - T_i$

$= -201 - 15$

$= \mathbf{-216\ °C}$

Ans: Fall in the temperature (ΔT) is **–216 °C**.

9. A cylinder containing one gram molecule of the gas was compressed adiabatically until its temperature rose from 27 °C to 97 °C. Calculate the work done and heat produced in the gas ($\gamma = 1.5$).

Solution:

Given: $n = 1, \gamma = 1.5$

$T_f - T_i = 97 - 27 = 70\ °C$

We know, $R = 8.31$ J/mol K

To find: i. Work done (W)

ii. Heat produced (Q)

Formula: $W = \dfrac{nR(T_f - T_i)}{(1-\gamma)}$

Calculation: From formula,

$$W = \frac{1 \times 8.31 \times 70}{1-1.5}$$

$$= \mathbf{-11.63 \times 10^2\ J}$$

As work done on the gas is converted into heat, rising temperature of the gas,

heat produced, $Q = \dfrac{11.63 \times 10^2}{4.18}$ cal ≈ **278 cal**

Ans: i. Work done is **-11.63×10^2 J.**

ii. Heat produced in the gas **278 cal.**

[Note:** While calculating heat produced in calories, standard value of conversion factor is considered.**]

Long Answer (LA) (4 Marks Each)

1. State first law of thermodynamics and derive the relation between the change in internal energy (ΔU), work done (W) and heat (Q).

Ans: Statement: *When the amount of heat Q is added to a system, its internal energy is increased by an amount ΔU and the remaining is lost in the form of work done W on the surrounding.*

Relation between the change in internal energy (ΔU), work done (W) and heat (Q):

i. When the amount of heat Q is added to the system and the system does not do any work during the process, its internal energy increases by the amount, $\Delta U = Q$.

ii. When the system does some work to increase its volume, and no heat is added to it while expanding, the system loses energy to its surrounding and its internal energy decreases.

∴ $\Delta U = -W$.

iii. As, the internal energy can be changed using both the ways, we can consider the total change in the internal energy as,

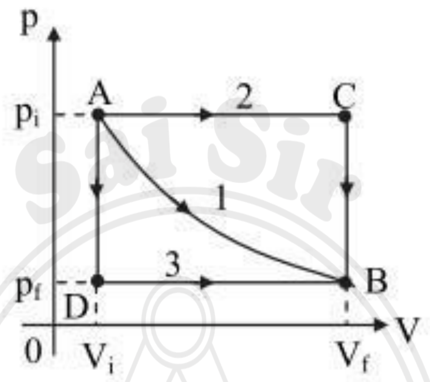
$\Delta U = Q - W$ ....(1)

This is the mathematical statement of the first law of thermodynamics.

2. Explain work done during a thermodynamic process.

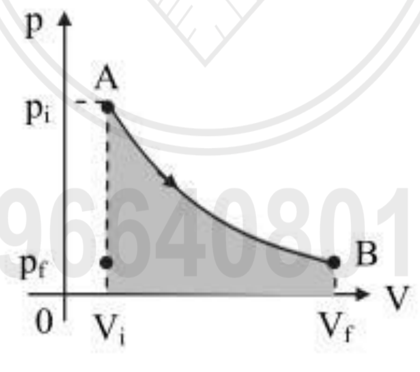
Ans:

i. Consider the system is initially at state A with its pressure is p_i and volume is V_i. Let the state be indicated as coordinates (V_i, p_i). It can attain final state (V_f, p_f) along different possible paths as shown in the p-V diagram below.



Different ways to change a system

ii. Consider a system changes its state from initial state A to final state B via path 1 as shown in figure.



Pressure and volume both change

a. When system changes itself from A to B, both its pressure and volume change. The pressure decreases while volume increases.

b. The work done by the system is given by the area under the curve. It is positive when the volume increases (as shown in figure) or negative when the volume decreases.

iii. Consider the system changes its state from A to B via path 2 as shown in figure (a).

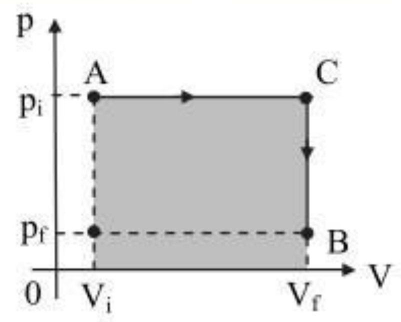


Figure (a)

a. In this case, the volume increases to V_f from the point A up to the point C at the constant pressure p_i.

b. After point C, the pressure of the system decreases to p_f at constant volume as shown in the figure (a).

c. The system thus, reaches its final state B with co-ordinates (V_f, p_f). Work done in this process is represented by the shaded area under the curve in figure (a).

iv. Consider the system changes its state from A to B via path 3 as shown in figure (b).

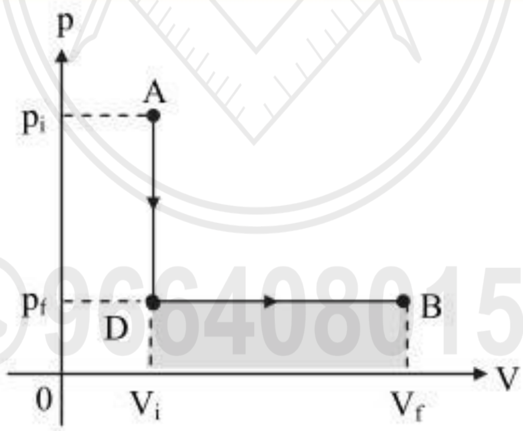


Figure (b)

a. In this case, the pressure decreases from p_i to p_f at constant volume V_i along the path AD.

b. After point D, the volume of the system increases to V_f at constant pressure p_f as shown in figure (b).

c. Work done in this process is represented by the shaded area under the curve in figure (b).

v. From figures (a) and (b) we can conclude that the work done is more when the system follows path ACB than the work done by the system along the path ADB.

Thus, the work done by a system in a thermodynamic process depends not only on the initial and the final states, but also on the intermediate states, i.e., on the paths along which the change takes place.

3. Explain thermodynamics of isobaric process.

Ans:

i. A thermodynamic process which is carried out at constant pressure i.e., $\Delta p = 0$ is called isobaric process.

ii. For an isobaric process, none of the quantities ΔU, Q and W is zero.

iii. Temperature of system changes, i.e., $\Delta T \neq 0$.

iv. Energy exchanged is used to do work as well as to change internal energy causing increase in temperature. Thus, $Q = \Delta U + W$.

v. As work is done volume changes during the process.

vi. Heat exchanged in case of an isobaric process:

a. Consider an ideal gas undergoing volume expansion at constant pressure.

b. If V_i and T_i are its volume and temperature in the initial state of a system and V_f and T_f are its final volume and temperature respectively, the work done in the expansion is given by,

$W = pdV = p(V_f - V_i) = nR(T_f - T_i)$ ….(1)

c. Also, the change in the internal energy of a system is given by,

$\Delta U = nC_V\Delta T = nC_V(T_f - T_i)$ ….(2)

Where, C_V is the specific heat at constant volume and $\Delta T = (T_f - T_i)$ is the change in its temperature during the isobaric process.

d. According to the first law of thermodynamics, the heat exchanged is given by, $Q = \Delta U + W$

Using equations (1) and (2) we get,

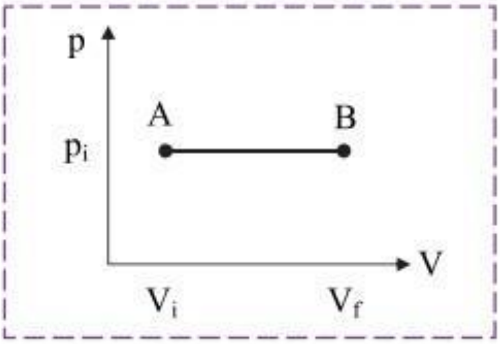
$Q = nC_V(T_f - T_i) + nR(T_f - T_i)$

$\therefore$ $Q = (nC_V + nR)(T_f - T_i)$

$\therefore$ $Q = nC_p(T_f - T_i)$ ….($\because C_p = C_V + R$)

Where, C_p is the specific heat at constant pressure.

vii. The p-V diagram for an isobaric process is called isobar. It is shown in figure below.



4. Explain thermodynamics of isochoric process.

Ans:

i. A thermodynamic process in which volume of the system is kept constant is called isochoric process.

ii. A system does no work on its environment during an isochoric process.

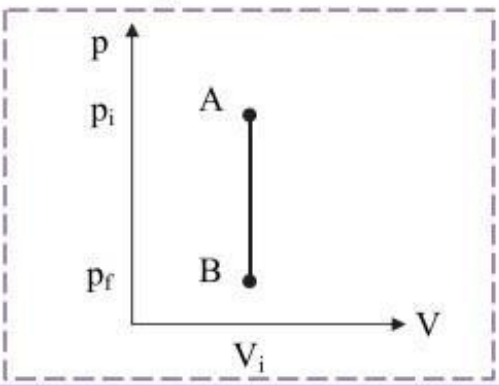
iii. For an isochoric process, $\Delta V = 0$, and from the first law of thermodynamics, $\Delta U = Q$.

iv. Temperature of the system changes,
i.e., $\Delta T \neq 0$.

v. This means that for an isochoric change, all the energy added in the form of heat remains in the system itself and causes an increase in its internal energy. Also, as volume is unchanged, no work is done.

vi. The first law of thermodynamics for isochoric process is,
$Q = \Delta U$ ….(1)
The change in internal energy is given by,
$\Delta U = nC_V\Delta T$ ….(2)
The work done is given by,
$W = p\Delta V = 0$ ….($\because \Delta V = 0$)
From equations (1) and (2),
The heat exchanged is given by,
$Q = \Delta U = nC_V\Delta T$

vii. p-V diagram of isochoric process is as shown below:



5. Explain thermodynamics of adiabatic process.

Ans:

i. Adiabatic process is a process during which there is no transfer of heat from or to the system.

ii. For adiabatic change, $Q = 0$.

$\therefore$ $\Delta U + W = 0$

$\therefore$ $\Delta U = -W$

iii. This implies, for adiabatic expansion, W is positive and ΔU is negative i.e., when work is done by the system adiabatically its internal energy decreases.

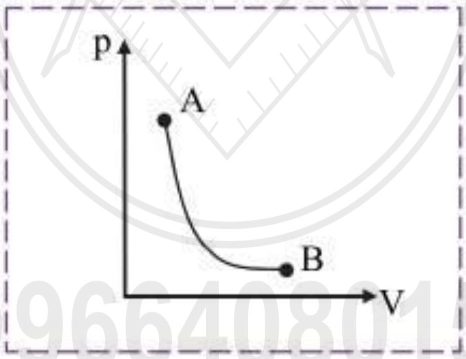
iv. Similarly, when a system is compressed adiabatically, W is negative and ΔU is positive i.e., the internal energy of the system increases in an adiabatic compression.

v. Heat transfer to and from system is prevented by either perfectly insulating the system from its environment or by carrying out the change rapidly so that there is no time for any exchange of heat.

vi. Temperature of system changes, i.e., $\Delta T \neq 0$

vii. Usually, it is a sudden change and system does not find any time to exchange heat with its environment.

viii. p-V diagram for adiabatic process is as shown below:



05 Oscillations

Multiple Choice Questions (1 Mark Each)

1. A particle is moving in a circle with uniform speed. Its motion is
   (A) Periodic and simple harmonic
   (B) Non periodic
   (C) Periodic but not simple harmonic
   (D) Non periodic but simple harmonic

2. A particle is performing simple harmonic motion with amplitude A and angular velocity ω. The ratio of maximum velocity to maximum acceleration is
   (A) ω **(B) $1/\omega$**
   (C) ω^2 (D) A/ω

Hint: $v_{max} = A\omega$

$a_{max} = A\omega^2$

$$\therefore \quad \frac{v_{max}}{a_{max}} = \frac{1}{\omega}$$

3. Acceleration of a particle executing S.H.M. at its mean position.
   (A) Is infinity (B) Varies
   (C) Is maximum **(D) Is zero**

4. In a second's pendulum, mass of bob is 50 g. If it is replaced by 100 g mass, then its period will be
   (A) 1 s **(B) 2 s**
   (C) 3 s (D) 4 s

Hint: Time period of second's pendulum is 2 seconds irrespective to mass.

5. The maximum speed of a particle executing S.H.M. is 10 m/s and maximum acceleration is 31.4 m/s^2. Its periodic time is
   (A) 1 s **(B) 2 s**
   (C) 4 s (D) 6 s

Hint: $v_{max} = A\omega = 10$ m/s

$a_{max} = A\omega^2 = 31.4$ m/s^2

∴ $\frac{a_{max}}{v_{max}} = \omega = 3.14$ rad/s

∴ $T = \frac{2\pi}{\omega} = \frac{2 \times 3.14}{3.14} = 2$ second

6. When the displacement of a simple harmonic oscillator is half of its amplitude, its P.E. is 3 J. Its total energy is
(A) 6 J (B) **12 J**
(C) 15 J (D) 20 J

Hint: When $x = \frac{A}{2}$, P.E. $= \frac{1}{2}kx^2 = 3$ J

$$\text{T.E.} = \frac{1}{2}kA^2 = \frac{1}{2}k(2x)^2 = 4 \times \frac{1}{2}kx^2 = 4 \times 3 = 12 \text{ J}$$

7. Two S.H.M.'s have zero phase difference and equal amplitudes A. The resultant amplitude on their composition will be
(A) **2 A** (B) zero
(C) $\sqrt{2}$ A (D) $\sqrt{2A}$

Very Short Answer (VSA) (1 Mark Each)

1. A simple pendulum moves from one end to the other in $\frac{1}{4}$ second. What is its frequency?

Ans: Time taken by simple pendulum to move from one end to the other is $\frac{T}{2}$ second.

Given: $\frac{T}{2} = \frac{1}{4}$ second

∴ $T = \frac{1}{2}$ second

∴ Frequency, $n = \frac{1}{T} = 2$ Hz

2. A particle executes S.H.M. of 2 cm. At the extreme position, the force is 4 N. What is the force at a point midway between mean and extreme positions?

Ans: Since F = kx ...(in magnitude)

$\therefore \quad \frac{F_2}{F_1} = \frac{x_2}{x_1}$

$\therefore \quad F_2 = \frac{1}{2} \times 4$ ...($x_1 = A = 2$ cm, $x_2 = \frac{A}{2} = 1$ cm, $F_1 = 4$ N)

$\therefore \quad = 2$ N

3. A simple pendulum is inside a spacecraft. What will be its periodic time?

Ans: Periodic time of simple pendulum is given by, $T = 2\pi\sqrt{\frac{L}{g}}$

But, inside a spacecraft, acceleration due to gravity is zero.

$\therefore \quad T = 2\pi\sqrt{\frac{L}{0}}$

$\therefore \quad T = \infty$

$\therefore$ Time period of simple pendulum inside a spacecraft is infinite.

4. What is amplitude of S.H.M.?

Ans: *The maximum displacement of a particle performing S.H.M. from its mean position is called the* ***amplitude of S.H.M.***

5. What is seconds pendulum?

Ans: *A simple pendulum whose period is two seconds is called* ***second's pendulum.***

6. State the formula for frequency of S.H.M in terms of force constant.

Ans: Frequency of S.H.M. is given by, $n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

7. What does the phase of π/2 indicate in linear S.H.M.?

Ans: In linear S.H.M, the phase π/2 indicate that the particle is at the positive extreme position during first oscillation.

Short Answer I (SA1) (2 Marks Each)

1. A particle is performing S.H.M. of amplitude 5 cm and period of 2 s. Find the speed of the particle at a point where its acceleration is half of its maximum value.

Solution:

Given: $A = 5 \text{ cm} = 5 \times 10^{-2} \text{ m}, T = 2 \text{ s},$

$$a = \frac{a_{max}}{2} = \frac{A\omega^2}{2} \quad \text{....(i)}$$

To find: speed (v)

Formula: $v = \omega\sqrt{A^2 - x^2}$

Calculation: Since, $a = \omega^2 x$

$$\therefore \quad x = \frac{a}{\omega^2} = \frac{A\omega^2}{2\omega^2} = \frac{A}{2} \quad \text{....[from (i)]}$$

From formula,

$$v = \omega\sqrt{A^2 - \frac{A^2}{4}} = \frac{\sqrt{3}}{2}A\omega$$

$$= \frac{\sqrt{3}}{2} \times 5 \times 10^{-2} \times \frac{2\pi}{T} \quad \text{....}\left[\because \omega = \frac{2\pi}{T}\right]$$

$$= \frac{\sqrt{3}}{2} \times 5 \times 10^{-2} \times \frac{2 \times 3.14}{2}$$

$\therefore$ $v = \mathbf{13.6 \times 10^{-2}\ m/s}$

Ans: The speed of the particle where its acceleration is half of its maximum value is **13.6×10^{-2} m/s.**

2. The acceleration due to gravity on the surface of moon is 1.7 m/s^2. What is the time period of a simple pendulum on the surface of moon if its time period on the surface of earth is 3.5 s?
(g on the surface of earth = 9.8 m/s^2)

Solution:

Given: $g_m = 1.7 \text{ m/s}^2, g_E = 9.8 \text{ m/s}^2, T_E = 3.5 \text{ s}$

To find: Time period on the surface of moon (T_m)

Formula: $T = 2\pi\sqrt{\frac{L}{g}}$

Calculation: From formula,

$$\frac{T_m}{T_E} = \sqrt{\frac{g_E}{g_m}}$$

$$\therefore \quad T_m = \sqrt{\frac{9.8}{1.7}} \times 3.5$$

$$= \sqrt{\frac{49}{8.5}} \times 3.5$$

$$= \frac{7 \times 3.5}{\sqrt{8.5}}$$

$$= \mathbf{8.4\ s}$$

Ans: Time period of a simple pendulum on the surface of moon is **8.4 s**.

3. The total energy of a body of mass 2 kg performing S.H.M. is 40 J. Find its speed while crossing the centre of the path.

Solution:

Given: m = 2 kg, T.E. = 40 J

To find: Speed while crossing the mean position (v_{max})

Formula: $T.E. = \frac{1}{2} m v_{max}^2$

Calculation: From formula,

$$v_{max} = \sqrt{\frac{2 \times T.E.}{m}}$$

$$= \sqrt{\frac{2 \times 40}{2}}$$

$$= 2\sqrt{10}$$

$$= 2 \times 3.162$$

$$= \mathbf{6.324\ m/s}$$

Ans: Speed of particle while crossing the mean position is **6.324 m/s.**

4. Derive differential equation of linear S.H.M.

Ans:

i. In a linear S.H.M., the force is directed towards the mean position and its magnitude is directly proportional to the displacement of the body from mean position.

$\therefore$ $f \propto -x$

$\therefore$ $f = -kx$ ….(1)

where, k is force constant and x is displacement from the mean position.

ii. According to Newton's second law of motion,

$f = ma$ ....(2)

From equations (1) and (2),

$ma = -kx$ ....(3)

iii. The velocity of the particle is given by, $v = \frac{dx}{dt}$

∴ Acceleration, $a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$ ....(4)

Substituting equation (4) in equation (3),

$$m\frac{d^2x}{dt^2} = -kx$$

∴ $\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$

iv. Substituting $\frac{k}{m} = \omega^2$, where ω is the angular frequency,

∴ $\frac{d^2x}{dt^2} + \omega^2 x = 0$

This is the differential equation of linear S.H.M.

5. Define linear S.H.M.

Ans: ***Linear S.H.M.*** *is defined as the linear periodic motion of a body, in which force (or acceleration) is always directed towards the mean position and its magnitude is proportional to the displacement from the mean position.*

6. State any two laws of simple pendulum.

Ans: Laws of simple pendulum:

i. The period of a simple pendulum is directly proportional to the square root of its length.

∴ $T \propto \sqrt{L}$ ....(when g = constant)

ii. The period of a simple pendulum is inversely proportional to the square root of acceleration due to gravity.

∴ $T \propto \frac{1}{\sqrt{g}}$ ....(when L = constant)

iii. The period of a simple pendulum does not depend on its mass.

iv. The period of a simple pendulum does not depend on its amplitude (for small amplitude).

[Any two laws]

Short Answer II (SA1) (3 Marks Each)

1. The period of oscillation of simple pendulum increases by 20% , when its length is increased by 44 cm. find its initial length.

Solution:

Given: $T_2 = \left(1 + \frac{20}{100}\right) T_1$

$\therefore \quad \frac{T_2}{T_1} = \frac{120}{100}, L_2 = (L_1 + 0.44)$ m

To find: Initial length (L_1)

Formula: $T = 2\pi\sqrt{\frac{L}{g}}$

Calculation: From formula,

$T_1 = 2\pi\sqrt{\frac{L_1}{g}}$ ....(1)

$T_2 = 2\pi\sqrt{\frac{L_2}{g}}$ ....(2)

Dividing equation (1) by equation (2),

$\therefore \quad \frac{T_1}{T_2} = \sqrt{\frac{L_1}{L_2}}$

$\therefore \quad \frac{100}{120} = \sqrt{\frac{L_1}{L_2}}$ ....(Given)

$\therefore \quad \frac{10}{12} = \sqrt{\frac{L_1}{L_1 + 0.44}}$

$\therefore \quad 1.44\ L_1 = L_1 + 0.44$

$\therefore \quad L_1 = \mathbf{1\ m}$

Ans: Initial length of pendulum is **1 m.**

2. A particle performing S.H.M. has velocities of 8 cm/s and 6 cm/s at displacements of 3 cm and 4 cm respectively. Calculate the amplitude and period of S.H.M.

Solution:

Given: $v_1 = 8$ cm/s, $v_2 = 6$ cm/s, $x_1 = 3$ cm, $x_2 = 4$ cm

To find: Amplitude (A), Period (T)

Formula: $v = \omega\sqrt{A^2 - x^2}$

Calculation: From the given condition,

$v_1^2 = \omega^2 (A^2 - x_1^2)$

$\therefore \quad 64 = \omega^2 (A^2 - 9) \quad ....(i)$

Also, $v_2^2 = \omega^2 (A^2 - x_2^2)$

$\therefore \quad 36 = \omega^2 (A^2 - 16) \quad ....(ii)$

Dividing (i) by (ii),

$$\frac{64}{36} = \frac{A^2 - 9}{A^2 - 16}$$

$$\therefore \quad \frac{16}{9} = \frac{A^2 - 9}{A^2 - 16}$$

$\therefore \quad 16A^2 - 256 = 9A^2 - 81$

$\therefore \quad 7A^2 = 175$

$\therefore \quad A =$ **5 cm**

Substituting value of A in equation (i), we get,

$64 = \omega^2 (25 - 9) = 16\,\omega^2$

$\therefore \quad \omega^2 = 4$

$\therefore \quad \omega = 2$ rad/s

$$\therefore \quad \frac{2\pi}{T} = 2$$

$\therefore \quad T = \pi$ s = **3.14 s**

Ans: The amplitude and period of S.H.M. of the particle are **5 cm** and **3.14 s** respectively.

3. A particle performs linear S.H.M. of period 4 seconds and amplitude 4 cm. Find the time taken by it to travel a distance of 1 cm from the positive extreme position.

Solution:.

Given: T = 4 s, A = 4 cm = 0.04 m,

x = 1 cm from extreme position = 4 – 1 = 3 cm = 0.03 m

To find: Time taken (t)

Formula: $x = A \sin(\omega t + \phi)$

Calculation: Particle starts from positive extreme position.

$$\therefore \quad \phi = \frac{\pi}{2}$$

From formula,

$$x = A\sin\left(\frac{2\pi t}{T}+\phi\right) \quad ....\left(\because \omega = \frac{2\pi}{T}\right)$$

$\therefore \quad 3 = 4\sin\left(\frac{2\pi t}{4}+\frac{\pi}{2}\right)$

$\therefore \quad \cos\left(\frac{2\pi}{4}\right)t = \frac{3}{4} \qquad ....[\because \sin\left(\frac{\pi}{2}+\theta\right) = \cos\theta]$

$\therefore \quad \left(\frac{\pi}{2}\right)t = \cos^{-1}\left(\frac{3}{4}\right)$

$\therefore \quad t = \frac{2}{\pi}\times 41.4°\times\frac{\pi}{180} = \frac{41.4}{90} = \mathbf{0.46\ s}$

Ans: Time taken by it to travel a distance of 1 cm from the positive extreme position is **0.46 s.**

4. Obtain an expression for resultant amplitude of , composition of two S.H.M.'s having same period along same path.

Ans:

i. Consider a particle simultaneously subjected to two S.H.M.s having the same period and along same path (let it be along the x-axis), but of different amplitudes and initial phases. The resultant displacement at any instant is equal to the vector sum of its displacements due to both the S.H.M.s at that instant.

ii. Let the two linear S.H.M's be given by equations,

$x_1 = A_1\sin(\omega t+\phi_1) \quad ....(1)$

$x_2 = A_2\sin(\omega t+\phi_2) \quad ....(2)$

where A_1, A_2 are amplitudes; ϕ_1, ϕ_2 are initial phase angles and x_1, x_2 are the displacement of two S.H.M's in time 't'. ω is same for both S.H.M's.

iii. The resultant displacement of the two S.H.M's is given by,

$x = x_1 + x_2 \quad ....(3)$

Using equations (1) and (2) , equation (3) can be written as,

$x = A_1\sin(\omega t+\phi_1) + A_2\sin(\omega t+\phi_2)$

$= A_1[\sin\omega t\cos\phi_1 + \cos\omega t\sin\phi_1] + A_2[\sin\omega t\cos\phi_2 + \cos\omega t\sin\phi_2]$

$= A_1\sin\omega t\cos\phi_1 + A_1\cos\omega t\sin\phi_1 + A_2\sin\omega t\cos\phi_2 + A_2\cos\omega t\sin\phi_2$

$= [A_1\sin\omega t\cos\phi_1 + A_2\sin\omega t\cos\phi_2]$

$+ [A_1\cos\omega t\sin\phi_1 + A_2\cos\omega t\sin\phi_2]$

$\therefore \quad x = \sin\omega t[A_1\cos\phi_1 + A_2\cos\phi_2] + \cos\omega t[A_1\sin\phi_1 + A_2\sin\phi_2] \quad ....(4)$

iv. As A_1, A_2, ϕ_1 and ϕ_2 are constants, we can combine them in terms of another convenient constants R and δ as

$A_1 \cos \phi_1 + A_2 \cos \phi_2 = R \cos\delta$ ....(5)

and $A_1 \sin \phi_1 + A_2 \sin \phi_2 = R \sin \delta$ ....(6)

v. Using equations (5) and (6), equation (4) can be written as,

$x = \sin \omega t.\ R \cos \delta + \cos \omega t.R \sin \delta = R [\sin \omega t \cos \delta + \cos \omega t \sin \delta]$

$\therefore$ $x = R \sin (\omega t + \delta)$ ....(7)

Equation (7) is the equation of an S.H.M. of the same angular frequency (hence, the same period) but of amplitude R and initial phase δ. It shows that the combination (superposition) of two linear S.H.M.s of the same period and occurring along the same path is also an S.H.M.

vi. Resultant amplitude is,

$R = \sqrt{(R \sin \delta)^2 + (R \cos \delta)^2}$

Squaring and adding equations (5) and (6) we get,

$(A_1 \cos \phi_1 + A_2 \cos \phi_2)^2 + (A_1 \sin \phi_1 + A_2 \sin \phi_2)^2 = R^2\cos^2\delta + R^2\sin^2\delta$

$\therefore$ $A_1^2\cos^2\phi_1 + A_2^2 \cos^2\phi_2 + 2A_1 A_2 \cos \phi_1 \cos\phi_2$
$+ A_1^2 \sin^2\phi_1 + A_2^2\sin^2\phi_2 + 2A_1A_2 \sin\phi_1 \sin\phi_2 = R^2 (\cos^2\delta + \sin^2\delta)$

$\therefore$ $A_1^2 (\cos^2\phi_1 + \sin^2\phi_1) + A_2^2 (\cos^2\phi_2 + \sin^2\phi_2)$
$+ 2A_1A_2 (\cos \phi_1 \cos \phi_2 + \sin\phi_1 \sin\phi_2) = R^2$

$\therefore$ $A_1^2 + A_2^2 + 2A_1A_2 \cos (\phi_1 - \phi_2) = R^2$

$\therefore$ $R = \pm\sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos(\phi_1 - \phi_2)}$ ....(8)

Equation (8) represents resultant amplitude of two S.H.M's.

5. Define angular S.H.M. and obtain its differential equation.

Ans: ***Angular S.H.M.*** *is defined as the oscillatory motion of a body in which the torque for angular acceleration is directly proportional to the angular displacement and its direction is opposite to that of angular displacement.*

i. Consider a metallic disc hanging from a rigid support, when twisted, it performs an oscillatory motion for which the restoring torque acting upon it, for angular displacement θ is,

$\tau \propto -\theta$

$\therefore$ $\tau = -c\theta$ ....(1)

ii. The constant of proportionality (c) is the restoring torque per unit angular displacement.

iii. If I is the moment of inertia of the disc, the torque acting on the disc is given by,

$\tau = I\alpha$ ....(2)

Where, α is the angular acceleration.

iv. From equations (1) and (2),
$I\alpha = -c\theta$

$\therefore$ $I\frac{d^2\theta}{dt^2} + c\theta = 0$ ....$\left(\because \alpha = \frac{d^2\theta}{dt^2}\right)$

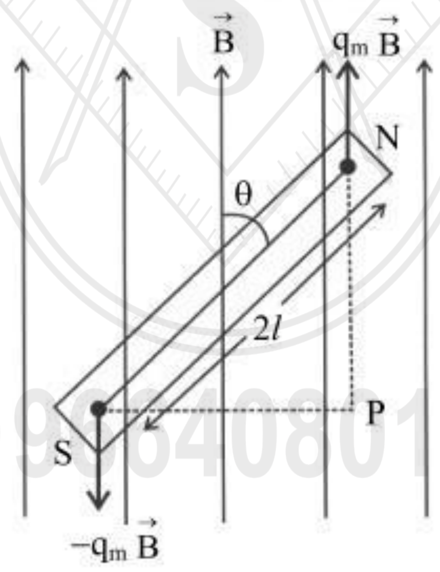
This is the differential equation for angular S.H.M.

6. Obtain the expression for the period of a magnet vibrating in a uniform magnetic field and performing S.H.M.

Ans:

i. If a bar magnet is freely suspended in the plane of a uniform magnetic field, it remains in equilibrium with its axis parallel to the direction of the field.
If it is given a small angular displacement θ about an axis passing through its centre, perpendicular to itself and to the field and released, it performs angular oscillations.

ii. Let μ be the magnetic dipole moment and B the magnetic field.
In the deflected position, a restoring torque acts on the magnet that tends to bring it back to its equilibrium position.



Magnet vibrating in a uniform magnatic field

iii. The magnitude of this torque is $\tau = \mu B\sin\theta$
If θ is small, $\sin\theta \approx \theta$

$\therefore$ $\tau = \mu B\theta$

iv. For clockwise angular displacement θ, the restoring torque is in the anticlockwise direction.

$\therefore$ $\tau = I\alpha = -\mu B\theta$

Where, I is the moment of inertia of the bar magnet and α is its angular acceleration.

$\therefore \quad \alpha = -\left(\frac{\mu B}{I}\right)\theta \qquad ....(1)$

v. Since μ, B and I are constants, equation (1) shows that angular acceleration is directly proportional to the angular displacement and directed opposite to the angular displacement. Hence the magnet performs angular S.H.M.

vi. The period of vibrations of the magnet is given by,

$$T = \frac{2\pi}{\sqrt{\text{angular acceleration per unit angular displacement}}}$$

$$= \frac{2\pi}{\sqrt{\alpha / \theta}}$$

Thus, by considering magnitude of angular acceleration (α),

$$T = 2\pi\sqrt{\frac{I}{\mu B}}$$

Long Answer (LA) (4 Marks Each)

1. Using differential equation of linear S.H.M., obtain an expression for acceleration, velocity and displacement of simple harmonic motion.

Ans:

i. **Expression for acceleration in linear S.H.M:**

a. From differential equation,

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

$$\frac{d^2x}{dt^2} = -\omega^2 x \qquad ....(1)$$

b. But, linear acceleration is given by,

$$\frac{d^2x}{dt^2} = a \qquad ....(2)$$

From equations (1) and (2),

$$a = -\omega^2 x \qquad ....(3)$$

Equation (3) gives acceleration in linear S.H.M.

ii. **Expression for velocity in linear S.H.M:**

a. From differential equation of linear S.H.M,

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

$\therefore \quad \frac{d}{dt}\left(\frac{dx}{dt}\right) = -\omega^2 x$

$\therefore \quad \frac{dv}{dt} = -\omega^2 x \qquad \ldots\left(\because \frac{dx}{dt} = v\right)$

$\therefore \quad \frac{dv}{dx} \cdot \frac{dx}{dt} = -\omega^2 x$

$\therefore \quad v\frac{dv}{dx} = -\omega^2 x \qquad \ldots\left(\because \frac{dx}{dt} = v\right)$

$\therefore \quad v\,dv = -\omega^2 x dx \qquad \ldots(4)$

b. Integrating both sides of equation (4),

$$\int v\,dv = \int -\omega^2 x dx$$

$$\frac{v^2}{2} = -\frac{\omega^2 x^2}{2} + C \qquad \ldots(5)$$

where, C is the constant of integration.

c. At extreme position, $x = \pm A$ and $v = 0$.

Substituting these values in equation (5),

$$0 = -\frac{\omega^2 A^2}{2} + C$$

$\therefore \quad C = \frac{\omega^2 A^2}{2} \qquad \ldots(6)$

d. Substituting equation (6) in equation (5),

$$\frac{v^2}{2} = -\frac{\omega^2 x^2}{2} + \frac{\omega^2 A^2}{2}$$

$\therefore \quad v^2 = \omega^2 A^2 - \omega^2 x^2$

$\therefore \quad v^2 = \omega^2(A^2 - x^2)$

$\therefore \quad v = \pm\,\omega\sqrt{A^2 - x^2}$

This is the required expression for velocity in linear S.H.M.

iii. Expression for displacement in linear S.H.M:

a. From differential equation of linear S.H.M, velocity is given by,

$v = \omega\sqrt{A^2 - x^2}$ ….(1)

But, in linear motion, $v = \frac{dx}{dt}$ ….(2)

From equation (1) and (2),

$\frac{dx}{dt} = \omega\sqrt{A^2 - x^2}$

∴ $\frac{dx}{\sqrt{A^2 - x^2}} = \omega\, dt$ ….(3)

b. Integrating both sides of equation (3),

$\int \frac{dx}{\sqrt{A^2 - x^2}} = \int \omega\, dt$

∴ $\sin^{-1}\left(\frac{x}{A}\right) = \omega t + \phi$

where, α is constant of integration which depends upon initial condition (phase angle)

∴ $\frac{x}{A} = \sin(\omega t + \phi)$

∴ $x = A\sin(\omega t + \phi)$

This is required expression for displacement of a particle performing linear S.H.M. at time t.

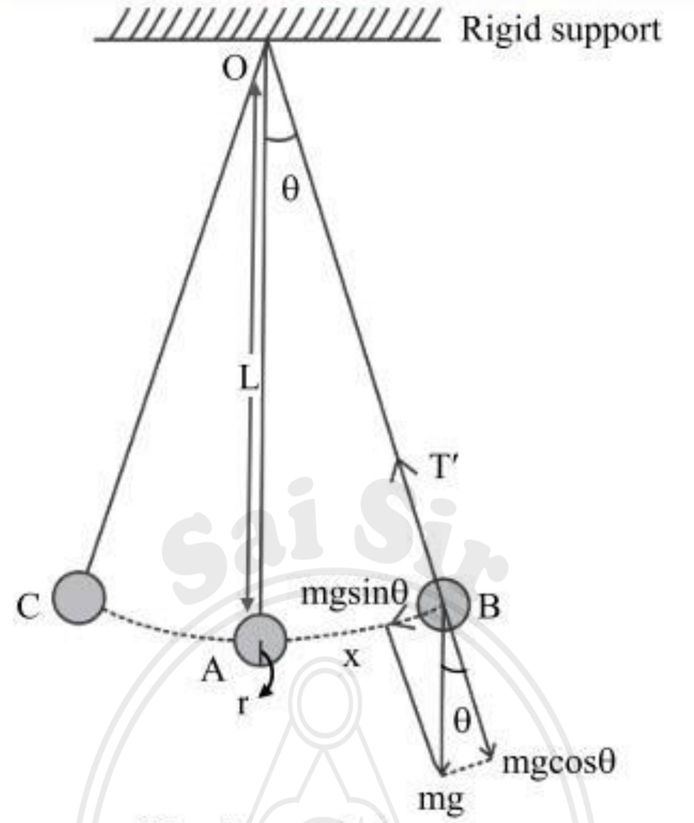
2. Define ideal simple pendulum and obtain an expression for its periodic time.

Ans: ***An ideal simple pendulum*** *is a heavy particle suspended by a massless, inextensible, flexible string from a rigid support.*

Expression for time period of simple pendulum:

i. Let 'm' be the mass of the bob and T′ be the tension in the string. The pendulum remains in equilibrium in the position OA, with the centre of gravity of the bob, vertically below the point of suspension O.

ii. If now the pendulum is displaced through a small angle θ, called angular amplitude, and released, it begins to oscillate on either side of the mean (equilibrium) position in a single vertical plane.



Simple pendulum

iii. In the displaced position (extreme position), two forces are acting on the bob.

a. Force T′ due to tension in the string, directed along the string, towards the support.

b. Weight mg, in the vertically downward direction.

iv. At the extreme positions, there should not be any net force along the string.

v. The component of mg can only balance the force due to tension.
Thus, weight mg is resolved into two components;

a. The component mg cosθ along the string, which is balanced by the tension T′.

b. The component mg sinθ perpendicular to the string is the restoring force acting on mass m tending to return it to the equilibrium position.

∴ Restoring force, $F = -mg\sin\theta$

vi. As θ is very small ($\theta < 10°$),
$\sin\theta \approx \theta^c$

∴ $F \approx -mg\theta$
From the figure,

For small angle, $\theta = \frac{x}{L}$

∴ $F = -mg\frac{x}{L}$ ....(1)

As m, g and L are constant, $F \propto -x$

vii. Thus, for small displacement, the restoring force is directly proportional to the displacement and is oppositely directed. Hence the bob of a simple pendulum performs linear S.H.M. for small amplitudes.

viii. The period T of oscillation of a pendulum is given by,

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\text{acceleration per unit displacement}}}$$

Using equation (1),

$$F = -mg\frac{x}{L}$$

∴ $ma = -mg\frac{x}{L}$ ....($\because$ F = ma)

∴ $a = -g\frac{x}{L}$

∴ $\frac{a}{x} = -\frac{g}{L} = \frac{g}{L}$ (in magnitude)

Substituting in the expression for T,

$$T = 2\pi\sqrt{\frac{L}{g}}$$

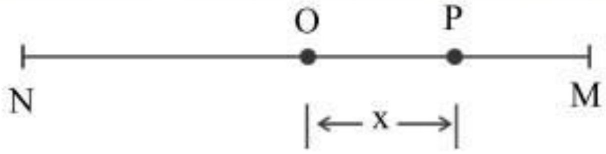
This gives the expression for the time period of a simple pendulum.

3. Deduce the expression for kinetic energy, potential energy and total energy of a particle performing S.H.M. State the factors on which total energy depends.

Ans:

i. **Expression for kinetic energy:**

a. Consider a particle of mass m, performing a linear S.H.M. along the path MN about the mean position O as shown in figure.



Energy in an S.H.M.

b. At a given instant, let the particle be at P, at a distance x from O.

c. Velocity of the particle in S.H.M. is given as

$$v = \omega\sqrt{A^2 - x^2} = A\omega\cos(\omega t + \phi)$$

where x is the displacement of the particle performing S.H.M. and A is the amplitude of S.H.M.

d. Thus, the kinetic energy,

$$E_k = \frac{1}{2}mv^2$$

$$= \frac{1}{2}m\omega^2\left(A^2 - x^2\right) \quad \text{....(1)}$$

$$= \frac{1}{2}k\left(A^2 - x^2\right)$$

This is the kinetic energy at displacement x.

e. Also, at time t, kinetic energy is,

$$E_k = \frac{1}{2}mv^2 = \frac{1}{2}mA^2\omega^2\cos^2(\omega t + \phi)$$

$$= \frac{1}{2}kA^2\cos^2(\omega t + \phi)$$

Thus, with time, it varies as $\cos^2\theta$.

ii. **Expression for potential energy:**

a. The restoring force acting on the particle at point P is given by, f = –kx where k is the force constant.

b. Suppose that the particle is displaced further by an infinitesimal displacement 'dx' against the restoring force 'f'.

c. The external work done (dW) during this displacement is

$dW = f(-dx) = -kx(-dx) = kxdx$

d. The total work done on the particle to displace it from O to P is given by, $W = \int_0^x dW = \int_0^x kx\,dx = \frac{1}{2}kx^2$

e. This work done is stored as the potential energy (P.E.) E_p of the particle at displacement x.

$$\therefore \quad E_p = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2x^2 \quad \text{....(2)}$$

f. At time t,

$$\therefore \quad E_p = \frac{1}{2}kx^2 = \frac{1}{2}kA^2\sin^2(\omega t + \phi) = \frac{1}{2}mA^2\omega^2\cos^2(\omega t + \phi)$$

Thus, with time, it varies as $\sin^2\theta$.

c. Velocity of the particle in S.H.M. is given as

$$v = \omega\sqrt{A^2 - x^2} = A\omega\cos(\omega t + \phi)$$

where x is the displacement of the particle performing S.H.M. and A is the amplitude of S.H.M.

d. Thus, the kinetic energy,

$$E_k = \frac{1}{2}mv^2$$

$$= \frac{1}{2}m\omega^2\left(A^2 - x^2\right) \qquad \text{....(1)}$$

$$= \frac{1}{2}k\left(A^2 - x^2\right)$$

This is the kinetic energy at displacement x.

e. Also, at time t, kinetic energy is,

$$E_k = \frac{1}{2}mv^2 = \frac{1}{2}mA^2\omega^2\cos^2(\omega t + \phi)$$

$$= \frac{1}{2}kA^2\cos^2(\omega t + \phi)$$

Thus, with time, it varies as $\cos^2\theta$.

ii. **Expression for potential energy:**

a. The restoring force acting on the particle at point P is given by, $f = -kx$ where k is the force constant.

b. Suppose that the particle is displaced further by an infinitesimal displacement 'dx' against the restoring force 'f'.

c. The external work done (dW) during this displacement is $dW = f(-dx) = -kx(-dx) = kxdx$

d. The total work done on the particle to displace it from O to P is given by, $W = \int_0^x dW = \int_0^x kx\,dx = \frac{1}{2}kx^2$

e. This work done is stored as the potential energy (P.E.) E_p of the particle at displacement x.

$$\therefore \quad E_p = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2x^2 \qquad \text{....(2)}$$

f. At time t,

$$\therefore \quad E_p = \frac{1}{2}kx^2 = \frac{1}{2}kA^2\sin^2(\omega t + \phi) = \frac{1}{2}mA^2\omega^2\cos^2(\omega t + \phi)$$

Thus, with time, it varies as $\sin^2\theta$.

iii. **Expression for total energy:**

a. The total energy of the particle is the sum of its kinetic energy and potential energy.

∴ $E = E_k + E_p$

b. Using equation (1) and equation (2), we get

$$E = \frac{1}{2}m\omega^2(A^2 - x^2) + \frac{1}{2}m\omega^2x^2$$

$$E = \frac{1}{2}m\omega^2A^2 = \frac{1}{2}kA^2 = \frac{1}{2}m(v_{max})^2$$

This expression gives the total energy of the particle at point P.

iv. Total energy in S.H.M. is

a. directly proportional to

1. the mass of the particle
2. the square of the amplitude
3. the square of the frequency
4. the force constant

b. inversely proportional to square of the period.

06 Superposition of Waves

Multiple Choice Questions (1 Mark Each)

1. A standing wave is produced on a string fixed at one end with the other end free. The length of the string
   (A) must be an odd integral multiple of λ
   (B) must be an odd integral multiple of $\lambda/2$
   (C) must be an odd integral multiple of $\lambda/4$
   (D) must be an even integral multiple of λ

2. The equation of a simple harmonic progressive wave is given by, $y = 5 \cos \pi [200t - x/150]$, where x and y are in cm and 't' is in second. Then the velocity of the wave is
   (A) 2 m/s (B) 150 m/s
   (C) 200 m/s **(D) 300 m/s**

Hint: Comparing $y = 5 \cos\left(200\pi t - \frac{\pi}{150}x\right)$ with y

$$= A \cos\left(\omega t - \frac{2\pi x}{\lambda}\right)$$

we have, $\omega = 200\pi$

$\therefore \quad 2\pi n = 200\pi$

$\therefore \quad n = 100$ Hz

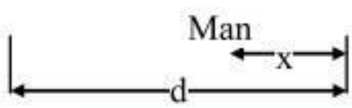
$\therefore \quad \lambda = 2 \times 150 = 300$ cm

$\therefore \quad v = n\lambda = 100 \times 300 \times 10^{-2} = 300$ m/s

***[Note:** Answer calculated above is in accordance with textual method of calculation.]*

3. A man standing unsymmetrical position between two mountains and fires a gun. He hears the first echo after 1.5 s and the second echo after 2.5 s. If the speed of sound in air is 340 m/s, then the distance between the mountains will be
   (A) 400 m (B) 520 m
   (C) 640 m **(D) 680 m**

Hint :



Let x be the nearest distance between man and mountain.

$\therefore \quad v = \dfrac{2x}{t_1}$

$\therefore \quad x = \dfrac{v \times t_1}{2} = \dfrac{340 \times 1.5}{2} = 255 \text{ m}$

Similarly, $v = \dfrac{2(d-x)}{t_2}$

$\therefore \quad d = \dfrac{v \times t_2}{2} + x = \dfrac{340 \times 2.5}{2} + 255 = 425 + 255 = 680 \text{ m}$

4. A set of tuning forks is arranged in ascending order of frequencies each tuning fork gives 8 beats/s with the preceding one. If frequency of the first tuning fork is 120 Hz and the last fork is 200 Hz, then the number of tuning forks arranged will be,

(A) 8 (B) 9
(C) 10 **(D) 11**

Hint: $f_1 = 120$, $f_2 = 120 + 8$ …... $f_n = 120 + (n-1)8$

$\therefore \quad 120 + (n-1)8 = 200$

$\therefore \quad (n-1)8 = 80$

$\therefore \quad n - 1 = 10$

$\therefore \quad n = 11$

5. In law of tension, the fundamental frequency of vibrating string is,

(A) inversely proportional to square root of tension
(B) directly proportional to the square of tension
(C) directly proportional to the square root of tension
(D) inversely proportional to density

6. The integral multiple of fundamental frequencies are

(A) beats (B) resonance
(C) overtones **(D) harmonics**

7. An organ pipe of length 0.4 m is open at both ends. The speed of sound in air is 340 m/s. The fundamental frequency is,

(A) 405 Hz (B) 415 Hz
(C) 425 Hz (D) 435 Hz

Hint: $n = \dfrac{v}{2L} = \dfrac{340}{2 \times 0.4} = 425 \text{ Hz}$

Very Short Answer (VSA) (1 Mark Each)

1. A wave is represented by an equation y = A sin(Bx + Ct). Given that the constants A, B and C are positive, can you tell in which direction the wave is moving?

Ans: The wave is moving in direction of negative X-axis.

2. Why wave motion is doubly periodic?

Ans: Wave motion is doubly periodic because it repeats itself after equal interval of time and space.

3. What is interference of sound waves?

Ans: If two longitudinal (sound) waves arrive at a point such that compression of one wave coincides with the compression of the other wave and rarefaction coincides with the rarefaction of the other wave and then the resultant amplitude of wave is maximum or if compression of one wave fall on the rarefaction of the other wave and vice versa and then amplitude of the resulting wave is minimum, then these effects are interference of longitudinal (sound) waves.

4. What are beats?

Ans: *One waxing and successive waning together constitute one* ***beat****.*

5. What are harmonics?

Ans: *The frequencies of a particular overtone which are the integral multiples of the fundamental frequency are known as* ***harmonics****.*

6. What are overtones?

Ans: *The tones whose frequencies are greater than the fundamental frequency are called* ***overtones****.*

7. State law of length.

Ans: **Law of length:** *The fundamental frequency of vibrations of a string is inversely proportional to the length of the vibrating string, if tension and mass per unit length are constant.*

$\therefore \quad n \propto \frac{1}{l}$ ....(if T and m are constant.)

8. State law of tension.

Ans: Law of tension: *The fundamental frequency of vibrations of a string is directly proportional to the square root of tension, if vibrating length and mass per unit length are constant.*

∴ $n \propto \sqrt{T}$ ....(if l and m are constant.)

9. State law of linear density.

Ans: Law of linear density: *The fundamental frequency of vibrations of a string is inversely proportional to the square root of mass per unit length (linear density), if the tension and vibrating length of the string are constant.*

∴ $n \propto \frac{1}{\sqrt{m}}$ ....(if T and l are constant.)

10. What is the resonance?

Ans: The phenomenon in which the body vibrates under action of external periodic force, whose frequency is equal to the natural frequency of the driven body, so that amplitude becomes maximum is called as resonance.

11. What are forced vibrations?

Ans: The vibrations of a body under the action of an external periodic force in which body vibrates with frequency equal to frequency of an external periodic force (driving frequency) other than natural frequency are called as forced vibrations.

12. A violin string vibrates with fundamental frequency of 510 Hz. What is the frequency of first overtone?

Ans: Frequency of first overtone, $n_1 = 2n$

$= 2 \times 510$

$= 1020$ Hz

13. A string 1 m long is fixed at one end. The other end is moved up and down with frequency 20 Hz. Due to this, a stationary wave with four complete loops, gets produced on the string. Find the speed of the progressive wave which produces the stationary wave. [Note: Remember that the moving end is a antinode.]

Ans: L = 1 m, n = 20 Hz

Since, an antinode is formed at the free end.

Thus, with four and a half loops of the string,

$$L = \frac{\lambda}{4} + 2\lambda = \frac{9}{4}\lambda$$

$\therefore \quad \lambda = \frac{4L}{9} = \frac{4}{9} \times 1 = \frac{4}{9}\text{m}$

$\therefore \quad v = n\lambda$

$= 20 \times \frac{4}{9}$

$= 8.88 \text{ m/s}$

*[**Note:** Answer calculated above is in accordance with textual method of calculation.]*

Short Answer I (SA1) (2 Marks Each)

1. For a stationary wave set up in a string having both ends fixed, what is the ratio of the fundamental frequency to the third harmonic?

Ans: Fundamental frequency of vibration, $n = \frac{1}{2l}\sqrt{\frac{T}{m}}$

Frequency of third harmonic, $n_1 = \frac{3}{2l}\sqrt{\frac{T}{m}}$

$$\therefore \quad \frac{n}{n_1} = \frac{\frac{1}{2l}\sqrt{\frac{T}{m}}}{\frac{3}{2l}\sqrt{\frac{T}{m}}} = \frac{1}{3}$$

$$\therefore \quad \frac{n}{n_1} = \frac{1}{3}$$

2. What are stationary waves? Why are they called stationary wave?

Ans:

i. *When two identical waves travelling along the same path in opposite directions interfere with each other, resultant wave is called **stationary wave.***

ii. Stationary waves are called so because the resultant harmonic disturbance of the particles does not travel in any direction and there is no transport of energy.

3. Distinguish between overtone and harmonic.

Ans:

Sr. No.	Overtone	Harmonic
i.	First harmonic is natural frequency of vibration.	First overtone is the next higher frequency of vibration.
ii.	Harmonics are simply integral multiples of fundamental frequency.	Overtones are not necessarily integral multiples of the fundamental frequency. They are frequencies other than the fundamental frequency.
iii.	All harmonics may or may not be present in vibration	All overtones are always present in the vibration.

[Any two differences]

4. State any four applications of beats.

Ans:

i. The phenomenon of beats is used for matching the frequencies of different musical instruments by artists.

ii. The speed of an airplane can be determined by using Doppler RADAR. Phenomenon of beats, arising due to the difference in frequencies produced by the source and received at the source after reflection from the air plane, allows us to calculate the velocity of the air plane.

iii. Doppler ultrasonography and echo cardiogram works on the principle of phenomenon of beats.

iv. Unknown frequency of a sound note can be determined by using the phenomenon of beats.

5. Prove that a pipe open at both end of length of 2L, has same fundamental frequency as another pipe of closed at one end of length L.

Ans:

i. Let L_o and L_c be the lengths of a pipe open at both ends and a pipe closed at one end, respectively.

ii. Let n_o and n_c be their corresponding fundamental frequencies.

iii. $n_o = \frac{v}{2L_o}$ and $n_c = \frac{v}{4L_c}$

where, v is the velocity of the sound in air.

iv. Since $L_c = L$ and $L_o = 2L$

∴ $n_o = \frac{v}{4L}$ and $n_c = \frac{v}{4L}$

∴ $n_o = n_c$

v. Hence, both the pipes have same fundamental frequencies.

6. How the frequency of vibrating wire is affected, if the load is fully immersed in water?

Ans:

i. According to the law of tension of a vibrating string, the fundamental frequency (n) is directly proportional to the square root of its tension. [when, l and m are kept constant]

∴ $n \propto \sqrt{T}$

ii. If load is fully immersed in water, then tension in the string decreases. Hence, frequency of vibration also decreases.

7. A sonometer wire of length 1 m is stretched by a weight of 10 kg. The fundamental frequency of vibration is 100 Hz. Determine the linear density of material of wire.

Solution:

Given: $L = 1$ m, $M = 10$ kg, $n_0 = 100$ Hz

To find: Linear density of wire (m)

Formulae: i. $T = Mg$ ii. $n_0 = \frac{1}{2L}\sqrt{\frac{T}{m}}$

Calculation: From formula (i),

$T = 10 \times 9.8 = 98$ N

From formula (ii),

$100 = \frac{1}{2 \times 1}\sqrt{\frac{98}{m}}$

Squaring both sides, we get

$10^4 = \frac{98}{4m}$

∴ $m = \frac{98}{4 \times 10^4}$

$= \mathbf{2.45 \times 10^{-3}}$ **kg/m**

Ans: The linear density of the wire is $\mathbf{2.45 \times 10^{-3}}$ **kg/m**.

[Note: Answer calculated above is in accordance with textual method of calculation.]

Short Answer II (SA1) (3 Marks Each)

1. Find the amplitude of the resultant wave produced due to interference of two waves given as,

$y_1 = A_1 \sin \omega t$, $y_2 = A_2 \sin (\omega t + \phi)$

Ans: Expression for amplitude of resultant wave:

i. Consider two waves having the same frequency but different amplitudes A_1 and A_2. Let these waves differ in phase by ϕ.

ii. The displacement of each wave at $x = 0$ is given as $y_1 = A_1 \sin \omega t$ and $y_2 = A_2 \sin (\omega t + \phi)$

iii. According to the principle of superposition of waves, the resultant displacement at $x = 0$ is

$y = y_1 + y_2$

$\therefore$ $y = A_1 \sin \omega t + A_2 \sin (\omega t + \phi)$

$y = A_1 \sin \omega t + A_2 \sin \omega t \cos \phi + A_2 \cos \omega t \sin \phi$

$y = (A_1 + A_2 \cos \phi) \sin \omega t + A_2 \sin \phi \cos \omega t$

Let, $A_1 + A_2 \cos \phi = A \cos \theta$ ....(1)

$A_2 \sin \phi = A \sin \theta$ ....(2)

$\therefore$ $y = A \cos \theta \sin \omega t + A \sin \theta \cos \omega t$

$\therefore$ $y = A \sin(\omega t + \theta)$

iv. This is the equation for displacement of the resultant wave. It has the same frequency as that of the interfering waves.

v. The resultant amplitude A is given by squaring and adding equations (1) and (2).

$A^2 \cos^2 \theta + A^2 \sin^2 \theta = (A_1 + A_2 \cos \phi)^2 + A_2^2 \sin^2 \phi$

$A^2 = A_2^2 + 2A_1A_2 \cos \phi + A_2^2 \cos^2 \phi + A_2^2 \sin^2 \phi$

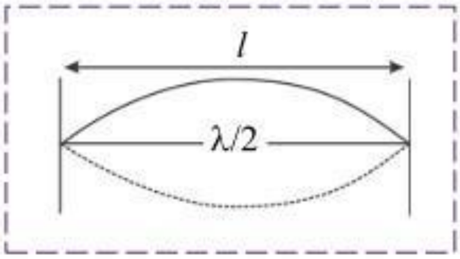
$\therefore$ $A = \sqrt{A_1^2 + 2A_1A_2 \cos \phi + A_2^2}$

2. Show that even as well as odd harmonics are present as overtone in modes of vibration of string.

Ans:

i. Fundamental mode:

a. If a string is stretched between two rigid supports and is plucked at its centre, the string vibrates as shown in figure.



b. It consists of an antinode formed at the centre and nodes at the two ends with one loop formed along its length.

c. If λ is the wavelength and l is the length of the string, then

Length of loop $= \frac{\lambda}{2} = l$

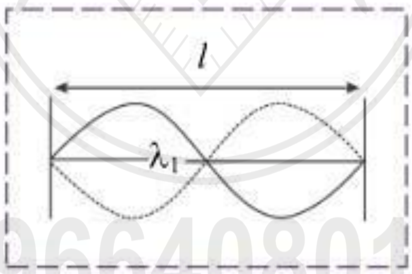
d. The frequency of vibrations of the string,

$$n = \frac{v}{\lambda} = \frac{1}{2l}\sqrt{\frac{T}{m}} \qquad \ldots.\left(\because v = \sqrt{\frac{T}{m}}\right)$$

This is the lowest frequency with which the string can vibrate. It is the fundamental frequency of vibrations or the first harmonic.

ii. For second mode or first overtone:

a. For first overtone or second harmonic, two loops are formed in this mode of vibrations.



b. There is a node at the centre of the string and at its both ends.

c. If λ_1 is wavelength of vibrations, the length of one loop $= \frac{\lambda_1}{2} = \frac{l}{2}$

∴ $\lambda_1 = l$

d. Thus, the frequency of vibrations is given as

$$n_1 = \frac{1}{\lambda_1}\sqrt{\frac{T}{m}}$$

∴ $n_1 = \frac{1}{l}\sqrt{\frac{T}{m}}$

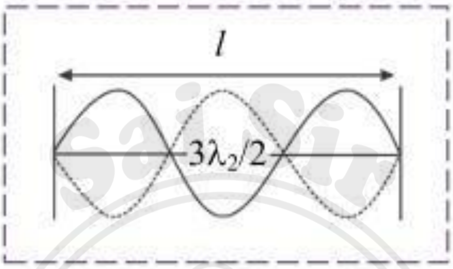
Comparing with fundamental frequency,

∴ $n_1 = 2n$.

Thus the frequency of the first overtone or second harmonic is equal to twice the fundamental frequency.

iii. For third mode or second overtone:

a. The string is made to vibrate in such a way that three loops are formed along the string as shown in figure.



b. If λ_2 is the wavelength, the length of one loop is $\frac{\lambda_2}{2} = \frac{l}{3}$

∴ $\lambda_2 = \frac{2l}{3}$

c. Therefore the frequency of vibrations is

$n_2 = \frac{1}{\lambda_2}\sqrt{\frac{T}{m}}$

∴ $n_2 = \frac{3}{2l}\sqrt{\frac{T}{m}}$

Comparing with fundamental frequency,

∴ $n_2 = 3n$.

Thus frequency of second overtone or third harmonic is equal to thrice the fundamental frequency.

iv. Similarly for higher modes of vibrations of the string, the frequencies of vibrations are as 4n, 5n, 6n…pn.

Thus even as well as odd harmonics are present as overtone in modes of vibration of string.

3. State and explain laws of vibrating strings.

Ans: The fundamental frequency of a vibrating string under tension is given as, $n = \frac{1}{2l}\sqrt{\frac{T}{m}}$

From this formula, three laws of vibrating string can be given as follows:

i. **Law of length:** *The fundamental frequency of vibrations of a string is inversely proportional to the length of the vibrating string, if tension and mass per unit length are constant.*

$\therefore$ $n \propto \frac{1}{l}$ ….(if T and m are constant.)

ii. **Law of tension:** *The fundamental frequency of vibrations of a string is directly proportional to the square root of tension, if vibrating length and mass per unit length are constant.*

$\therefore$ $n \propto \sqrt{T}$ ….(if l and m are constant.)

iii. **Law of linear density:** *The fundamental frequency of vibrations of a string is inversely proportional to the square root of mass per unit length (linear density), if the tension and vibrating length of the string are constant.*

$\therefore$ $n \propto \frac{1}{\sqrt{m}}$ ….(if T and l are constant.)

If r is the radius and ρ is the density of material of string, linear density is given as

Linear density = mass per unit length

= volume per unit length ×density

$= \frac{\pi r^2 l}{l} \times \rho$

$= \pi r^2 \rho$

As $n \propto \frac{1}{\sqrt{m}}$, if T and l are constant, we have,

$n \propto \frac{1}{\sqrt{\pi r^2 \rho}}$

i.e., $n \propto \frac{1}{\sqrt{\rho}}$ and $n \propto \frac{1}{r}$

Thus the fundamental frequency of vibrations of a stretched string is inversely proportional to the radius of string and the square root of the density of the material of vibrating string.

4. Two wires of the same material and same cross section are stretched on a sonometer. One wire is loaded with 1 kg and another is loaded with 9 kg. The vibrating length of first wire is 60 cm and its fundamental frequency of vibration is the same as that of the second wire. Calculate vibrating length of the other wire.

Solution:

Given: $M_1 = 1$ kg, $M_2 = 9$ kg,
$A_1 = A_2$,
$m_1 = m_2$ (where m is linear density of wire)
$L_1 = 60$ cm $= 0.6$ m
$(n_0)_1 = (n_0)_2$

To find: Vibrating length of wire (L_2)

Formula: $n = \frac{1}{2L}\sqrt{\frac{T}{m}}$

Calculation: From formula,

$$(n)_1 = \frac{1}{2L_1}\sqrt{\frac{T_1}{m_1}}$$

$$(n)_2 = \frac{1}{2L_2}\sqrt{\frac{T_2}{m_2}}$$

$\because$ $(n)_1 = (n)_2$

$\therefore$ $$\frac{1}{2L_1}\sqrt{\frac{T_1}{m_1}} = \frac{1}{2L_2}\sqrt{\frac{T_2}{m_2}}$$

$\therefore$ $$\frac{1}{4L_1^2}\frac{T_1}{m_1} = \frac{1}{4L_2^2}\frac{T_2}{m_2}$$

$\therefore$ $$L_2 = \sqrt{L_1^2 \times \frac{T_2}{m_2} \times \frac{m_1}{T_1}}$$

$$= \sqrt{(0.6)^2 \times \frac{9\text{ g}}{1\text{ g}}}$$

$$= 0.6 \times 3$$

$$= \mathbf{1.8\ m}$$

Ans: The vibrating length of the wire is **1.8 m**.

*[**Note:** Answer calculated above is in accordance with textual method of calculation.]*

5. The equation of simple harmonic progressive wave is, y = sin π/2 (4t/0.025 – x/0.25). Where all quantities are in S.I. system. Find amplitude, frequency, wavelength and velocity of wave.

Solution:

i. Given equation: $y = \sin\frac{\pi}{2}\left(\frac{4t}{0.025} - \frac{x}{0.25}\right)$

$$y = \sin\left(\frac{2\pi t}{0.025} - \frac{\pi x}{0.5}\right)$$

Comparing above equation with $y = A\sin\left(\omega t - \frac{2\pi x}{\lambda}\right)$

∴ $A = \mathbf{1\ m}$

ii. $\omega = \frac{2\pi}{0.025}$

∴ $2\pi n = \frac{2\pi}{0.025}$

∴ $n = \mathbf{40\ Hz}$

iii. $\frac{2\pi}{\lambda} = \frac{\pi}{0.5}$

∴ $\lambda = \mathbf{1\ m}$

iv. $v = n\lambda$

$= 40 \times 1$

$= \mathbf{40\ m/s}$

Ans: The amplitude, frequency, wavelength and velocity of the wave are **1 m**, **40 Hz**, **1 m** and **40 m/s** respectively.

6. A stretched sonometer wire is in unison with a tuning fork. When the length is increased by 4%, the number of beats heard per second is 6. Find the frequency of the fork.

Solution:

Given: $l_2 = l_1 + 0.04\, l_1 = 1.04\, l_1$

$n_1 - n_2 = 6$

To find: Initial frequency of tuning fork (n_1)

Formula: $n_1 l_1 = n_2 l_2$

Calculation: From formula,

$n_1 l_1 = n_2(1.04\, l_1)$

∴ $n_1 = 1.04\, n_2$

∴ $n_2 = \frac{1}{1.04} n_1$

But $n_1 - n_2 = 6$

$\therefore \quad n_1 - \dfrac{n_1}{1.04} = 6$

$\therefore \quad 0.04\, n_1 = 6.24$

$\therefore \quad n_1 = \mathbf{156\ Hz}$

Ans: The initial frequency of tuning fork is **156 Hz**.

Long Answer (LA) (4 Marks Each)

1. **Explain the formulation of stationary waves by analytical method. What are nodes and antinodes? Show that the distance between two successive nodes or antinodes is λ/2.**

Ans: Expression for equation of stationary wave on a stretched string:

i. Consider two simple harmonic progressive waves of equal amplitudes (a) and wavelength (λ) propagating on a long uniform string in opposite directions.

ii. The equation of wave travelling along the X-axis in the positive direction is given by,

$$y_1 = a \sin\left[2\pi\left(nt - \frac{x}{\lambda}\right)\right]$$

The equation of wave travelling along the X-axis in the negative direction is given by,

$$y_2 = a \sin\left[2\pi\left(nt + \frac{x}{\lambda}\right)\right]$$

iii. When these waves interfere, the resultant displacement of particles of string is given by the principle of superposition of waves as

$y = y_1 + y_2$

$$\therefore \quad y = a \sin\left[2\pi\left(nt - \frac{x}{\lambda}\right)\right] + a \sin\left[2\pi\left(nt + \frac{x}{\lambda}\right)\right]$$

iv. By using trigonometry formula,

$$\sin C + \sin D = 2 \sin\left(\frac{C + D}{2}\right) \cos\left(\frac{C - D}{2}\right)$$

$$\therefore \quad y = 2a \sin(2\pi nt) \cos\frac{2\pi x}{\lambda}$$

$$y = 2a \cos\frac{2\pi x}{\lambda} \sin(2\pi nt) \qquad \ldots(1)$$

v. Substituting $2a \cos\frac{2\pi x}{\lambda} = A$ in equation (1),

$y = A \sin(2\pi nt)$

∴ $y = A \sin\omega t$ ....($\because \omega = 2\pi n$)

This is the equation of a stationary wave which gives resultant displacement due to two simple harmonic progressive waves.

vi. **Nodes:** *The points of a medium, which vibrate with minimum amplitude are called nodes.*

Condition for node: Amplitude is minimum, i.e., A = 0.

∴ $2a \cos\frac{2\pi x}{\lambda} = 0$

∴ $\cos\frac{2\pi x}{\lambda} = 0$

∴ $\frac{2\pi x}{\lambda} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, ...$

∴ $x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, ...$

i.e., $x = (2p - 1)\frac{\lambda}{4}$ where p = 1, 2, 3,....

Distance between two successive nodes:

$\frac{3\lambda}{4} - \frac{\lambda}{4} = \frac{\lambda}{2}$;

$\frac{5\lambda}{4} - \frac{3\lambda}{4} = \frac{\lambda}{2}$

∴ Distance between two successive nodes is $\frac{\lambda}{2}$.

vii. **Antinodes:**

The points of a medium, which vibrate with maximum amplitude are called antinodes.

Condition for antinode: Amplitude is maximum, i.e., A = ±2a

∴ $2a \cos\frac{2\pi x}{\lambda} = \pm 2a$

∴ $\cos\frac{2\pi x}{\lambda} = \pm 1$

∴ $\frac{2\pi x}{\lambda} = 0, \pi, 2\pi, 3\pi, ...$

∴ $x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, ...$

i.e., $x = \frac{\lambda p}{2}$ where p = 0, 1, 2, 3,…

∴ Distance between two successive antinodes:

$\frac{\lambda}{2} - 0 = \frac{\lambda}{2}$;

$\lambda - \frac{\lambda}{2} = \frac{\lambda}{2}$

∴ Distance between two successive antinodes is $\frac{\lambda}{2}$.

2. Explain the production of beats and deduce analytically the expression for beats frequency.

Ans:

i. Production of beats:

a. When there is superposition of two sound waves, having same amplitude but slightly different frequencies, travelling in the same direction, the intensity of sound varies periodically with time. This phenomenon is known as production of beats.

b. The variation in the loudness of sound that goes up and down is the phenomenon of formation of beats.

c. It can be considered as superposition of waves and formation of standing waves in time at one point in space where waves of slightly different frequencies are passing. The two waves are in and out of phase giving constructive and destructive interference.

ii. Expression for beats frequency:

a. Consider two sound waves, having same amplitude and slightly different frequencies n_1 and n_2.

b. Let x be a point of the medium where they arrive in phase.

c. The displacement due to each wave at any instant of time at that point is given as

$$y_1 = a \sin\left[2\pi\left(n_1 t - \frac{x}{\lambda_1}\right)\right]$$

$$y_2 = a \sin\left[2\pi\left(n_2 t - \frac{x}{\lambda_2}\right)\right]$$

d. Let us assume for simplicity that the listener is at x = 0.

∴ $y_1 = a \sin(2\pi n_1 t)$ and $y_2 = a \sin(2\pi n_2 t)$

e. According to the principle of superposition of waves,

$y = y_1 + y_2$

∴ $y = a \sin(2\pi n_1 t) + a \sin(2\pi n_2 t)$

∴ $$y = 2a \sin\left[2\pi\left(\frac{n_1 + n_2}{2}\right)t\right]\cos\left[2\pi\left(\frac{n_1 - n_2}{2}\right)t\right]$$

[By using formula,

$$\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)]$$

Rearranging the above equation, we get

$$y = 2a \cos\left[\frac{2\pi(n_1 - n_2)}{2}t\right]\sin\left[\frac{2\pi(n_1 + n_2)}{2}t\right]$$

f. Let $2a \cos\left[\frac{2\pi(n_1 - n_2)}{2}t\right] = A$

$\frac{n_1 + n_2}{2} = n$

∴ $y = A \sin(2\pi nt)$

This is the equation of a progressive wave having frequency 'n' and amplitude 'A'. The frequency 'n' is the mean of the frequencies n_1 and n_2 of arriving waves while the amplitude A varies periodically with time.

g. The intensity of sound is proportional to the square of the amplitude. Hence the resultant intensity will be maximum when the amplitude is maximum.

h. For maximum amplitude (waxing),

$A = \pm 2a$

∴ $$2a \cos\left[\frac{2\pi(n_1 - n_2)}{2}\right]t = \pm 2a$$

∴ $$\cos\left[\frac{2\pi(n_1 - n_2)}{2}\right]t = \pm a$$

i.e., $$\left[2\pi\left(\frac{n_1 - n_2}{2}\right)t\right] = 0, \pi, 2\pi, 3\pi, \ldots$$

∴ $$t = 0, \frac{1}{n_1 - n_2}, \frac{2}{n_1 - n_2}, \frac{3}{n_1 - n_2}, \ldots$$

Thus, the time interval between two successive maxima of sound is always $\frac{1}{n_1 - n_2}$.

Hence, the period of beats is $T = \frac{1}{n_1 - n_2}$.

The number of waxing heard per second is the reciprocal of period of waxing.

∴ Beat frequency in waxing, $N = n_1 - n_2$

i. The intensity of sound will be minimum when amplitude is zero (waning):

For minimum amplitude, $A = 0$,

∴ $2a\cos\left[2\pi\left(\frac{n_1 - n_2}{2}\right)t\right] = 0$

∴ $\cos\left[2\pi\left(\frac{n_1 - n_2}{2}\right)t\right] = 0$

∴ $\left[2\pi\left(\frac{n_1 - n_2}{2}\right)t\right] = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$

∴ $t = \frac{1}{2(n_1 - n_2)}, \frac{3}{2(n_1 - n_2)}, \frac{5}{2(n_1 - n_2)}, \ldots$

Therefore, time interval between two successive minima is also $\frac{1}{(n_1 - n_2)}$.

The number of waning heard per second is the reciprocal of period of waning.

Period of beat, $T = \frac{1}{n_1 - n_2}$

Beat frequency in waning, $N = n_1 - n_2$

3. State and verify the laws of vibrating strings using sonometer.

Ans:

i. Law of length: *The fundamental frequency of vibrations of a string is inversely proportional to the length of the vibrating string, if tension and mass per unit length are constant.*

∴ $n \propto \frac{1}{l}$ ....(If T and m are constant)

Verification of first law:

a. By measuring length of wire and its mass, the mass per unit length (m) of wire is determined. Then the wire is stretched on the sonometer and the hanger is suspended from its free end.

b. A suitable tension (T) is applied to the wire by placing slotted weights on the hanger.

c. The length of wire (l_1) vibrating with the same frequency (n_1) as that of the tuning fork is determined as follows.

d. A light paper rider is placed on the wire midway between the bridges. The tuning fork is set into vibrations by striking on a rubber pad.

e. The stem of tuning fork is held in contact with the sonometer box. By changing distance between the bridges without disturbing paper rider, frequency of vibrations of wire is changed.

f. When the frequency of vibrations of wire becomes exactly equal to the frequency of tuning fork, the wire vibrates with maximum amplitude and the paper rider is thrown off.

g. In this way a set of tuning forks having different frequencies n_1, n_2, n_3, …………are used and corresponding vibrating lengths of wire are noted as l_1, l_2, l_3……….by keeping the tension constant (T) .

h. It is observe that $n_1l_1 = n_2l_2 = n_3l_3 = \ldots\ldots = \text{constant}$, for constant value of tension (T) and mass per unit length (m).

∴ $nl = \text{constant}$

i.e., $n \propto \frac{1}{l}$, if T and m are constant.

Thus, the first law of a vibrating string is verified by using a sonometer.

ii. **Law of tension:** *The fundamental frequency of vibrations of a string is directly proportional to the square root of tension, if vibrating length and mass per unit length are constant.*

∴ $n \propto \sqrt{T}$ ….(If l and m are constant)

Verification of second law:

a. The vibrating length (l) of the given wire of mass per unit length (m) is kept constant for verification of second law.

b. By changing the tension, the same length is made to vibrate in unison with different tuning forks of various frequencies.

c. If tensions T_1, T_2, T_3……..correspond to frequencies n_1, n_2, n_3,………etc. It is observed that $\frac{n_1}{\sqrt{T_1}} = \frac{n_2}{\sqrt{T_2}} = \frac{n_3}{\sqrt{T_3}} = \ldots\ldots = \text{constant}$

$\therefore \quad \frac{n}{\sqrt{T}} = \text{constant}$

$\therefore \quad n \propto \sqrt{T}$ if l and m are constant. Thus, the law of tension of a vibrating string is verified by using a sonometer.

iii. **Law of linear density:** *The fundamental frequency of vibrations of a string is inversely proportional to the square root of mass per unit length (linear density), if the tension and vibrating length of the string are constant.*

$\therefore \quad n \propto \sqrt{\frac{1}{m}}$ ….(If l and T are constant)

Verification of third law:

a. For verification of third law of a vibrating string, two wires having different masses per unit lengths m_1 and m_2 (linear densities) are used.

b. The first wire is subjected to suitable tension and made to vibrate in unison with given tuning fork.

c. The vibrating length is noted as (l_1). Using the same fork, the second wire is made to vibrate under the same tension and the vibrating length (l_2) is determined.

d. Thus the frequency of vibration of the two wires is kept same under same applied tension T. It is found that,

$$l_1\sqrt{m_1} = l_2\sqrt{m_2}$$

$$l\sqrt{m} = \text{constant}$$

e. But by first law of a vibrating string, $n \propto \frac{1}{l}$

Therefore, $n \propto \frac{1}{\sqrt{m}}$, if T and l are constant. Thus, the third law of vibrating string is verified by using a sonometer.

4. Waves produced by two vibrators in a medium have wavelength 2 m and 2.1 m respectively. When sounded together they produce 8 beats/second. Calculate wave velocity and frequencies of the vibrators.

Solutions:

Given: $\lambda_1 = 2$ m, $\lambda_2 = 2.1$, $n_1 - n_2 = 8$

To find: i. Wave velocity (v)

ii. Frequencies (n_1 and n_2)

Formula: $v = n\lambda$

Calculation: From formula,

$v = n_1\lambda_1 = n_2\lambda_2$

But $n_1 - n_2 = 8$ ….(Given)

$\therefore \quad \frac{v}{\lambda_1} - \frac{v}{\lambda_2} = 8$

$\therefore \quad v\left(\frac{1}{2} - \frac{1}{2.1}\right) = 8$

$\therefore \quad v = \frac{8 \times 2 \times 2.1}{0.1}$

$\therefore \quad v = \mathbf{336\ m/s}$

$\therefore \quad n_1 = \frac{v}{\lambda_1} = \frac{336}{2} = \mathbf{168\ Hz}$

$\therefore \quad n_2 = \frac{v}{\lambda_2} = \frac{336}{2.1} = \mathbf{160\ Hz}$

Ans: i. The wave velocity is **336 m/s**.

ii. The frequency are **168 Hz** and **160 Hz**.

07 Wave Optics

Note: The (R) marked questions are the part of reduced/non-evaluative portion for academic year 2020-21 only.

Multiple Choice Questions (1 Mark Each)

(R) 1. When light travels from an optically rarer medium to an optically denser medium, the speed decreases because of change in:
 (A) Wavelength (B) Frequency
 (C) Amplitude (D) Phase

2. Light of wavelength 5000 A.U. falls on a plane reflecting surface. The frequency of reflected light is…
 (A) 6×10^{14} Hz (B) 5×10^{14} Hz
 (C) 2×10^{14} Hz (D) 1.666×10^{14} Hz

Hint: $\nu = \frac{c}{\lambda} = \frac{3\times10^8}{5000\times10^{-10}} = 6 \times 10^{14}$ Hz

3. Light follows wave nature because…
 (A) light rays travel in a straight line.
 (B) light exhibits the phenomenon of reflection and refraction.
 (C) light exhibits the phenomenon of interference.
 (D) light causes the phenomenon of photoelectric effect.

4. Young's double slit experiment is carried out using green, red and blue light, one colour at a time. The fringe widths recorded are W_G, W_R, and W_B respectively then…
 (A) $W_G > W_B > W_R$ (B) $W_B > W_G > W_R$
 (C) $W_R > W_B > W_G$ **(D) $W_R > W_G > W_B$**

Hint: Fringe width, $W \propto \lambda$ and $\lambda_R > \lambda_G > \lambda_B$

$\therefore$ $W_R > W_G > W_B$

5. The path difference between two waves meeting at a point is $(11/4)\lambda$. The phase difference between the two waves is…

(A) $11\ \pi/4$ **(B) $11\ \pi/2$**
(C) $11\ \pi$ (D) $22\ \pi$

Hint: Phase difference, $\Delta\phi = \frac{2\pi}{\lambda}\Delta l$

$\therefore \quad \Delta\phi = \frac{2\pi}{\lambda}\times\frac{11}{4}\lambda = \frac{11\pi}{2}$

6. Which of the following cannot produce two coherent sources?

(A) Lloyd's mirror (B) Fresnel biprism
(C) Young's double slit **(D) Prism**

7. The bending of beam of light around corners of obstacle is called…

(A) reflection **(B) diffraction**
(C) refraction (D) interference

8. In a single slit diffraction pattern, first minima obtained with red light of wavelength 6600 A.U. coincides with first maxima of some other wavelength λ then is…

(A) 5500 A.U. (B) 5000 A.U.
(C) 4800 A.U. **(D) 4400 A.U.**

Hint: For first minima in diffraction pattern,

$a \sin\theta = 1 \times \lambda_{Red}$

For first maxima in diffraction pattern,

$a \sin\theta = \frac{3}{2}\lambda$

As both coincide, $\lambda_{Red} = \frac{3}{2}\lambda$

$\therefore \quad \lambda = \lambda_{Red} \times \frac{2}{3}$

$= 6600 \times \frac{2}{3}$

$= 4400$ A.U.

Very Short Answer (VSA) (1 Mark Each)

1. What is the shape of the wave front on Earth for Sunlight?

Ans: The shape of the wavefront on Earth for Sunlight is plane.

2. In Young's double slit experiment, if there is no initial phase difference between the light from the two slits, a point on the screen corresponds to the 5^{th} minimum. What is the path difference?

Ans: Path difference $\Delta l = (2n-1)\frac{\lambda}{2} = (2 \times 5 - 1)\frac{\lambda}{2} = \frac{9\lambda}{2}$

3. Two coherent sources whose intensity ratio is 25:1 produce interference fringes. Calculate the ratio of amplitudes of light waves coming from them.

Ans: $\frac{I_1}{I_2} = \frac{25}{1} = \frac{a_1^2}{a_2^2} \Rightarrow \frac{a_1}{a_2} = \frac{5}{1}$

4. Why two light sources must be of equal intensity to obtain a well-defined interference pattern?

Ans: This is because, only if the intensities of two light sources are equal, the intensity of dark fringes (destructive interference) is zero and the contrast between bright and dark fringes will be maximum, thereby giving rise to well-defined interference pattern.

5. What is the relation between phase difference and optical path in terms of speed of light in vacuum?

Ans: Phase difference, $\Delta\phi' = \frac{\omega}{c} \times n\Delta x$

where, ω is angular frequency, c is speed of light in vacuum and $n\Delta x$ is optical path for a wave travelling in medium of refractive index n.

6. What should be the slit width to obtain pronounced diffraction with a single slit illuminated by light of wavelength λ?

Ans: To obtain pronounced diffraction with a single slit, the slit width should be of the order of wavelength λ.

7. **What must be ratio of the slit width to the wavelength for a single slit, to have the first diffraction minimum at 45°?**

Ans: For 1st minimum, $\sin\theta_1 = \frac{\lambda}{a}$. For $\theta_1 = 45°$, $\frac{\lambda}{a} = \sin 45° = \frac{1}{\sqrt{2}}$

∴ Ratio of slit width to wavelength, $a : \lambda = \sqrt{2} : 1$

Short Answer I (SA1) (2 Marks Each)

1. **What are Secondary sources? State Huygens' principle.**

Ans: Secondary sources are those sources which do not produce light of their own but receive light from some other source and either reflect or scatter it around.

Examples: the moon, the planets, objects.

Statement of Huygens' principle: *Each point on a wavefront acts as a secondary source of light emitting secondary light waves called wavelets in all directions which travel with the speed of light in the medium. The new wavefront can be obtained by taking the envelope of these secondary wavelets travelling in the forward direction and is thus, the envelope of the secondary wavelets in forward direction. The wavelets travelling in the backward direction are ineffective.*

2. **A plane wavefront of light of wavelength 5500 A.U. is incident on two slits in a screen perpendicular to the direction of light rays. If the total separation of 10 bright fringes on a screen 2 m away is 2 cm. Find the distance between the slits.**

Solution:

Given: $\lambda = 5500$ A.U.$= 5500 \times 10^{-10}$ m, D = 2 m

Distance between 10 fringes = 2 cm = 0.02 m.

Fringe width $W = 0.02/10 = 0.002 \text{ m} = 2 \times 10^{-3}$ m

To find: Distance between slits (d)

Formula: $W = \frac{\lambda D}{d}$

Calculation: From formula,

$$2 \times 10^{-3} = \frac{5500 \times 10^{-10} \times 2}{d}$$

$$\therefore \quad d = \frac{5.5 \times 10^{-7} \times 2}{2 \times 10^{-3}} = \mathbf{5.5 \times 10^{-4}\ m}$$

Ans: The distance between two slits is $\mathbf{5.5 \times 10^{-4}}$ **m.**

3. State any four conditions for obtaining well – defined and steady interference pattern.

Ans: Conditions for obtaining well – defined and steady interference pattern:

i. The two sources of light must be coherent.

ii. The two sources of light must be monochromatic.

iii. The two interfering waves must have the same amplitude.

iv. The separation between the two slits (d) must be small in comparison to the distance between the plane containing the slits and the observing screen (D).

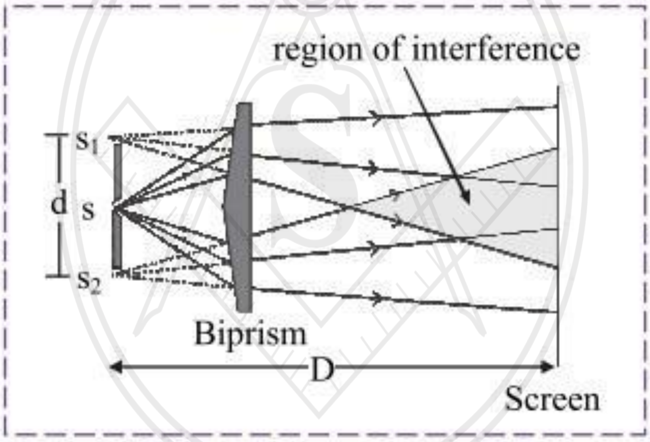
v. The two slits should be narrow.

vi. The two waves should be in the same state of polarization.

[Any four points]

4. Draw a neat labelled ray diagram of Fresnel biprism experiment showing the region of interference.

Ans: Fresnel biprism experiment:



5. What is Optical path length? How is it different from actual path length?

Ans:

i. When a wave travels a distance Δx through a medium having refractive index of n, its phase changes by the same amount as it would if the wave had travelled a distance $n\Delta x$ in vacuum.

ii. Thus, a path length of Δx in a medium of refractive index n is equivalent to a path length of $n\Delta x$ in vacuum.

iii. $n\Delta x$ is called the optical path travelled by a wave.

iv. This means, optical path through a medium is the effective path travelled by light in vacuum to generate the same phase difference.

v. Optical path in a medium can also be defined as the corresponding path in vacuum that the light travels in the same time as it takes in the given medium.

i.e., time = $\frac{d_{medium}}{v_{medium}} = \frac{d_{vacuum}}{v_{vacuum}}$

∴ $d_{vacuum} = \frac{v_{vacuum}}{v_{medium}} \times d_{medium} = n \times d_{medium}$

But d_{vacuum} = Optical path

∴ Optical path = $n \times d_{medium}$

vi. Thus, a distance d travelled in a medium of refractive index n introduces a path difference = nd – d = d(n – 1) over a ray travelling equal distance through vacuum.

6. What is difference between Fresnel and Fraunhofer diffraction?

Ans:

No.	Fresnel diffraction	Fraunhofer diffraction
i.	Source of light and screen are kept at finite distance.	Source of light and screen are at infinite distance.
ii.	Spherical or cylindrical wavefronts are considered.	Only plane wavefronts are considered.
iii.	It is observed in straight edge, narrow slit etc.	It is observed in single slit, double slit etc.
iv.	Lenses are not used.	Convex lenses are used.

7. Compare Young's double slit interference pattern and single slit diffraction pattern.

Ans:

	Young's double slit interference pattern:	Single slit diffraction pattern
i.	**Dimension of slit:** For a common laboratory set up, the slits in the Young's double slit experiment are much thinner than their separation. They are usually obtained by using a biprism or a Lloyd's mirror. The separation between the slits is a few mm only.	**Dimension of slit:** The single slit used to obtain the diffraction pattern is usually of width less than 1 mm.
ii.	**Size of pattern obtained:** With best possible set up, observer can usually see about 30 to 40 equally spaced bright and dark fringes of nearly same brightness.	**Size of pattern obtained:** Taken on either side, observer can see around 20 to 30 fringes with central fringe being the brightest.

iii.	**Fringe width W:** $W = \frac{\lambda D}{d}$	**Fringe width W:** $W = \frac{\lambda D}{a}$ Except for the central bright fringe
iv.	**For n^{th} bright fringe**	
a.	Phase difference, ϕ between extreme rays: $n(2\pi)$	Phase difference, ϕ between extreme rays: $\left(n+\frac{1}{2}\right)(2\pi)$ OR $(2n+1)\pi$
b.	Angular position, θ: $n\left(\frac{\lambda}{d}\right)$	Angular position, θ: $\left(n+\frac{1}{2}\right)\left(\frac{\lambda}{a}\right)$ OR $\frac{(2n+1)\lambda}{2a}$
c.	Path difference, Δl between extreme rays: $n\lambda$	Path difference, Δl between extreme rays: $n\lambda$
d.	Distance from the central bright spot, y: $n\left(\frac{\lambda D}{d}\right) = nW$	Distance from the central bright spot, y: $\left(n+\frac{1}{2}\right)\left(\frac{\lambda D}{a}\right) = \left(n+\frac{1}{2}\right)W$
v.	**For n^{th} dark fringe**	
a.	Phase difference, ϕ between extreme rays: $\left(n-\frac{1}{2}\right)(2\pi)$ OR $(2n-1)\pi$	Phase difference, ϕ between extreme rays: $n(2\pi)$
b.	Angular position, θ: $\left(n-\frac{1}{2}\right)\left(\frac{\lambda}{d}\right)$ OR $(2n-1)\frac{\lambda}{2d}$	Angular position, θ: $n\left(\frac{\lambda}{a}\right)$
c.	Path difference, Δl between extreme rays: $\left(n-\frac{1}{2}\right)\lambda$ OR $(2n-1)\frac{\lambda}{2}$	Path difference, Δl between extreme rays: $n\lambda$
d.	Distance from the central bright spot, y′: $\left(n-\frac{1}{2}\right)\left(\frac{\lambda D}{d}\right) = \left(n-\frac{1}{2}\right)W$	Distance from the central bright spot, y′: $n\left(\frac{\lambda D}{a}\right) = nW$

8. White light consists of wavelengths from 400 nm to 700 nm. What will be the wavelength range seen, when white light is passed through glass of refractive index 1.55?

Solution:

Given: $n = 1.55$

Smallest wavelength = 400 nm,

Largest wavelength = 700 nm

To find: Range of wavelength of light when passed through glass

Formula: $\lambda_{med} = \frac{\lambda_{vac}}{n}$

Calculation: For smallest wavelength (in glass),

From formula

$\lambda_{med} = \frac{400}{1.55}$

$= 2.5806 \times 10^2$ nm

$=$ **258.06 nm**

For largest wavelength

From formula,

$\lambda_{med} = \frac{700}{1.55}$

$= 4.5161 \times 10^2$ nm

$=$ **451.61 nm**

Ans: The wavelength range when white- light is passed through glass is **258.06 nm to 451.61 nm.**

9. The optical path of a ray of light of a given wavelength travelling a distance of 3 cm in flint glass having refractive index 1.6 is same as that on travelling a distance x cm through a medium having refractive index 1.25. Determine the value of x.

Solution:

Given: $d_1 = 3$ cm, $n_1 = 1.6$, $n_2 = 1.25$

To find: Optical path in medium 2 (d_2)

Formula: $n_1d_1 = n_2d_2$

Calculation: From formula,

$1.6 \times 3 = 1.25 \times d_2$

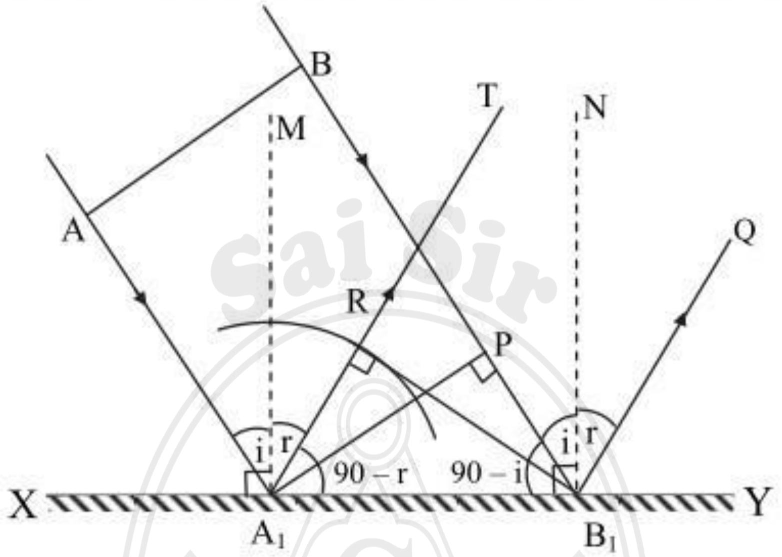
$\therefore \quad d_2 = \frac{1.6 \times 3}{1.25} =$ **3.84 cm**

Ans: The value of x is **3.84 cm.**

Short Answer II (SA1) (3 Marks Each)

1. Explain reflection of light at a plane surface with the help of a neat ray diagram.

Ans:



Reflection of light

XY : Plane reflecting surface
AB : Plane wavefront
RB_1 : Reflecting wavefront
A_1M, B_1N : Normal to the plane
$\angle AA_1M = \angle BB_1N = \angle i$ = Angle of incidence
$\angle TA_1M = \angle QB_1N = \angle r$ = Angle of reflection

Explanation:

i. A plane wavefront AB is advancing obliquely towards plane reflecting surface XY. AA_1 and BB_1 are incident rays.

ii. When 'A' reaches XY at A_1, then ray at 'B' reaches point 'P' and it has to cover distance PB_1 to reach the reflecting surface XY.

iii. Let 't' be the time required to cover distance PB_1. During this time interval, secondary wavelets are emitted from A_1 and will spread over a hemisphere of radius A_1R, in the same medium.

Distance covered by secondary wavelets to reach from A_1 to R in time t is same as the distance covered by primary waves to reach from P to B_1.

Thus $A_1R = PB_1 = ct$.

iv. All other rays between AA_1 and BB_1 will reach XY after A_1 and before B_1. Hence, they will also emit secondary wavelets of decreasing radii.

v. The surface touching all such hemispheres is RB_1 which is reflected wavefront, bounded by reflected rays A_1R and B_1Q.

vi. Draw $A_1M \perp XY$ and $B_1N \perp XY$.

Thus, angle of incidence is $\angle AA_1M = \angle BB_1N = i$ and angle of reflection is $\angle MA_1R = \angle NB_1Q = r$.

$\therefore$ $\angle RA_1B_1 = 90 - r$ and $\angle PB_1A_1 = 90 - i$

vii. In ΔA_1RB_1 and ΔA_1PB_1

$\angle A_1RB_1 \cong \angle A_1PB_1$

$A_1R = PB_1$ (Reflected waves travel equal distance in same medium in equal time).

$A_1B_1 = A_1B_1$ ….(common side)

$\therefore$ $\Delta A_1RB_1 \cong \Delta A_1PB_1$

$\therefore$ $\angle RA_1B_1 = \angle PB_1A_1$

$\therefore$ $90 - r = 90 - i$

$\therefore$ $i = r$

viii. Also, from the figure, it is clear that incident ray, reflected ray and normal lie in the same plane.

ix. Assuming rays AA_1 and BB_1 to be coming from extremities of the object, A_1B_1 is the size of object. Distance between corresponding reflected rays A_1T and B_1Q will be same as A_1B_1 as they are corresponding parts of congruent triangles.

This implies size of object in reflected image is same as actual size of object.

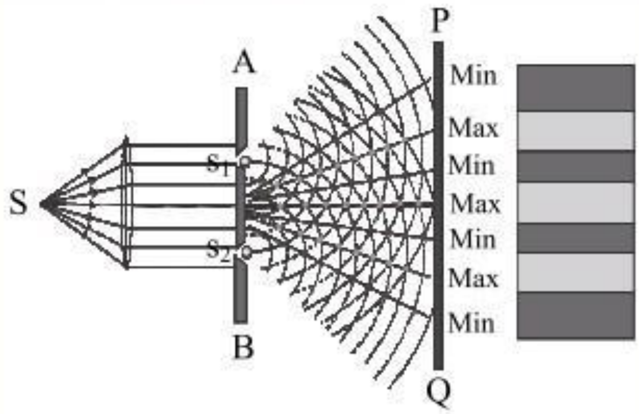
x. Also, taking A and B to be right and left sides of the object respectively, after reflection right side at A is seen at T and left side at B is seen at Q. This explains lateral inversion.

2. Describe Young's double slit experiment with a neat diagram showing points of maximum and minimum intensity.

Ans:

i. In Young's double slit interference experiment, a plane wavefront is made to fall on an opaque screen AB having two similar narrow slits S_1 and S_2.

ii. The figure below shows a cross section of the experimental set up and the slits have their lengths perpendicular to the plane of the paper.



Young's double slit experiment

iii. The slits are about 2 - 4 mm apart from each other.

iv. An observing screen PQ is placed behind of AB.

v. Assuming that the slits S_1 and S_2 are equidistant from the S, the wavefronts starting from S and reaching the S_1 and S_2 at every instant of time are in phase.

vi. When the rays fall on S_1 and S_2, the two slits act as secondary sources of light emitting cylindrical wavelets (with axis along the slit length) to the right of AB.

vii. The two secondary sources emit waves in phase with each other.

viii. The crests/troughs of the secondary wavelets superpose as shown in the figure and interfere constructively high intensity giving rise to bright band.

ix. When the crest of one wave coincides with the trough of the other causing zero intensity, dark images of the slits are produced on the screen PQ.

x. The dark and bright regions are called fringes and the whole pattern is called interference pattern.

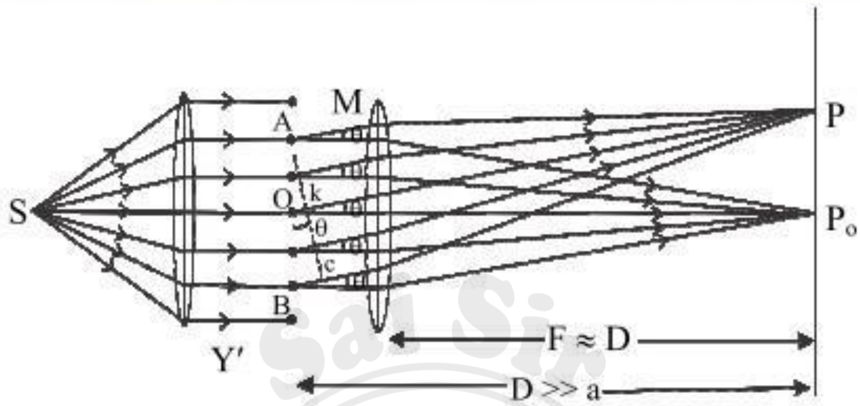
3. Explain experimental setup for Fraunhofer diffraction with neat diagram.

Ans:

i. Set up for Fraunhofer diffraction has a monochromatic source of light S at the focus of a converging lens. Ignoring aberrations, the emerging beam will consist of plane parallel rays resulting in plane wavefronts.

ii. These are incident on the diffracting element such as a slit, a circular aperture, a double slit, a grating, etc.

iii. In the case of a circular aperture, S is a point source and the lenses are bi-convex. For linear elements like slits, grating, etc., the source is linear and the lenses are cylindrical in shape so that the focussed image is also linear.



Set up for Fraunhofer diffraction

iv. Emerging beam is incident on another converging lens that focuses the beam on a screen.

4. The distance between two bright fringes in a biprism experiment using light of wavelength 6000 A.U. 0.32 mm. By how much will the distance change, if light of wavelength 4800 A.U. is used?

Solution:

Given: Distance between consecutive bright fringes,

$y_A = 0.32 \text{ mm} = 0.32 \times 10^{-3} \text{ m}$

$\lambda_A = 6000 \text{ A.U.} = 6 \times 10^{-7} \text{ m}, \lambda_B = 4800 \text{ A.U.} = 4.8 \times 10^{-7} \text{ m}$

Let y_B be distance between consecutive bright fringes when wavelength λ_B is used

To find: Change in distance between the fringes $|y_A - y_B|$

Formula: $y_A\lambda_B = y_B\lambda_A$

Calculation: From formula,

$$\therefore \quad y_B = \frac{y_A\lambda_B}{\lambda_A} = \frac{0.32\times10^{-3}\times4.8\times10^{-7}}{6\times10^{-7}}$$

$$= 0.256 \times 10^{-3}$$

$$\therefore \quad \text{Change} = |y_A - y_B|$$

$$= |0.320 \times 10^{-3} - 0.256 \times 10^{-3}|$$

$$= 0.064 \times 10^{-3} \text{ m}$$

$$= \mathbf{0.064 \text{ mm}}$$

Ans: The change in distance between the fringes is **0.064 mm.**

5. A parallel beam of green light of wavelength 546 nm passes through a slit of width 0.4 mm. The intensity pattern of the transmitted light is seen on a screen which is 40 cm away. What is the distance between the two first order minima?

Solution:

Given: $\lambda = 546 \text{ nm} = 546 \times 10^{-9}$ m, a = 0.4 mm = 0.4×10^{-3} m

D = 40 cm = 40×10^{-2} m

To find: Distance between two first order minima

Formula: Width of central maxima, $W_c = \frac{2\lambda D}{a}$

Calculation: From formula,

$$W_c = \frac{2 \times 546 \times 10^{-9} \times 40 \times 10^{-2}}{0.4 \times 10^{-3}}$$

$= 2 \times 546 \times 10^{-6}$

$= 1092 \times 10^{-6}$

$= 1.092 \times 10^{-3}$ m

≈ 1.1 mm

Distance between two first order minima

= Width of central maxima = **1.1 mm**

Ans: Distance between two first order minima is **1.1 mm.**

6. In Fraunhofer diffraction by a narrow slit, a screen is placed at a distance of 2 m from the lens to obtain the diffraction pattern. If the slit width is 0.2 mm and the first minimum is 5 mm on either side of central maximum. Find the wavelength of light.

Solution:

Given: D = 2 m, a = 0.2 mm = 2×10^{-4} m, y_{1d} = 5 mm

Width of central maxima = $2y_{1d} = 2 \times 5$ mm

= 10 mm = 10×10^{-3} m

To find: Wavelength of light (λ)

Formula: Width of central maxima, $W_c = \frac{2\lambda D}{a}$

Calculation: From formula,

$$10 \times 10^{-3} = \frac{2 \times \lambda \times 2}{2 \times 10^{-4}}$$

$\therefore$ $$\lambda = \frac{10 \times 10^{-3} \times 2 \times 10^{-4}}{2 \times 2} = 5 \times 10^{-7} \text{ m}$$

= 5000 Å

Ans: Wavelength of the light used is **5000 Å.**

Long Answer (LA) (4 Marks Each)

1. Describe geometry of the Young's double slit experiment with the help of ray diagram. What is fringe width? Obtain an expression of it. Write the conditions for constructive as well as destructive interference.

Ans:

i. Let S_1 and S_2 be the two coherent monochromatic sources which are separated by short distance d. They emit light waves of wavelength λ.

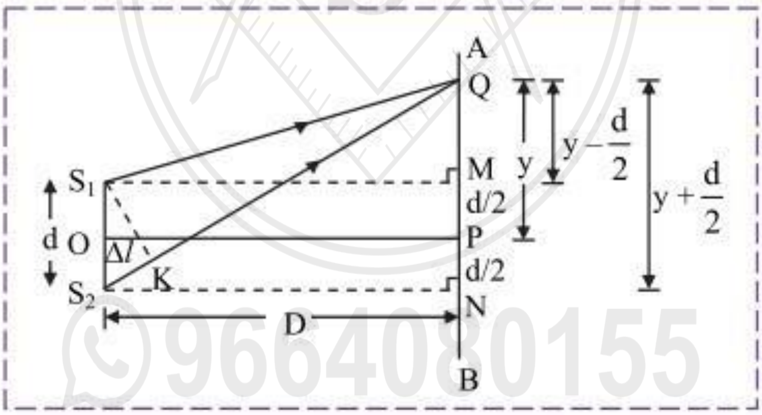
ii. Let D = horizontal distance between screen and source.

iii. Draw S_1M and $S_2N \perp AB$

OP = perpendicular bisector of slit.

Since $S_1P = S_2P$, the path difference between waves reaching P from S_1 and S_2 is zero, therefore there is a bright point at P.

iv. Consider a point Q on the screen which is at a distance y from the central point P on the screen. Light waves from S_1 and S_2 reach at Q simultaneously by covering path S_1Q and S_2Q, where they superimpose.



Derivation:

In Δ S_1MQ, $(S_1Q)^2 = (S_1M)^2 + (MQ)^2$

$$(S_1Q)^2 = D^2 + \left[y - \frac{d}{2}\right]^2 \quad \text{....(1)}$$

In Δ S_2NQ, $(S_2Q)^2 = (S_2N)^2 + (NQ)^2$

$$\therefore \quad (S_2Q)^2 = D^2 + \left[y + \frac{d}{2}\right]^2 \quad \text{....(2)}$$

Subtract equation (1) from (2),

$$(S_2Q)^2 - (S_1Q)^2 = \left[D^2 + \left(y + \frac{d}{2}\right)^2\right] - \left[D^2 + \left(y - \frac{d}{2}\right)^2\right]$$

$$= D^2 + \left(y + \frac{d}{2}\right)^2 - D^2 - \left(y - \frac{d}{2}\right)^2$$

$$= \left(y + \frac{d}{2}\right)^2 - \left(y - \frac{d}{2}\right)^2$$

$$= \left(y^2 + \frac{d^2}{4} + yd\right) - \left(y^2 + \frac{d^2}{4} - yd\right)$$

$$= y^2 + \frac{d^2}{4} + yd - y^2 - \frac{d^2}{4} + yd(S_2Q)^2 - (S_1Q)^2$$

$$= 2yd$$

∴ $(S_2Q + S_1Q)\,(S_2Q - S_1Q) = 2yd$

∴ $S_2Q - S_1Q = \dfrac{2yd}{S_2Q + S_1Q}$ ….(3)

If $y << D$ and $d << D$ then, $S_1Q \approx S_2Q \approx D$

$S_2Q + S_1Q = 2D$

∴ Equation (3) becomes,

$S_2Q - S_1Q = \dfrac{2yd}{2D}$

∴ $S_2Q - S_1Q = \dfrac{yd}{D}$

∴ $\Delta l = \dfrac{yd}{D}$ ….(4)

Equation (4) gives the path difference of two interfering light waves.

Point Q will be bright if,

$\Delta l = n\lambda = 2n\dfrac{\lambda}{2}$

where n = 0, 1, 2,….

∴ $\dfrac{y_n d}{D} = n\lambda = 2n\dfrac{\lambda}{2}$ ….[From equation (4)]

∴ $y_n = n\dfrac{\lambda D}{d}$ ….(5)

Equation (5) represents distance of n^{th} bright fringe from central bright fringe.

Point Q will be dark point if,

$$\Delta l = (2n - 1)\frac{\lambda}{2}$$

where n = 1, 2, 3,……

$\therefore$ $$\frac{y'_n d}{D} = (2n - 1)\frac{\lambda}{2}$$

$\therefore$ $$y'_n = (2n - 1)\frac{\lambda D}{2d} = \left(n - \frac{1}{2}\right)\frac{\lambda D}{d} \quad ....(6)$$

Equation (6) represents distance of n^{th} dark fringe from central maximum.

Fringe width:

The distance between any two successive dark or any two successive bright fringes is equal. This is called the **fringe width** and is given by,

Fringe width = $W = \Delta y = y_{n+1} - y_n = y'_{n+1} - y'_n$

$$W = \lambda\frac{D}{d}$$

Thus, both dark and bright fringes are equidistant and have equal widths.

Conditions for constructive and destructive interference:

The phase difference between the two waves reaching P, from S_1 and S_2 is given by,

$$\Delta\phi = y\frac{d}{D}\left(\frac{2\pi}{\lambda}\right) \quad ....\left(\because \Delta l = \frac{yd}{D}\right)$$

The condition for constructive interference in terms of phase difference is given by,

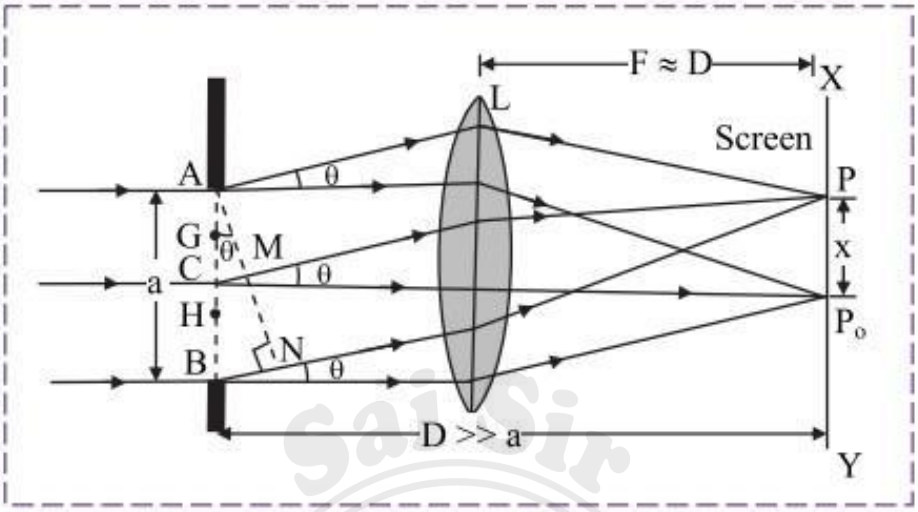
$\Delta\phi = n2\pi$, where, $n = 0, \pm 1 \pm 2$

$\therefore$ The condition for destructive interference in terms of phase difference is given by

$$\Delta\phi = \left(n - \frac{1}{2}\right)2\pi, \text{ where, } n = \pm1, \pm2$$

2. Explain Fraunhofer diffraction at a single slit with neat ray diagram. Obtain expression for width of the central bright fringe.

Ans: Fraunhofer diffraction due to single slit:



i. Consider a narrow slit AB of width 'a', kept perpendicular to the plane of the paper. The slit can be imagined to be divided into extremely thin slits or slit elements. It is illuminated by a parallel beam of monochromatic light of wavelength λ i.e., a plane wavefront is incident on AB.

ii. The diffracted light is focused by a converging lens L, on a screen XY.

iii. The screen is kept in the focal plane of the lens and is perpendicular to the plane of the paper.

iv. Let D be the distance between the slit and the screen.

v. According to Huygens' principle, each and every point of the slit acts as a source of secondary wavelets, spreading in all directions.

Position of central maxima:

i. Let C be the centre of the slit AB. The secondary wavelets travelling parallel to CP_o come to a focus at P_o. The secondary wavelets from points equidistant from C in the upper and lower halves of the slit travel equal paths before reaching P_o.

ii. The optical path difference between all these wavelets is zero and hence they interfere in the same phase forming a bright image at P_o.

iii. The intensity of light is maximum at the point P_o. It is called the central or the principal maxima of the diffraction pattern.

iv. For the line CP_o, angle $\theta = 0°$.

Position of secondary minima:

i. Consider a point P on the screen at which waves travelling in a direction making an angle θ with CP are brought to focus at P by the lens. This point P will be of maximum or minimum intensity because the waves reaching at P will cover unequal distance.

ii. Draw AN perpendicular to the direction of diffracted rays from point A. BN is the path difference between secondary waves coming from A and B.

iii. From ΔABN, $\sin\theta = \frac{BN}{AB}$

∴ $BN = AB\sin\theta = a\sin\theta$

Since θ is very small

∴ $\sin\theta \approx \theta$

∴ $BN = a\theta$

In figure, suppose $BN = \lambda$ and $\theta = \theta_1$ then $\sin\theta_1 = \frac{\lambda}{a}$

iv. Such a point on the screen will be the position of first secondary minimum. It is because, if the slit is assumed to be divided into two equal halves AC and BC, then the waves from corresponding points of two halves of the slit will have a path difference of $\lambda/2$.

It gives rise to destructive interference at P which has minimum intensity.

v. If point P on the screen is such that $BN = 2\lambda$ and angle $\theta = \theta_2$, then, $\sin\theta_2 = \frac{2\lambda}{a}$. Such a point on the screen will be the position of the second secondary minimum.

In general, for n^{th} minimum, $\sin\theta_n = \frac{n\lambda}{a}$

where, $n = \pm1, \pm2, \pm3, \ldots$

vi. If y_{n_d} is the distance of n^{th} minimum from P_o, on the screen, then

$\left(\tan\theta_{n_d}\right) = \frac{y_{n_d}}{D}$.

vii. If θ_{n_d} is very small,

$\tan\theta_{n_d} \approx \sin\theta_{n_d} = \frac{n\lambda}{a}$

∴ $\frac{y_{n_d}}{D} = \frac{n\lambda}{a}$

∴ $y_{n_d} = \frac{n\lambda D}{a} = nW$ ....(1)

where, W is fringe width

Equation (1) gives distance of n^{th} secondary minima from central maxima.

viii. The central bright fringe is spread between the first dark fringes on either side. Hence, width of the central bright fringe is the distance between the centres of first dark fringe on either side.

∴ Width of the central bright fringe,

$$W_c = 2y_{1d} = 2W = 2\left(\frac{\lambda D}{a}\right)$$

This is the required expression for width of the central bright fringe.

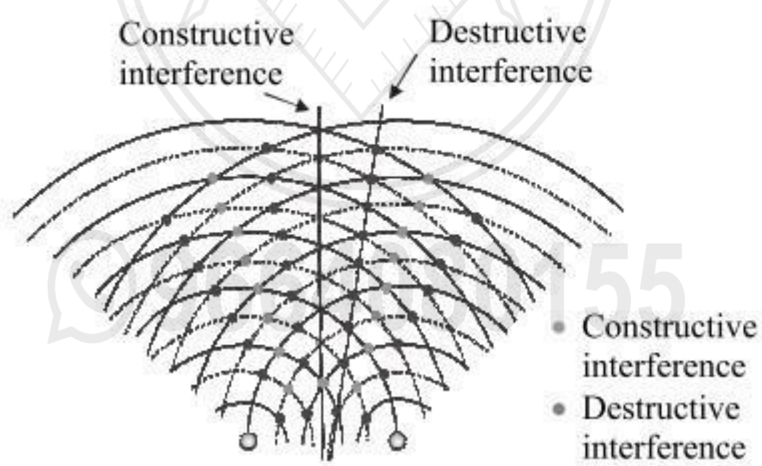
3. What is interference? Explain constructive and destructive interference with the help of diagram. What are coherent sources of light?

Ans: ***Interference of light*** *is defined as the modification in the intensity of light (larger at some places and smaller at some places) produced by the superposition of two or more light waves.*

Constructive and destructive interference:

i. Points where the crest of one wave coincides with the crest of another wave and where the trough of one wave coincides with the trough of another wave are points with the maximum displacement. At these points, displacement is twice that for each wave. These are points of constructive interference.

ii. Points where the crest of one wave is coincident with the trough of another are points with the zero displacement. These are points of destructive interference.



Interference for waves

Two sources which emit light waves of the same frequency having a constant phase difference, independent of time, are called ***coherent sources*** *of light.*

08 Electrostatics

Multiple Choice Questions (1 Mark Each)

1. A metal foil of negligible thickness is introduced between two plates of a capacitor at the centre. The capacitance of capacitor will be
   (A) Half (B) Double
   (C) Same (D) K times

2. Capacitance (in F) of a spherical conductor of radius 1 m is
   (A) 1.1×10^{-10} (B) 9×10^{-9}
   (C) 10^{-6} (D) 10^{-3}

Hint: For isolated sphere, $C = 4\pi\varepsilon_0 r$

$\therefore \quad C = \dfrac{r}{1/4\pi\varepsilon_0} = \dfrac{1}{9 \times 10^9} = 1.1 \times 10^{-10}$ F

3. A dipole is placed in a uniform electric field, its potential energy will be minimum when the angle between its axis and field is
   (A) 2π (B) π
   (C) $\dfrac{\pi}{2}$ **(D) Zero**

4. If an electron is brought towards another electron, the electric potential energy of the system
   (A) decreases **(B) increases**
   (C) Becomes zero (D) Remains same

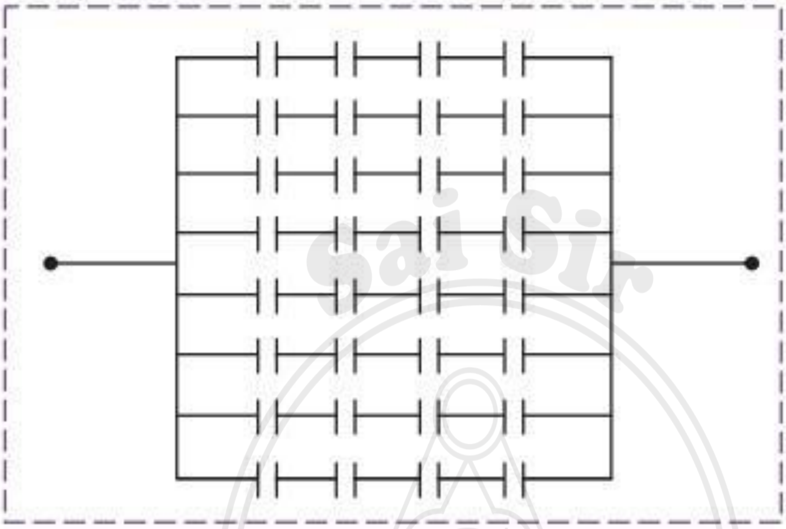
5. The work done in carrying a charge Q once round a circle of radius r with charge q at the centre of the circle is
   (A) $\dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{Q}{r}$ (B) $\dfrac{Q \cdot q}{4\pi\varepsilon_0 r}$
   (C) Zero (D) $\dfrac{Q \cdot q}{2r}$

6. You are given a number of capacitors labelled as 8μF - 250V. Find the number of capacitors needed to get an arrangement equivalent of 16μF - 1000V.

(A) 4 (B) 16

(C) 32 (D) 64

Hint:



When 4 capacitors (8 μF, 250 V) are connected in series, the voltage sums upto 1000 V.

But due to series combination, the effective capacitance reduces to 2 μF.

Hence, such 8 parallel combinations gives the resultant as 16 μF.

∴ Total number of capacitors = 4 × 8 = 32

7. A parallel plate capacitor with oil between the plates (dielectric constant of oil, k = 2) has a capacitance C. If the oil is removed, then the capacitance of the capacitor becomes

(A) 2C (B) $C\sqrt{\pi}$

(C) $\frac{C}{\sqrt{2}}$ **(D) $\frac{C}{2}$**

Hint: Since, $C \propto k$

∴ $\frac{C}{C'} = \frac{2}{1}$

∴ $C' = \frac{C}{2}$

Very Short Answer (VSA) (1 Mark Each)

1. What do you mean by dielectric polarization?

Ans: Dielectric polarization is the term given to describe the behavior of a material when an external electric field is applied on it. It occurs when a dipole moment is formed in an insulating material because of an externally applied electric field.

2. Which physical quantity has its unit as J/C? Is it a scalar or vector?

Ans: Electrostatic potential has its unit as J/C and it is a scalar quantity.

3. What are linear isotropic dielectrics?

Ans: Linear isotropic dielectrics are those substances in which induced dipole moment are in the direction of field and is proportional to field strength.

4. What happens to the energy stored in a capacitor, if the plates of a charged capacitor are drawn apart, the battery remaining connected?

Ans: The plates of the capacitor is still connected to the battery, hence moving the plates further apart decreases the capacitance, hence energy stored in the capacitor decreases.

5. The mean free path of electrons in a metal is 4×10^{-8} m. Find the electric field , in units of V/m, which can give on an average 2 eV energy to an electron in the metal.

Ans: Electric field, $E = \frac{V}{d} = \frac{2}{4 \times 10^{-8}} = 5 \times 10^{7}$ V/m

6. Find the electric potential at the surface of an atomic nucleus(Z=50) of radius 9×10^{-13} cm.

Ans: Electric potential, $V = \frac{1}{4\pi\varepsilon_0} \frac{Ze}{r}$

$$= \frac{9 \times 10^{9} \times 50 \times 1.6 \times 10^{-19}}{9 \times 10^{-13} \times 10^{-2}}$$

$$= 8 \times 10^{6} \text{ V}$$

7. **The capacity of a parallel plate capacitor is 10 μF when the distance between its plates is 9 cm. What will be its capacity if the distance between the plates is reduced by 6 cm.**

Ans: Since, $C \propto \frac{1}{d}$

$$\therefore \quad \frac{C_1}{C_2} = \frac{d_2}{d_1}$$

$$\therefore \quad C_2 = \frac{C_1 \times d_1}{d_2} = \frac{10 \times 10^{-6} \times 9 \times 10^{-2}}{3 \times 10^{-2}} = 30\ \mu F$$

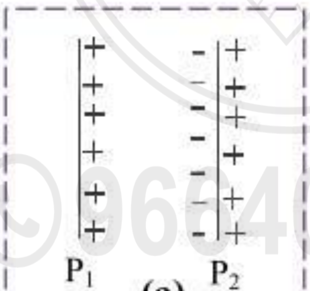
Short Answer I (SA1) (2 Marks Each)

1. **Explain the principle of a capacitor.**

Ans:

i. Consider a metal plate P_1 having area A with some positive charge +Q be given to the plate. Let its potential be V. Its capacity is given by, $C_1 = \frac{Q}{V}$

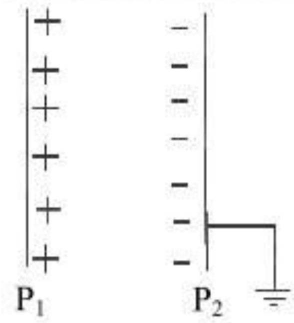
ii. Now consider another insulated metal plate P_2 held near the plate P_1. By induction a negative charge is produced on the nearer face and an equal positive charge develops on the farther face of P_2 as shown in the figure (a) below.



(a)

iii. The induced negative charge lowers the potential of plate P_1, while the induced positive charge raises its potential.

iv. As the induced negative charge is closer to P_1 it is more effective, and thus there is a net reduction in potential of plate P_1.
If the outer surface of P_2 is connected to earth, the induced positive charges on P_2 being free, flows to earth. The induced negative charge on P_2 stays on it, as it is bound to positive charge of P_1. This greatly reduces the potential of P_2 as shown in the figure (b) below.



(b)

v. If V_1 is the potential on plate P_2 due to charge (–Q) then the net potential of the system will now be $+V - V_1$.

Hence the capacity $C_2 = \frac{Q}{V - V_1}$

∴ $C_2 > C_1$

vi. Thus, capacity of metal plate P_1, is increased by placing an identical earth connected metal plate P_2 near it.

2. **Obtain an expression for the electric field intensity at a point outside uniformly charged infinite plane sheet.**

Ans:

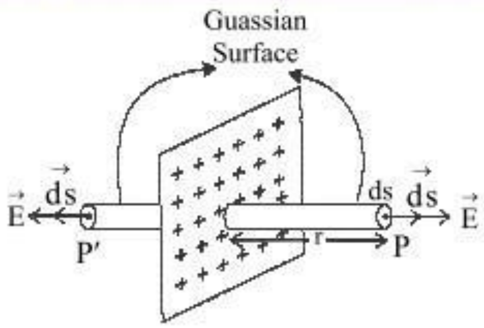
i. Consider a uniformly charged infinite plane sheet with uniform surface charge density (charge per unit area) σ kept in a medium of permittivity $\varepsilon(\varepsilon = \varepsilon_0 k)$.

ii. By Gauss' theorem, the net flux through a closed surface

$\phi = \frac{q}{\varepsilon_0}$ (for air/vacuum k = 1) ….(1)

where q is the total charge inside the closed surface.

iii. To find the electric field due to a charged infinite plane sheet at P at a distance r from sheet, imagine a Gaussian surface around P in the form of a cylinder having cross sectional area A and length 2r with its axis perpendicular to the plane sheet. The plane sheet passes through the middle of the length of the cylinder such that the ends of the cylinder (called end caps P and P′) are equidistant (at a distance r) from the plane sheet as shown in the figure below.



Charged infinite plane sheet

iv. By symmetry electric field is perpendicular to plane sheet and directed outwards, having same magnitude at a given distance on either sides of the sheet. The electric field is at right angles to the end caps and away from the plane. Its magnitude is the same at P and P′. The flux passing through the curved surface is zero as the electric field is tangential to this surface.

∴ The total flux through the closed surface is given by,

$$\phi = \left[\oint Eds\right]_P + \left[\oint Eds\right]_{P'} \quad ....(\because \theta = 0, \cos\theta = 1)$$

$$= EA + EA$$

∴ $\phi = 2EA$ ....(2)

v. From equations (1) and (2),`

$$\frac{q}{\varepsilon_0} = 2EA$$

∴ $\dfrac{\sigma A}{\varepsilon_0} = 2EA$ ....$\left(\because \sigma = \dfrac{q}{A}\right)$

∴ $E = \dfrac{\sigma}{2\varepsilon_0}$

This is the required expression.

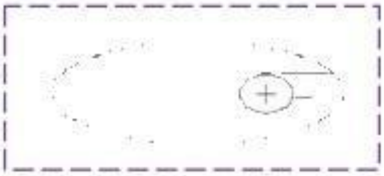
3. What are polar and non polar dielectrics?

Ans:

i. Polar dielectric

a. A dielectric molecule in which the centre of mass of positive charges (protons) does not coincide with the centre of mass of negative charges (electrons), because of the asymmetric shape of the molecules is called polar dielectric.

b. Representation:



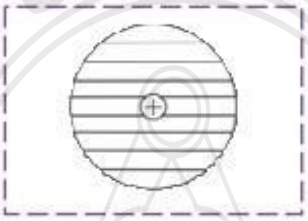
c. They have permanent dipole moments of the order of 10^{-30} Cm. They act as tiny electric dipoles, as the charges are separated by a small distance.

d. Examples: HCl, water, alcohol, NH_3

ii. Non-polar dielectric

a. A dielectric in which the centre of mass of the positive charges coincides with the centre of mass of the negative charges is called a non-polar dielectric.

b. Representation:

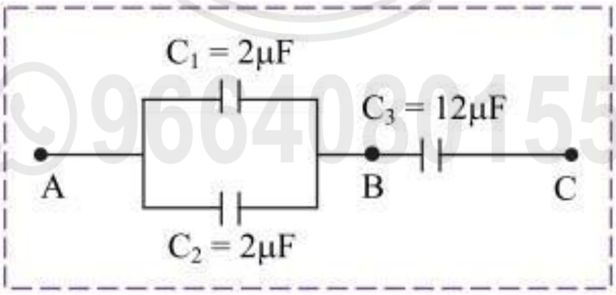


c. These have symmetrical shapes and have zero dipole moment in the normal state.

d. Examples: H_2, N_2, O_2, CO_2, benzene, methane

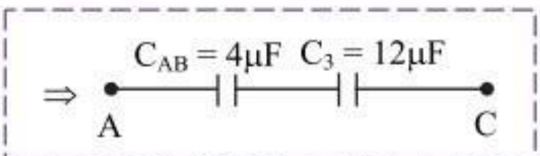
4. Two capacitors each of capacity 2 μF are connected in parallel. This system is connected in series with a third capacitor of 12 μF capacity. Find the equivalent capacity of the system.

Solution:



Here, C_1 and C_2 are in parallel.

∴ $C_{AB} = C_1 + C_2 = 2\ \mu F + 2\ \mu F = 4\ \mu F$



Here, C_{AB} and C_3 are in series,

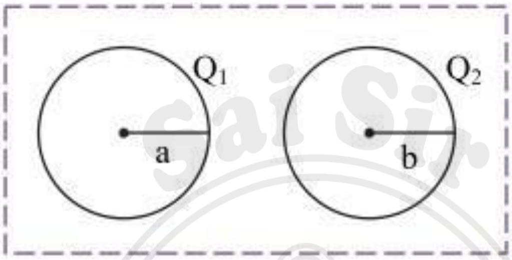
$\therefore \quad \frac{1}{C_{AC}} = \frac{1}{4\mu} + \frac{1}{12\mu}$

$\therefore \quad C_{AC} = \mathbf{3\mu F}$

Ans: The equivalent capacity of the system is **3μF**.

5. Two spheres A and B of radius a and b respectively are at the same potential. Find the ratio of the surface charge densities of A and B.

Solution:



Since, the electric potential is same, $\frac{1}{4\pi\varepsilon_0}\frac{Q_1}{a} = \frac{1}{4\pi\varepsilon_0}\frac{Q_2}{b}$

$\therefore \quad \frac{Q_1}{Q_2} = \frac{a}{b}$

But $\sigma_1 = \frac{Q_1}{4\pi a^2}$ and $\sigma_2 = \frac{Q_2}{4\pi b^2}$

$\therefore \quad \frac{\sigma_1}{\sigma_2} = \frac{Q_1}{Q_2} \times \frac{b^2}{a^2} = \frac{a}{b} \times \frac{b^2}{a^2} = \frac{\mathbf{b}}{\mathbf{a}}$

Ans: The ratio of the surface charge densities of A and B is **b : a.**

6. A molecule with a dipole moment p is placed in an electric field of strength E. Initially the dipole is aligned parallel to the field. If the dipole is to be rotated to be anti-parallel to the field, find the work required to be done by an external agent.

Solution:

When the dipole is aligned parallel to the field, $\theta_0 = 0°$

When the dipole is aligned antiparallel to the field, $\theta = 180°$

$\therefore$ Work done by external torque,

$W = pE\,[\cos\theta_0 - \cos\theta]$

$= pE[\cos 0 - \cos 180]$

$= pE\,[1 - (-1)] = \mathbf{2\ pE}$

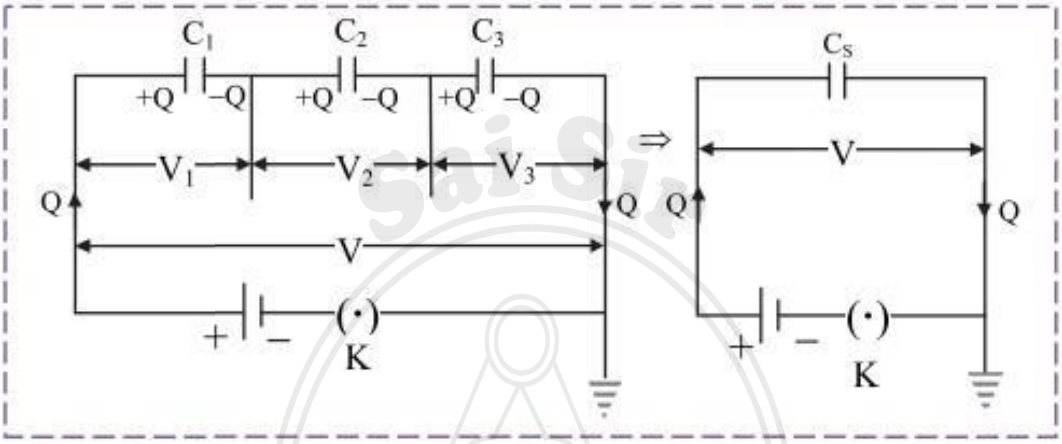
Ans: Work required to be done by an external agent is **2 pE.**

Short Answer II (SA1) (3 Marks Each)

1. Derive an expression for the effective capacitance of three parallel plate capacitors connected in series.

Ans:

i. Diagram:



ii. Explanation:

a. Capacitors are said to be connected in series if they are connected one after the other in the form of a chain.

b. Let capacitors of capacitances C_1, C_2, C_3 be connected in series as shown in the figure.

c. Let V_1, V_2, V_3 be the corresponding potential differences in the capacitors.

d. Suppose a potential difference 'V' is applied across the combination. The left plate of capacitor C_1 has a charge +Q. An equal but opposite charge –Q is induced on the right plate of this capacitor. This charge –Q induces a charge +Q on the left plate of C_2 and so on.

e. Thus, each capacitor receives a magnitude of charge Q. Hence, when the capacitors are connected in series, same current flows through them and all have the same charge +Q induced on the plate.

Thus, potential difference induced across capacitors is given by,

$$V_1 = \frac{Q}{C_1},\ V_2 = \frac{Q}{C_2},\ V_3 = \frac{Q}{C_3} \qquad \text{....(1)}$$

f. Total potential difference 'V' across the combination is given by,

$V = V_1 + V_2 + V_3$

$$\therefore \quad V = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} \qquad \text{....[From equation (1)]}$$

$\therefore \quad V = Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$ ....(2)

g. If these capacitors are replaced by a single capacitor of capacity C_S, such that effective capacity remains same then

$C_S = \frac{Q}{V}$

$\therefore \quad V = \frac{Q}{C_S}$ ....(3)

From equation (2) and (3),

$\frac{Q}{C_S} = Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$

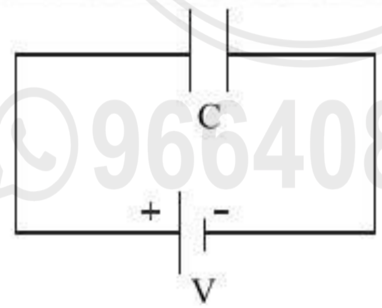
$\therefore \quad \frac{1}{C_S} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$

$\therefore \quad C_S = \frac{C_1C_2C_3}{C_1C_2 + C_2C_3 + C_3C_1}$

2. Obtain an expression for the energy stored in a charged condenser. Express it in different forms.

Ans:

i. Consider a capacitor of capacitance C being charged by a DC source of V volts as shown in figure below.



Capacitor charged by a DC source

ii. During the process of charging, let q′ be the charge on the capacitor and V be the potential difference between the plates. Hence $C = \frac{q}{V}$

iii. A small amount of work is done if a small charge dq is further transferred between the plates.

$\therefore \quad dW = V\,dq = \dfrac{q'}{C}\,dq$

iv. Total work done in transferring the charge

$$W = \int dW = \int_0^Q \frac{q'}{C}dq = \frac{1}{C}\int_0^Q q'\,dq$$

$$= \frac{1}{C}\left[\frac{(q')^2}{2}\right]_0^Q = \frac{1}{2}\frac{Q^2}{C}$$

v. This work done is stored as electrical potential energy U of the capacitor. This work done can be expressed in different forms as follows.

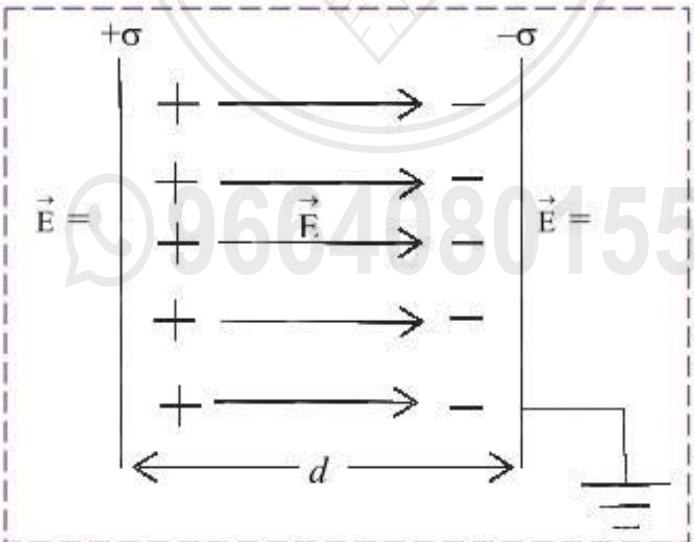
$\therefore \quad U = \dfrac{1}{2}\dfrac{Q^2}{C} = \dfrac{1}{2}CV^2 = \dfrac{1}{2}QV \qquad \text{....}(\because Q = CV)$

3. Obtain an expression for the capacitance of a parallel plate capacitor without a dielectric .

Ans:

i. A parallel plate capacitor consists of two thin conducting plates each of area A, held parallel to each other, at a suitable distance d apart.

ii. The plates are separated by an insulating medium like paper, air, mica, glass etc. One of the plates is insulated and the other is earthed as shown in figure below.



iii. When a charge +Q is given to the insulated plate, then a charge –Q is induced on the inner face of earthed plate and +Q is induced on its farther face. But as this face is earthed the charge +Q being free, flows to earth.

iv. In the outer regions the electric fields due to the two charged plates cancel out. Making net field is zero.

$$E = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = 0$$

v. In the inner regions between the two capacitor plates the electric fields due to the two charged plates add up. The net field is thus

$$E = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0} \quad \text{....(1)}$$

The direction of E is from positive to negative plate.

vi. Let V be the potential difference between the two plates. Then electric field between the plates is given by

$$E = \frac{V}{d} \text{ or } V = Ed \quad \text{....(2)}$$

Substituting equation (1) in equation (2),

$$V = \frac{Q}{A\varepsilon_0} d$$

vii. Thus, capacitance of the parallel plate capacitor is given by,

$$C = \frac{Q}{V} = \frac{Q}{\left(\frac{Qd}{A\varepsilon_0}\right)} = \frac{A\varepsilon_0}{d}$$

This is the required expression.

4. Two parallel plate capacitors X and Y have the same area of the plates and same separation between them, are connected in series to a battery of 15 V. X has air between the plates while Y contains a dielectric of constant k = 2.

i. Calculate the capacitance of each capacitor if equivalent capacitance of the combination is 2 μF.

ii. Calculate the potential difference between the plates of X and Y.

iii. What is the ratio of the electrostatic energy stored in X and Y?

Solution:

Given: $V = 15$ volt, $k_X = 1$, $k_Y = 2$, $C_s = C_{eq} = 2\ \mu F$

To Find:
i. C_X and C_Y
ii. V_X and V_Y
iii. $\frac{E_X}{E_Y}$

Formulae: i. $C = \dfrac{\varepsilon_0 kA}{d}$ ii. $\dfrac{1}{C_S} = \dfrac{1}{C_1} + \dfrac{1}{C_2}$

iii. $Q = CV$ iv. $E = \dfrac{Q^2}{2C}$

Calculation: From formula (i),

$C \propto k$

$\therefore \dfrac{C_X}{C_Y} = \dfrac{k_X}{k_Y} = \dfrac{1}{2}$

$\therefore C_Y = 2C_X$ ....(1)

$\therefore$ From formula (ii),

$\dfrac{1}{C_{eq}} = \dfrac{1}{C_X} + \dfrac{1}{C_Y}$

$\dfrac{1}{2\mu F} = \dfrac{1}{C_X} + \dfrac{1}{2C_X}$

$\therefore C_x = \mathbf{3\ \mu F}$

From equation (1),

$C_Y = \mathbf{6\ \mu F}$

From formula (iii),

$Q = C_{eq} \times V$

$= 2 \times 10^{-6} \times 15$

$= 30\ \mu C$

$\therefore V_x = \dfrac{Q}{C_X} = \dfrac{30 \times 10^{-6}}{3 \times 10^{-6}} = \mathbf{10\ V}$

$\therefore V_y = \dfrac{Q}{C_Y} = \dfrac{30 \times 10^{-6}}{6 \times 10^{-6}} = \mathbf{5\ V}$

From formula (iv),

$\dfrac{E_X}{E_Y} = \dfrac{Q^2}{2C_X} \times \dfrac{2C_Y}{Q^2}$

$= \dfrac{C_Y}{C_X} = \dfrac{6 \times 10^{-6}}{3 \times 10^{-6}} = \mathbf{2}$

Ans: i. Capacitance of each capacitor are **3 μF** and **6 μF**.

ii. Potential difference between the plates of X and Y are **10 V** and **5 V** respectively.

iii. The ratio of electrostatic energy stored in X and Y is **2 : 1**.

5. Find the amount of work done in rotating an electric dipole of dipole moment 3.2×10^{-8} Cm from its position of stable equilibrium to the position of unstable equilibrium in a uniform electric field if intensity is 10^4 N / C.

Solution:

Given: $p = 3.2 \times 10^{-8}$ Cm, $E = 10^4$ N/C

To Find: Work done in rotating dipole (W)

Formula: $W = pE (\cos \theta_0 - \cos \theta)$

Calculation: At stable equilibrium, $\theta_0 = 0°$

At unstable equilibrium, $\theta = 180°$

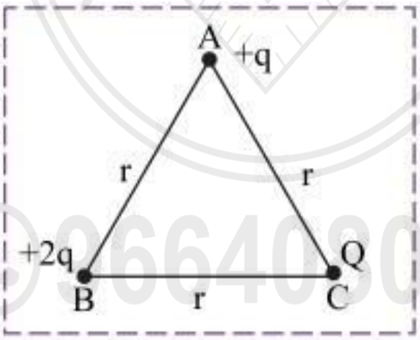
From formula,

$$W = pE (\cos \theta_0 - \cos \theta)$$
$$= 3.2 \times 10^{-8} \times 10^4 (\cos 0 - \cos 180)$$
$$= 3.2 \times 10^{-4} [1 - (-1)]$$
$$= \mathbf{6.4 \times 10^{-4} \ J}$$

Ans: Work done in rotating an electric dipole is $\mathbf{6.4 \times 10^{-4}}$ **J**.

6. Three point charges +q, +2q and Q are placed at the three vertices of an equilateral triangle. Find the value of charge Q (in terms of q), so that electric potential energy of the system is zero.

Solution:



Let 'r' be the side of the equilateral traingle,

$$(U)_{AB} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q.2q}{r}$$

$$(U)_{BC} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2q.Q}{r}$$

$$(U)_{AC} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q.Q}{r}$$

Since, electric potential energy of the system is zero.

∴ $(U)_{AB} + (U)_{BC} + (U)_{AC} = 0$

∴ $$\frac{1}{4\pi\varepsilon_0}\cdot\frac{2q^2}{r}+\frac{1}{4\pi\varepsilon_0}\cdot\frac{2qQ}{r}+\frac{1}{4\pi\varepsilon_0}\cdot\frac{qQ}{r}=0$$

∴ $2q^2 + 2qQ + qQ = 0$

∴ $3qQ = -2q^2$

∴ $$Q = \mathbf{\frac{-2q}{3}}$$

Ans: The value of charge Q (in terms of q) is $\mathbf{\frac{-2q}{3}}$.

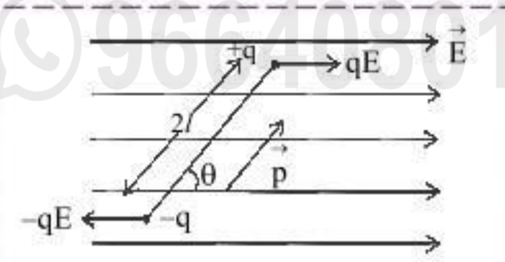
Long Answer (LA) (4 Marks Each)

1. Obtain an expression for the potential energy of a dipole in an external field.

Ans:

i. Consider a dipole with charges –q and +q separated by a finite distance $2l$, placed in a uniform electric field $\vec{E}$.

ii. It experiences a torque $\vec{\tau}$ which tends to rotate it as shown in figure below, $\vec{\tau} = \vec{p} \times \vec{E} = pE \sin\theta$



Couple acting on a dipole

iii. To neutralize this torque, let us assume an external torque $\vec{\tau}_{ext}$ be applied, which rotates it in the plane of the paper from angle θ_0 to angle θ, without angular acceleration and at an infinitesimal angular speed.

iv. Work done by the external torque

$$W = \int_{\theta_0}^{\theta} \tau_{ext}(\theta)d\theta = \int_{\theta_0}^{\theta} pE\sin\theta d\theta$$

$$= pE[-\cos\theta]_{\theta_0}^{\theta}$$

$$= pE[-\cos\theta - (-\cos\theta_0)]$$

$$= pE[-\cos\theta + \cos\theta_0]$$

$$= pE[\cos\theta_0 - \cos\theta]$$

This work done is stored as the potential energy of the system in the position when the dipole makes an angle θ with the electric field.

v. Thus, potential energy of electric dipole in external electric field is,
$U(\theta) - U(\theta_0) = pE(\cos\theta_0 - \cos\theta)$

2. Find the capacitance of a parallel plate capacitor with dielectric slab between the plates.

Ans:

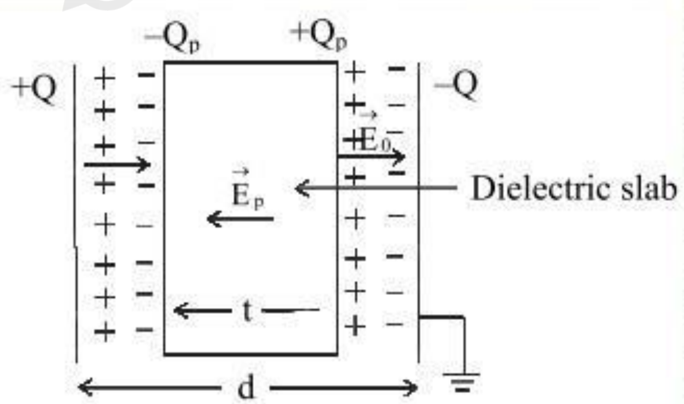
i. Consider a parallel plate capacitor with the two plates each of area A separated by a distance d. The capacitance of the capacitor is given by

$$C_0 = \frac{A\varepsilon_0}{d}$$

ii. Let E_0 be the electric field intensity between the plates before the introduction of the dielectric slab. Then the potential difference between the plates is given by $V_0 = E_0 d$,

where $E_0 = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$, and σ is the surface charge density on the plates.

iii. Let a dielectric slab of thickness t (t < d) be introduced between the plates of the capacitor as shown in the figure below.



Dielectric slab in the capacitor

iv. The field E_0 polarizes the dielectric, inducing charge $-Q_p$ on the left side and $+Q_p$ on the right side of the dielectric.

v. These induced charges set up a field E_p inside he dielectric in the opposite direction of E_0. The induced field is given by

$$E_p = \frac{\sigma_p}{\varepsilon_0} = \frac{Q_p}{A\varepsilon_0} \qquad \ldots\left(\because \sigma_p = \frac{Q_p}{A}\right)$$

vi. The net field (E) inside the dielectric reduces to $E_0 - E_p$.

$$\therefore \quad E = E_0 - E_p = \frac{E_0}{k} \qquad \ldots\left(\because \frac{E_0}{E_0 - E_p} = k\right)$$

where k is a constant called the dielectric constant.

$$\therefore \quad E = \frac{Q}{A\varepsilon_0 k} \text{ or } Q = Ak\varepsilon_0 E \qquad \ldots(2)$$

vii. The field E_p exists over a distance t and E_0 over the remaining distance (d – t) between the capacitor plates. Hence the potential difference between the capacitor plates is

$$V = E_0(d - t) + E(t)$$

$$= E_0(d - t) + \frac{E_0}{k}(t) \qquad \ldots\left(\because E = \frac{E_0}{k}\right)$$

$$= E_0\left[(d - t) + \frac{t}{k}\right]$$

$$= \frac{Q}{A\varepsilon_0}\left[d - t + \frac{t}{k}\right]$$

viii. The capacitance of the capacitor on the introduction of dielectric slab becomes

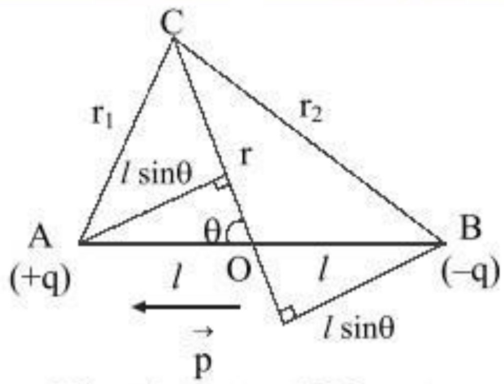
$$C = \frac{Q}{V} = \frac{Q}{\frac{Q}{A\varepsilon_0}\left(d - t + \frac{d}{k}\right)} = \frac{A\varepsilon_0}{\left(d - t + \frac{t}{k}\right)}$$

This is the required expression.

3. Derive an expression for the electric potential due to an electric dipole.

Ans:

i. Consider an electric dipole. Let origin be at the centre of the dipole as shown in figure below.



Electric potential due to an electric Dipole

ii. Let C be any point near the electric dipole at a distance r from the centre O inclined at an angle θ with axis of the dipole.

Let r_1 and r_2 be the distances of point C from charges +q and −q, respectively.

iii. Potential at C due to charge +q at A is,

$$V_1 = \frac{+q}{4\pi\varepsilon_0 r_1}$$

Potential at C due to charge −q at B is,

$$V_2 = \frac{-q}{4\pi\varepsilon_0 r_2}$$

iv. The potential at C due to the dipole is,

$$V_C = V_1 + V_2 = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_1} - \frac{1}{r_2}\right] \quad ....(1)$$

v. By geometry,

$$r_1^2 = r^2 + l^2 - 2rl\cos\theta$$

$$r_2^2 = r^2 + l^2 + 2rl\cos\theta$$

$$r_1^2 = r^2\left(1 + \frac{l^2}{r^2} - 2\frac{l}{r}\cos\theta\right)$$

$$r_2^2 = r^2\left(1 + \frac{l^2}{r^2} + 2\frac{l}{r}\cos\theta\right)$$

For a short dipole, $2l << r$ and

If $r >> l$; $\frac{l}{r}$ is small

$\therefore \quad \frac{l^2}{r^2}$ can be neglected

$$\therefore \quad r_1^2 = r^2\left(1 - 2\frac{l}{r}\cos\theta\right)$$

$$r_2^2 = r^2\left(1 + \frac{2l}{r}\cos\theta\right)$$

$$\therefore \quad r_1 = r\left(1 - \frac{2l}{r}\cos\theta\right)^{\frac{1}{2}}$$

$$r_2 = r\left(1 + \frac{2l}{r}\cos\theta\right)^{\frac{1}{2}}$$

$$\therefore \quad \frac{1}{r_1} = \frac{1}{r}\left(1 - \frac{2l}{r}\cos\theta\right)^{\frac{-1}{2}} \text{ and}$$

$$\frac{1}{r_2} = \frac{1}{r}\left(1 + \frac{2l}{r}\cos\theta\right)^{\frac{-1}{2}} \qquad \text{....(2)}$$

vi. Using equations (1) and (2),

$$V_C = V_1 + V_2$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r}\left(1 - \frac{2l\cos\theta}{r}\right)^{\frac{-1}{2}} - \frac{1}{r}\left(1 + \frac{2l\cos\theta}{r}\right)^{\frac{-1}{2}}\right]$$

vii. Using binomial expansion,

$(1 + x)^n = 1 + nx$, $x << 1$ and retaining terms up to the first order of $\frac{l}{r}$ only, we get

$$V_C = \frac{q}{4\pi\varepsilon_0}\frac{1}{r}\left[\left(1 + \frac{l}{r}\cos\theta\right) - \left(1 - \frac{l}{r}\cos\theta\right)\right]$$

$$= \frac{q}{4\pi\varepsilon_0 r}\left[1 + \frac{l}{r}\cos\theta - 1 + \frac{l}{r}\cos\theta\right]$$

$$= \frac{q}{4\pi\varepsilon_0 r}\left[\frac{2l}{r}\cos\theta\right]$$

$$\therefore \quad V_C = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2} \qquad \text{....}(\because p = q \times 2l)$$

viii. Electric potential at C, can also be expressed as,

$$V_C = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}\cdot\vec{r}}{r^3}$$

$$V_C = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}\cdot\hat{r}}{r^2}, \left(\hat{r} = \frac{\vec{r}}{r}\right) \qquad ....\left(\hat{r} = \frac{\vec{r}}{r}\right)$$

where $\hat{r}$ is a unit vector along the position vector $\overrightarrow{OC} = \hat{r}$

09 Current Electricity

Multiple Choice Questions (1 Mark Each)

1. Kirchhoff's second law (voltage law) is based on
   (A) conservation of charge
   (B) conservation of mass
   (C) conservation of energy
   (D) conservation of momentum

2. When unknown resistance is determined by meter bridge, the error due to contact resistance is minimised by
   (A) connecting both the resistances only in one gap
   (B) interchanging the position of known and unknown resistances
   (C) using uniform wire
   (D) obtaining the null point near the ends of the wire

3. The SI unit of potential gradient is
   (A) V/cm (B) V-m
   (C) V/m (D) V-cm

4. Instrument which can measure terminal potential difference as well as electromotive force (emf) is
   (A) Wheatstone's meter bridge
   (B) voltmeter
   (C) potentiometer
   (D) galvanometer

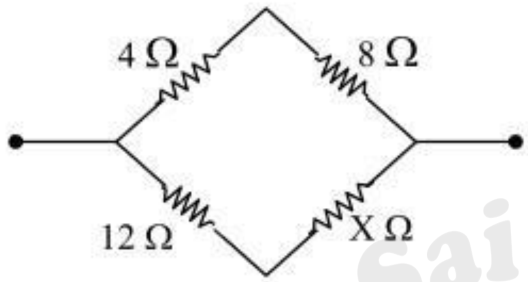
5. When null point is obtained in the potentiometer, the current is drawn from the
   (A) main battery
   (B) cell battery
   (C) both main and cell battery
   (D) neither main nor cell battery

6. If potential gradient of a wire decreases, then its length
   (A) remains constant (B) decreases
   (C) increases (D) none of the above

7. Four resistances 4 Ω, 8 Ω, X Ω and 12 Ω are connected in a series to form Wheatstone's network. If the network is balanced, the value of X is

(A) 24 (B) 18
(C) 12 (D) 8

Hint: Since the network is balanced,



$\therefore \quad \frac{4}{8} = \frac{12}{X}$

$\therefore \quad X = \frac{8 \times 12}{4} = 24\ \Omega$

Very Short Answer (VSA) (1 Mark Each)

Q.1. State Kirchhoff's first (current) law.

Ans: Statement: *The algebraic sum of the currents at a junction is zero in an electrical network.*

i.e., $\sum_{i=1}^{n} I_i = 0$

where I_i is the current in the i^{th} conductor at a junction having n conductors.

Q.2. State Kirchhoff's second (voltage) law.

Ans: Statement: *The algebraic sum of the potential differences (products of current and resistance) and the electromotive forces (emfs) in a closed loop is zero.*

i.e., $\sum IR + \sum E = 0$

Q.3. What is the basis of Kirchhoff's current law and voltage law?

Ans:

i. Kirchhoff's current law is based on the law of conservation of charge.

ii. Kirchhoff's voltage law is based on the law of conservation of energy.

Q.4. Are Kirchhoff's laws applicable to both AC and DC circuits?

Ans: Kirchhoff's laws are applicable for DC as well as AC circuits. They can be accurately used for DC circuits and low frequency AC circuits. In case of AC though, summation of current should be done in vector form or using instantaneous value for the AC components of the circuit.

Q.5. Define potential gradient.

Ans: ***Potential gradient*** *is defined as potential difference per unit length of wire.*

Q.6. On what factors does the potential gradient of the wire depend?

Ans: Potential gradient of the wire depends upon the potential difference between two point on the wire and length of the wire between two points.

Q.7. What is the SI unit of potential gradient?

Ans: SI unit of potential gradient is volt/metre.

Q.8. State any one use of a potentiometer.

Ans:

i. The potentiometer can be used as a voltage divider to continuously change the output voltage of a voltage supply.

ii. Potentiometer can be used as an audio control.

iii. Potentiometer can be used as a sensor.

[Any one use]

Q.9. A voltmeter has resistance of 100 Ω. What will be its reading when it is connected across a cell of emf 6 V and internal resistance 20 Ω?

Ans: Terminal potential, $V = \frac{E}{R + r} \times R = \frac{6}{100 + 20} \times 100 = 5\ V$

Q.10. In a meter bridge, two unknown resistances R and S, when connected between the two gaps, gives a null point is 60 cm from one end. What is the ratio of R and S?

Ans: Here, $l_R = 60$ cm

$\therefore \quad l_S = (100 - 60)$ cm $= 40$ cm

$\therefore \quad \frac{R}{S} = \frac{l_R}{l_S} = \frac{60}{40} = \frac{3}{2}$

Short Answer I (SA1) (2 Marks Each)

Q.1. What are the disadvantages of a potentiometer over a voltmeter?

Ans:

i. Potentiometer is not portable.

ii. Direct measurement of potential difference or emf is not possible.

Q.2. Distinguish between a potentiometer and a voltmeter.

Ans:

No.	Potentiometer	Voltmeter
i.	Its resistance is infinite.	Its resistance is high but finite.
ii.	It does not draw any current from the source of known e.m.f.	It draws some current from the source of e.m.f.
iii.	The potential difference measured by it is equal to actual potential difference (p.d.).	The potential difference measured by it is less than the actual potential difference (p.d.).
iv.	It has high sensitivity.	It has low sensitivity.
v.	It measures e.m.f as well as p.d.	It measures only p.d.
vi.	It is used to measure internal resistance of a cell.	It cannot be used to measure the internal resistance of a cell.
vii.	It is more accurate.	It is less accurate.
viii.	It does not give direct reading.	It gives direct reading.
ix.	It is not portable.	It is portable.
x.	It is used to measure lower voltage values only.	It is used to measure lower as well as higher voltage values.

[Any four differences]

Q.3. Distinguish between an ammeter and a voltmeter.

Ans:

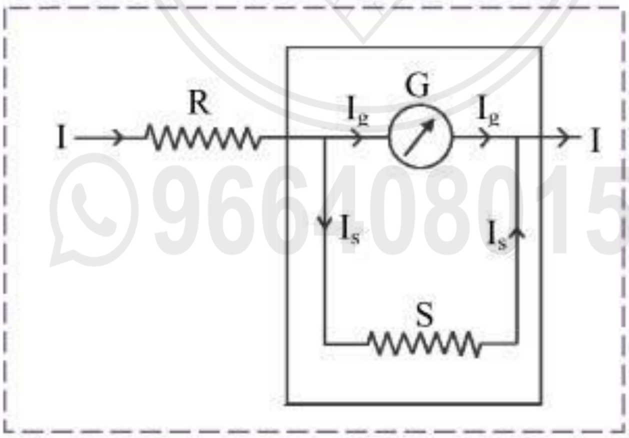
	Ammeter	Voltmeter
i.	It measures current.	It measures potential difference
ii.	It is connected in series.	It is connected in parallel.
iii.	It is an MCG with low resistance. (Ideally zero)	It is an MCG with high resistance. (Ideally infinite)
iv.	Smaller the shunt, greater will be the current measured.	Larger its resistance greater will be the potential difference measured.
v.	Resistance of ammeter, $R_A = \frac{SG}{S+G} = \frac{G}{n}$	Resistance of voltmeter, $R_v = G + X = Gn_v$

[Any four differences]

Q.4. How do you calculate the shunt required to increase the range n times?

Ans:

i. In the arrangement as shown in the figure, I and I_g are the current through the circuit and galvanometer respectively.
Therefore, the current through shunt S is, $I_s = (I - I_g)$



ii. Since S and G are parallel, potential difference across them is same.

∴ $GI_g = S\,I_s$

∴ $GI_g = S\,(I - I_g)$

∴ $S = \left(\frac{I_g}{I - I_g}\right)G$ ....(1)

Equation (1) is useful to calculate the range of current that the galvanometer can measure.

iii. If the current I is n times current I_g, then $I = n\ I_g$.
Using this in equation (1),

$$S = \left(\frac{GI_g}{nI_g - I_g}\right)$$

$\therefore$ $$S = \frac{G}{n-1}$$

This is the required shunt to increase the range n times.

*[**Note:** The framing of question is modified to remove the ambiguity from question.]*

Q.5. Define: i. electrical circuit ii. Junction

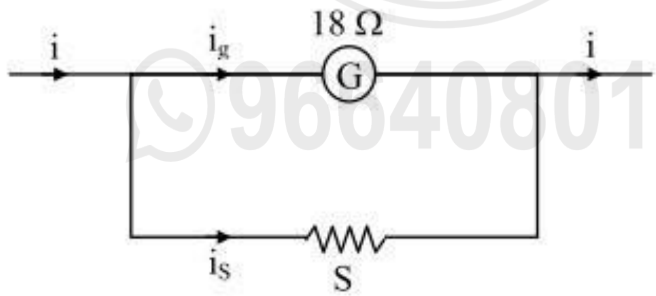
Ans:

i. **Electrical circuit:** *A continuous conducting path consisting of wires and other resistances and a switch between the two terminals of a cell or a battery along which an electric current flow is called a electrical circuit.*

ii. **Junction:** *Any point in an electric circuit where two or more conductors are joined together is a junction.*

Q.6. Calculate the value of the shunt resistance when connected across a galvanometer of resistance 18 Ω will allow 1/10 th of the current to pass through the galvanometer.

Solution:



Here, $i_g = \left(\frac{S}{S+18}\right)i$

Given: $i_g = \frac{i}{10}$

$\therefore$ $$\left(\frac{S}{S+18}\right)i = \frac{i}{10}$$

∴ $10\,S = S + 18$

∴ $9\,S = 18$

∴ $S = \mathbf{2\,\Omega}$

Ans: The value of the shunt resistance is **2 Ω**.

Q.7. Four resistances 6 Ω, 6 Ω, 6 Ω and 18 Ω form a Wheatstone bridge. Find the resistance which connected across the 18 Ω resistance will balance the network.

Solution:

Given: $P = Q = R = 6\,\Omega$

To find: Resistance (X)

Formula: $\frac{P}{Q} = \frac{R}{S}$

Calculation: Let resistance connected across 18 Ω be X.

Equivalent resistance for 18 Ω and X in parallel is given by,

$$X' = S = \frac{18X}{18 + X}$$

From formula,

$$\frac{6}{6} = \frac{6}{\frac{18X}{18+X}}$$

∴ $1 = \frac{6(18+X)}{18X}$

∴ $18X = 108 + 6X$

∴ $12X = 108$

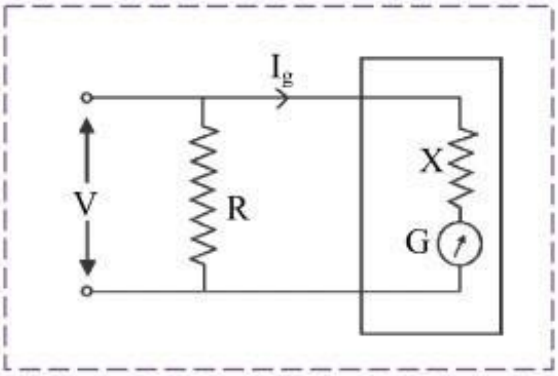
∴ $X = \mathbf{9\,\Omega}$

Ans: The resistance connected across 18 Ω resistance to balance the network is **9 Ω**.

Q.8. The maximum safe voltage that can be measured using a galvanometer of resistance G is V_m . Find the resistance to be connected in series with the galvanometer so that it becomes a voltmeter of range nV_m.

Ans:

i. Let 'X' be the resistance connected in series with the galvanometer as shown in figure.



ii. If V is the voltage to be measured, then

$V = I_g X + I_g G$

$\therefore$ $I_g X = V - I_g G$

$\therefore$ $X = \frac{V}{I_g} - G$ ....(1)

where I_g is the current flowing through the galvanometer.

iii. If voltage V is n times voltage V_m (voltage across galvanometer) then,

$V = n V_m = n(I_g G)$

Using this in equation (1),

$\therefore$ $X = \frac{nI_g G}{I_g} - G$

$\therefore$ $X = G(n - 1)$

Short Answer II (SA2) (3 Marks Each)

Q.1. Explain with a neat circuit diagram. How you will determine the unknown resistances using a meter bridge.

Ans: Construction:

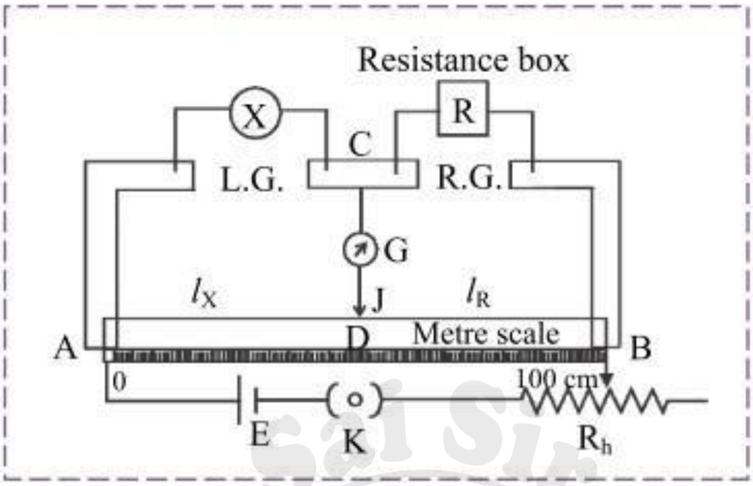
i. Metrebridge consists of a one metre long wire of uniform cross section, stretched on a metre scale which is fixed on a wooden table.

ii. The ends of the wire are fixed below two L shaped metallic strips. A single metallic strip separates the two L shaped strips leaving two gaps, left gap and right gap.

iii. Usually, an unknown resistance X is connected in the left gap and a resistance box is connected in the other gap.

iv. One terminal of a galvanometer is connected to the point C on the central strip, while the other terminal of the galvanometer carries the jockey (J). Temporary contact with the wire AB can be established with the help of the jockey.

v. A cell of emf E along with a key and a rheostat are connected between the points A and B.



Working:

i. A suitable resistance R is selected from resistance box.

ii. The jockey is brought in contact with AB at various points on the wire AB and the balance point (null point), D is obtained. The galvanometer shows no deflection when the jockey is at the balance point (point D).

iii. Let the respective lengths of the wire between A and D, and that between D and C be l_x and l_R .

iv. Then using the balancing conditions,

$$\frac{X}{R} = \frac{R_{AD}}{R_{DB}} \qquad ....(1)$$

where R_{AD} and R_{DB} are resistance of the parts AD and DB of the wire respectively.

v. If l is length of the wire, ρ is its specific resistance, and A is its area of cross section then

$$R_{AD} = \frac{\rho l_x}{A} \qquad ....(2)$$

$$R_{DB} = \frac{\rho l_R}{A} \qquad ....(3)$$

From equations (1), (2) and (3),

$$\frac{X}{R} = \frac{R_{AD}}{R_{DC}} = \frac{\rho l_x / A}{\rho l_R / A}$$

$$\therefore \quad \frac{X}{R} = \frac{l_x}{l_R}$$

$\therefore \quad X = \frac{l_x}{l_R} R$

Thus, knowing R, l_x and l_R, the value of the unknown resistance can be determined.

Q.2. State any two sources of errors in the metre bridge experiment. Explain how they can be minimised.

Ans: Sources of errors:

i. The cross section of the wire may not be uniform.

ii. The ends of the wire are soldered to the metallic strip where contact resistance is developed, which is not taken into account.

iii. The measurements of l_x and l_R may not be accurate.

To minimize the errors

i. The value of R is so adjusted that the null point is obtained around middle one third of the wire (between 34 cm and 66 cm) so that percentage error in the measurement of l_x and l_R are minimum and nearly the same.

ii. The experiment is repeated by interchanging the positions of unknown resistance X and known resistance box R.

iii. The jockey should be tapped on the wire and not slided. The jockey is used to detect whether there is a current through the central branch. This is possible only by tapping the jockey.

[Any two sources and their minimization]

Q.3. What is potential gradient? How is it measured? Explain.

Ans:

i. Potential gradient (K) is defined as potential difference per unit length of wire.

ii. It is measured as, $\frac{V}{L} = \frac{ER}{L(R+r)}$

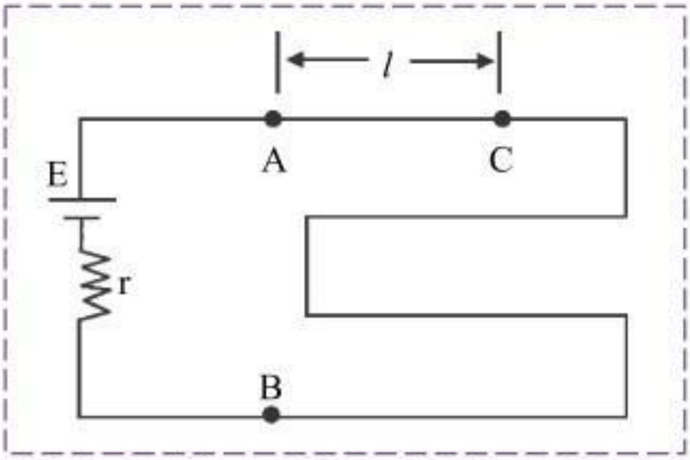
where, V = Potential difference between two points

L = Length (distance) between two points

iii. Explanation:

a. A potentiometer consists of a long wire AB of length L and resistance R having uniform cross sectional area A.

b. A cell of emf E having internal resistance r is connected across AB as shown in the figure.



c. When the circuit is switched on, current I pass through the wire.

Current through AB, $I = \dfrac{E}{R + r}$ ....(1)

d. Potential difference across AB,

$V_{AB} = IR$

$V_{AB} = \dfrac{ER}{(R + r)}$ ....[From equation (1)]

e. Therefore, the potential difference per unit length of the wire is,

$\dfrac{V_{AB}}{L} = \dfrac{ER}{L(R + r)}$

As long as E remains constant, $\dfrac{V_{AB}}{L}$ will remain constant.

f. $\dfrac{V_{AB}}{L}$ is known as potential gradient along AB and is denoted by K. Potential gradient can be defined as potential difference per unit length of wire.

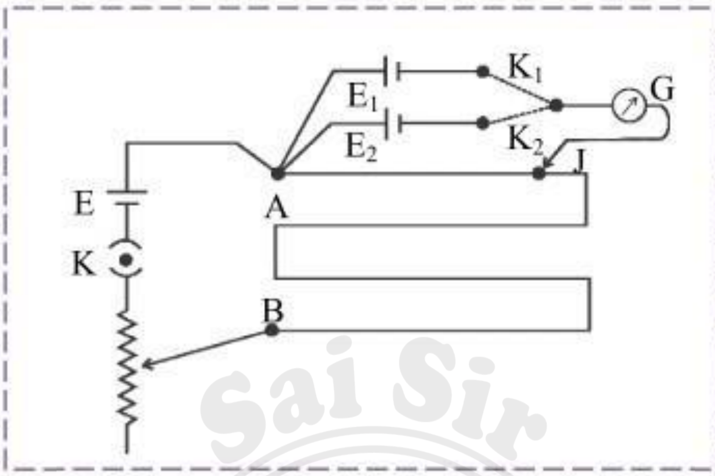
Q.4. Describe how a potentiometer is used to compare the emf's of two cells by connecting the cells individually.

Ans:

i. A potentiometer circuit is set up by connecting a battery of emf E, with a key K and a rheostat such that point A is at higher potential than point B.

ii. The cells whose emfs are to be compared are connected with their positive terminals at point A and negative terminals to the extreme terminals of a two way key K_1 and K_2.

iii. The central terminal of the two ways key is connected to a galvanometer. The other end of the galvanometer is connected to a jockey (J).



iv. Key K is closed and then, key K_1 is closed and key K_2 is kept open. Therefore, the cell of emf E_1 comes into circuit.

v. The null point is obtained by touching the jockey at various points on the potentiometer wire AB.

vi. Let l_1 be the length of the wire between the null point and the point A. Here, l_1 corresponds to emf E_1 of the cell. Therefore,

$E_1 = K\, l_1$ ....(1)

where K is the potential gradient along the potentiometer wire.

vii. Now key K_1 is kept open and key K_2 is closed. The cell of emf E_2 now comes in the circuit. Again, the null point is obtained with the help of the Jockey.

viii. Let l_2 be the length of the wire between the null point and the point A. Here l_2 corresponds to the emf E_2 of the cell.

∴ $E_2 = K\, l_2$ ....(2)

ix. Dividing equation (1) by equation (2),

$$\frac{E_1}{E_2} = \frac{l_1}{l_2}$$

Thus, emfs of the two cells can be compared and if any one of the emfs is known, the other can be determined.

Q.5. A cell of e.m.f 1.5 V and negligible internal resistance is connected in series with a potentiometer of length 10 m and total resistance 20 Ω. What resistance should be introduced in the resistance box such that the potential drop across the potentiometer is one microvolt per cm of the wire?

Solution:

Given: $R = 20\ \Omega$, $L = 10$ m, $E = 1.5$ V,

$K = 1\ \mu V/cm = 1 \times (10^{-6}/10^{-2})\ V/m$

$= 10^{-4}\ V/m$

To find: External resistance (R_E)

Formula: $K = \dfrac{V}{L}$

Calculation: Since, $I = \dfrac{E}{R + R_E}$

Also, $V = IR = \dfrac{ER}{R + R_E}$

From formula,

$$K = \frac{ER}{(R + R_E)L}$$

$$R + R_E = \frac{ER}{KL}$$

$$\therefore \quad R_E = \frac{1.5 \times 20}{10^{-4} \times 10} - 20 = 30000 - 20$$

$$\therefore \quad R_E = \mathbf{29980\ \Omega}$$

Ans: The external resistance should be **29980 Ω**.

Q.6. In a meter bridge, the balance point is found to be at 39.5 cm from the end A when the resistor R is of 12.5 Ω (right gap).

i. Determine the resistance of X (left gap).

ii. Determine the balance point of the bridge if X and R are interchanged?

iii. What happens if the galvanometer and cell are interchanged at the balance point of the bridge?

Ans: Given: $l_X = 39.5$ cm, $R = 12.5\ \Omega$,

$l_R = (100 - l_X) = 100 - 39.5 = 60.5$ cm.

i. Since, the bridge is balanced,

$$\frac{X}{R} = \frac{l_X}{l_R}$$

$$\therefore \quad X = \frac{l_X}{l_R} \times R$$

$$= \frac{39.5}{60.5} \times 12.5 = 8.16\ \Omega$$

ii. When X and R are interchanged then their respective lengths also gets interchange.

∴ The new balance point will be at 60.5 cm

iii. If the galvanometer and the cell are interchanged then the balance point remains unchanged.

Q.7. The emf of a standard cell is 1.5V and is balanced by a length of 300 cm of a potentiometer with 10 m long wire. Find the percentage error in a voltmeter which balances at 350 cm when its reading is 1.8 V.

Solution:

Given: $E_1 = 1.5$ V, $l_1 = 300$ cm, $l_2 = 350$ cm

From individual cell method of potentiometer,

$$\frac{E_1}{E_2} = \frac{l_1}{l_2}$$

∴ $E_2 = E_1 \times \frac{l_2}{l_1}$

$$= 1.5 \times \frac{350}{300} = 1.75 \text{ V}$$

But given reading is 1.8 V

∴ Error = 1.8 – 1.75 = 0.05 V

∴ Percentage error $= \frac{0.05}{1.75} \times 100$

$= \mathbf{2.8571\%}$

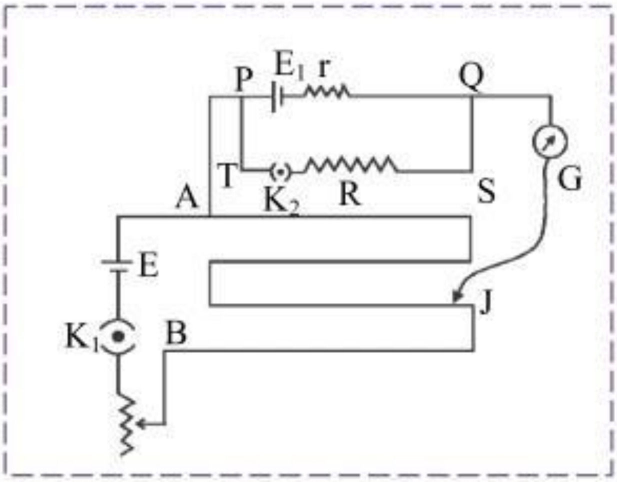
Ans: The percentage error in a voltmeter is **2.8571%**.

Long Answer (LA) (4 Marks Each)

Q.1. Describe with the help of a neat circuit diagram how you will determine the internal resistance of a cell by using a potentiometer. Derive the necessary formula.

Ans:

i. The experimental set up for this method consists of a potentiometer wire AB connected in series with a cell of emf E, the key K_1, and rheostat as shown in figure.



ii. The terminal A is at higher potential than terminal B. A cell of emf E_1 whose internal resistance r is to be determined is connected to the potentiometer wire through a galvanometer G and the jockey J.

iii. A resistance box R is connected across the cell E_1 through the key K_2. The key K_1 is closed and K_2 is open.

iv. The circuit now consists of the cell E, cell E_1, and the potentiometer wire. The null point is then obtained.

v. Let l_1 be length of the potentiometer wire between the null point and the point A. This length corresponds to emf E_1.

∴ $E_1 = Kl_1$ ….(1)

where K is potential gradient of the potentiometer wire which is constant.

vi. Now both the keys K_1 and K_2 are closed so that the circuit consists of the cell E, the cell E_1, the resistance box, the galvanometer and the jockey. Some resistance R is selected from the resistance box and null point is obtained.

vii. The length of the wire l_2 between the null point and point A is measured. This corresponds to the voltage between the null point and point A.

∴ $V = Kl_2$ ….(2)

Dividing equation (1) by equation (2),

∴ $\frac{E_1}{V} = \frac{Kl_1}{Kl_2} = \frac{l_1}{l_2}$ ….(3)

viii. Consider the loop PQSTP,

$E_1 = IR + Ir$ and $V = IR$

∴ $\frac{E_1}{V} = \frac{IR + Ir}{IR} = \frac{R + r}{R} = \frac{l_1}{l_2}$ ….[From equation (3)]

$$\Rightarrow r = R\left(\frac{l_1}{l_2} - 1\right)$$

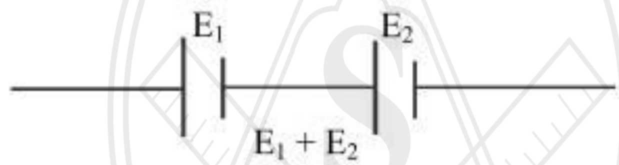
The above equation is used to determine the internal resistance of the cell.

Q.2. Describe how a potentiometer is used to compare the emf's of two cells by the combination method.

Ans:

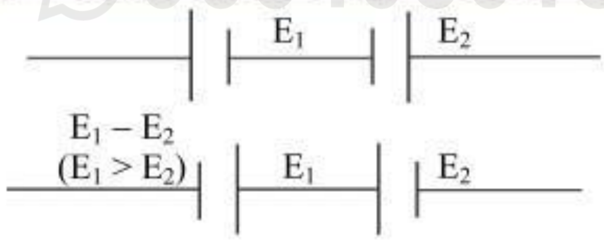
i. The emfs of two cells can be compared using sun and difference method.

ii. When two cells are connected such that the positive terminal of the first cell is connected to the negative terminal of the second cell, the emf of the two cells are added up and the effective emf of the combination of two cells is $E_1 + E_2$. This method of connecting two cells is called the sum method.



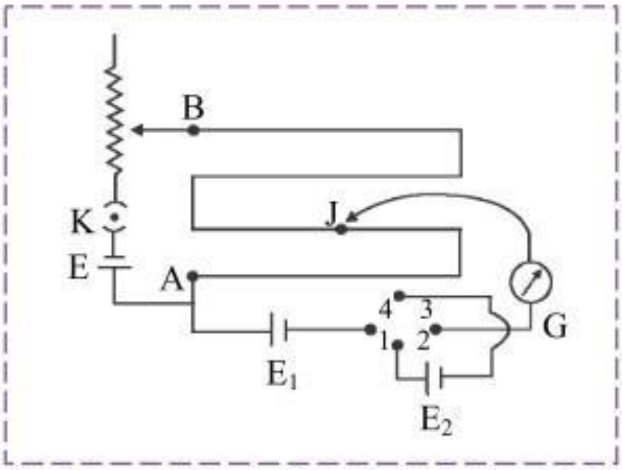
Sum method

iii. When two cells are connected such that their negative terminals are together or their positive terminals are connected together, then their emf oppose each other and effective emf of the combination of two cells is $E_1 - E_2$ (Considering $E_1 > E_2$). This method of connecting two cells is called the difference method.



Difference method

iv. Circuit for sum and difference method is connected as shown in below figure When keys K_1 and K_3 are closed the cells E_1 and E_2 are in the sum mode. The null point is obtained using the jockey.



v. Let l_1 be the length of the wire between the null point and the point A. This corresponds to the emf $(E_1 + E_2)$.

∴ $E_1 + E_2 = K\,l_1$ ....(1)

where K is the potential gradient along the potentiometer wire.

vi. Now the key K_1 and K_3 are kept open and keys K_2 and K_4 are closed. In this case the two cells are in the difference mode.

vii. Again the null point is obtained. Let l_2 be the length of the wire between the null point and the point A. This corresponds to emf $(E_1 - E_2)$.

∴ $E_1 - E_2 = Kl_2$ ....(2)

viii. Dividing equation (1) by equation (2),

$$\frac{E_1 + E_2}{E_1 - E_2} = \frac{l_1}{l_2}$$

By componendo and dividendo method,

$$\frac{(E_1 + E_2) + (E_1 - E_2)}{(E_1 + E_2) - (E_1 - E_2)} = \frac{l_1 + l_2}{l_1 - l_2}$$

∴ $$\frac{E_1}{E_2} = \frac{l_1 + l_2}{l_1 - l_2}$$

Thus, emf of two cells can be compared using sum and difference method.

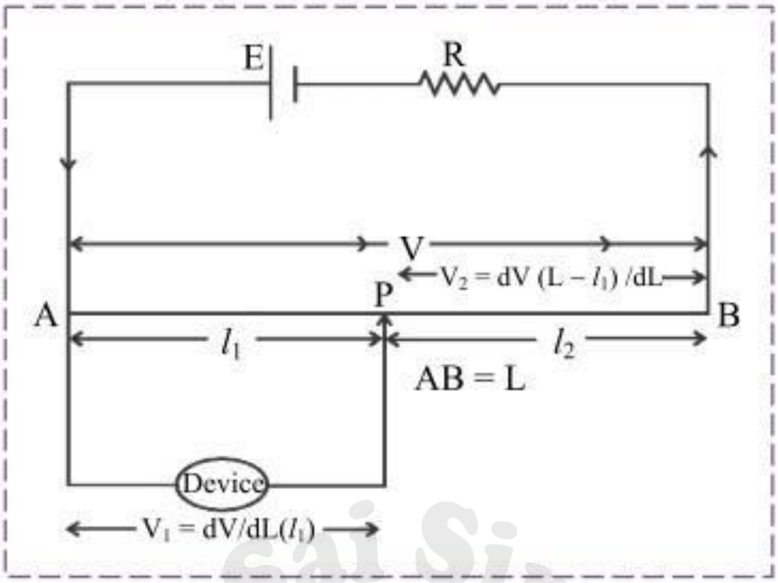
Q.3. State the uses of a potentiometer. Why is a potentiometer preferred over a voltmeter for measuring emf?

Ans:

i. Uses of a potentiometer:

a. Potentiometer as a voltage Divider:

1. The potentiometer can be used as a voltage divider to continuously change the output voltage of a voltage supply.



2. As shown in the above figure, potential V is set up between points A and B of a potentiometer wire.
3. One end of a device is connected to positive point A and the other end is connected to a slider that can move along wire AB.
4. The voltage V gets divided in proportion of lengths l_1 and l_2, such that

$$V_1 = \frac{dV(l)}{dL} \quad \text{and}$$

$$V_2 = \frac{dV(L - l_1)}{dL}$$

b. Potentiometer as an audio control:

1. Sliding potentiometers are commonly used in modern low-power audio systems as audio control devices.
2. Both sliding (faders) and rotary potentiometers (knobs) are regularly used for frequency attenuation, loudness control and for controlling different characteristics of audio signals.

c. Potentiometer as a sensor:

1. If the slider of a potentiometer is connected to the moving part of a machine, it can work as a motion sensor.
2. A small displacement of the moving part causes changes in potential which is further amplified using an amplifier circuit.
3. The potential difference is calibrated in terms of the displacement of the moving part.

ii. Potentiometer preferred over a voltmeter for measuring emf due to following reasons:

a. Potentiometer is more sensitive than a voltmeter.

b. A potentiometer can be used to measure a potential difference as well as an emf of a cell. A voltmeter always measures terminal potential difference, and as it draws some current, it cannot be used to measure an emf of a cell.

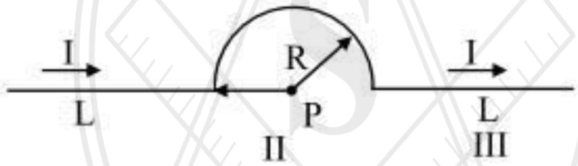
c. Measurement of potential difference or emf is very accurate in the case of a potentiometer. A very small potential difference of the order 10^{-6} volt can be measured with it. Least count of a potentiometer is much better compared to that of a voltmeter.

10 Magnetic Fields due to Electric Current

Multiple Choice Questions (1 Mark Each)

1. According to right hand rule, the direction of magnetic induction if the current is directed in anticlockwise direction is
   (A) perpendicular and inwards.
   (B) perpendicular and outwards.
   (C) same as current.
   (D) opposite to that of current.

2. A conductor has three segments; two straights of length L and a semi-circular with radius R. It carries a current I. What is the magnetic field B at point P?



(A) $\frac{\mu_0 I}{4\pi R}$ (B) $\frac{\mu_0}{4\pi}\frac{I}{R^2}$

(C) $\frac{\mu_0 I}{4R}$ (D) $\frac{\mu_0 I}{4\pi}$

Hint: Applying Biot-Savart law to the 3 sections of the wire.

For the section (i) and (iii) the angle between the current-length elements $I\,d\vec{l}$ and $\vec{R}$ is 180° and 0°, respectively.

$\therefore \quad dB = \frac{\mu_0}{4\pi}\frac{I\,dl\sin(180)^\circ}{R^2} = \frac{\mu_0}{4\pi}\frac{I\,dl\sin(0)^\circ}{R^2} = 0$

$\Rightarrow B_I = B_{III} = 0$

For section (ii), $d\vec{l}$ is always perpendicular to $\vec{R}$.

$\therefore \quad (dB)_{II} = \frac{\mu_0}{4\pi}\frac{I\,dl\sin(90)^\circ}{R^2} = \frac{\mu_0}{4\pi}\frac{I\,dl}{R^2}$

Integrating, $(B)_{II} = \frac{\mu_0}{4\pi}\frac{I}{R^2}\int_0^{\pi R} dl = \frac{\mu_0}{4\pi}\frac{I}{R^2}\pi R$

$\therefore \quad (B)_{II} = \frac{\mu_0}{4}\frac{I}{R}$

Total $B = B_I + B_{II} + B_{III} = 0 + \frac{\mu_0 I}{4R} + 0 = \frac{\mu_0 I}{4R}$

Direction of B at O is coming out of the plane of the paper.

*[**Note:** Answer calculated above is in accordance with textual methods of calculation.]*

3. A strong magnetic field is applied on a stationary electron. Then the electron
   (A) moves in the direction of the field.
   (B) remains stationary.
   (C) moves perpendicular to the direction of the field.
   (D) moves opposite to the direction of the field.

Hint: Magnetic force acts on a moving charge. Since, electron is stationary, no magnetic force will act upon it.

4. The force between two parallel current carrying conductors is F. If the current in each conductor is doubled, then the force between them becomes
   (A) 4F (B) 2F
   (C) F (D) F/4

Hint: $F = \frac{\mu_0}{4\pi}\frac{2I_1I_2}{r}$

After doubling current,

$F' = \frac{\mu_0}{4\pi}\frac{2\times 2I_1 \times 2I_2}{r} = 4F$

5. Which of the following is not a unit of magnetic induction?
   (A) gauss (B) tesla
   (C) oersted (D) Wb/m^2

6. The magnetic dipole moment of current loop is independent of
   (A) number of turns.
   (B) area of loop.
   (C) current in the loop.
   (D) magnetic field in which it is lying.

7. Circular loop of radius 0.0157 m carries a current 2 A. The magnetic field at the centre of the loop is

(A) 1.57×10^{-3} Wb/m^2

(B) 8.0×10^{-5} Wb/m^2

(C) 2.0×10^{-3} Wb/m^2

(D) 3.14×10^{-1} Wb/m^2

Hint: $B_c = \dfrac{\mu_0 I}{2R} = \dfrac{4\pi \times 10^{-7} \times 2}{2 \times 1.57 \times 10^{-2}} = \dfrac{4 \times 3.14 \times 10^{-7} \times 2}{2 \times 1.57 \times 10^{-2}} = 8.0 \times 10^{-5}$ Wb/m^2

Very Short Answer (VSA) (1 Mark Each)

Q.1. What is Lorentz force?

Ans: When a charged particle moves through a region in which both electric and magnetic fields are present, then the net force experienced by that charged particle is sum of electrostatic force and magnetic force and is called as Lorentz force.

Q.2. What is solenoid?

Ans: A solenoid is a long, insulated copper wire closely wound on a hollow cylindrical glass or plastic tube in the form of a helix.

Q.3. What is toroid?

Ans: A toroid is a solenoid of finite length bent into a hollow circular tube.

Q.4. Calculate the value of magnetic field at a distance of 2 cm from a very long straight wire carrying a current 5 A.

Ans: $B = \dfrac{\mu_0 I}{2\pi R}\ \dfrac{4\pi \times 10^{-7} \times 5}{2\pi \times 2 \times 10^{-2}} = 5 \times 10^{-5}$ T

Q.5. What happens to the magnetic field at the centre of a circular current carrying coil if we double the radius of the coil keeping the current unchanged?

Ans: Magnetic field at the centre of the circular coil, $B = \dfrac{\mu_0 I}{2r}$. Hence, if we double the radius, magnetic field at the centre of coil will become half its original value.

Q.6. A solenoid of length 50 cm of inner radius of 1 cm and is made up of 500 turns of copper wire for a current of 5 A in it. What will be magnitude of magnetic field inside the solenoid?

Ans: Magnitude of magnetic field inside the solenoid,

$$B = \mu_0 \frac{N}{l} i = 4\pi \times 10^{-7} \times \frac{500}{0.5} \times 5 = 6.284 \times 10^{-3}\ T$$

Q.7. State the orientation of magnetic dipole with respect to magnetic field, which possess maximum magnetic potential energy.

Ans: When magnetic dipole moment and magnetic field are antiparallel to each other, magnetic potential energy of a magnetic dipole is maximum.

Short Answer I (SA1) (2 Marks Each)

Q.1. A toroid of 4000 turns has outer radius of 26 cm and inner radius of 25 cm. If the current in the wire is 10 A. Calculate the magnetic field of the toroid.

Solution:

Given: $R_1 = 0.25$ m; $R_2 = 0.26$ m;
$N = 4000$; $i = 10$ A

To find: Magnetic field of the toroid (B)

Formula: $B = \mu_0 \times \frac{N}{l} \times i$

where, l = mean length of toroid = $2\pi \frac{(R_1 + R_2)}{2}$

Calculation: From formula,

$l = \pi (R_1 + R_2)$

$= \pi (0.25 + 0.26)$

$= \pi \times 0.51$ m

$$\therefore \quad B = \frac{(4\pi \times 10^{-7}) \times 4000 \times 10}{\pi \times 0.51} = \frac{16}{0.51} \times 10^{-3}$$

$$\therefore \quad B_{in} = \mathbf{3.137 \times 10^{-2}\ T}$$

Ans: The magnetic field of the toroid is $\mathbf{3.137 \times 10^{-2}\ T}$.

Q.2. Magnetic field lines can be entirely confined within the core of a toroid, but not within a straight solenoid. Why?

Ans:

i. The magnetic field around the toroid consists of concentric circular lines of force around it. Thus, magnetic field in the interior of toroid is tangential to each loop.

ii. Whereas, the magnetic field produced by a solenoid is similar to the magnetic field of a bar magnet. One end of the solenoid coil acts as south pole and the other end acts as north pole with the field lines inside the solenoid remaining parallel. Thus, the magnetic field B is parallel to the axis of the solenoid.

As a result, magnetic field lines can be entirely confined within the core of a toroid, but not within a straight solenoid.

Q.3. A solenoid of length π m and 5 cm in diameter has winding of 1000 turns and carries a current of 5 A. Calculate the magnetic field at its centre along the radius.

Solution:

Given: $l = \pi$ m, diameter = 5 cm,

N = 1000 turns, i = 5 A

We know that, $\mu_0 = 4\pi \times 10^{-7}$ Tm/A

To find: Magnetic field (B)

Formulae: i. $n = \dfrac{N}{l}$

ii. $B = \mu_0 ni$

Calculation: From formula (i),

$$n = \frac{1000}{\pi}\ \frac{\text{turns}}{\text{m}}$$

From formula (ii),

$$B = 4\pi \times 10^{-7} \times \frac{1000}{\pi} \times 5$$

$$= 20 \times 10^{-7+3}$$

$$= \mathbf{2 \times 10^{-3}\ T}$$

Ans: The magnetic field is $\mathbf{2 \times 10^{-3}}$ **T**.

Q.4. Currents of equal magnitude pass through two long parallel wires having separation of 1.35 cm. If the force per unit length on each wire is 4.76×10^{-2} N/m, what is I?

Solution:

Given: $I_1 = I_2 = I, \frac{F}{L} = 4.76 \times 10^{-2}$ N

$d = 1.35$ cm $= 1.35 \times 10^{-2}$ m

To find: Electric current

Formula: $\frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi d}$

Calculation: From formula,

$$4.76 \times 10^{-2} = \frac{4\pi \times 10^{-7} \times I \times I}{2 \times \pi \times 1.35 \times 10^{-2}}$$

$$\therefore \quad I^2 = \frac{4.76 \times 10^{-2} \times 1.35 \times 10^{-2}}{2 \times 10^{-7}} = 1.35 \times 2.38 \times 10^{-2-2+7}$$

$$I = \sqrt{1.35 \times 2.38 \times 10^3}$$

$$= \sqrt{13.5 \times 2.38} \times 10$$

$$= \left\{\text{anti}\log\left(\frac{1}{2}(\log 13.5 + \log 2.38)\right)\right\} \times 10$$

$$= \left\{\text{anti}\log\left(\frac{1}{2}(1.1303 + 0.3766)\right)\right\} \times 10$$

$= \{\text{antilog}(0.7535)\} \times 10$

$= 5.669 \times 10$

$=$ **56.69 A**

Ans: The electric current is **56.69 A**.

Q.5. Explain "Magnetic force never does any work on moving charges".

Ans:

i. Magnetic force is given by $\vec{F}_m = q\left(\vec{v} \times \vec{B}\right)$.

ii. This makes direction of magnetic force $\left(\vec{F}_m\right)$ perpendicular to direction of velocity of charged particles $\left(\vec{v}\right)$.

iii. Thus, magnetic force is in turn perpendicular to displacement of charged particles.

iv. According to properties of dot product, $\vec{F}_m \cdot \vec{v} = 0$, for any magnetic field $\vec{B}$.

Hence, magnetic force never does any work on moving charges.

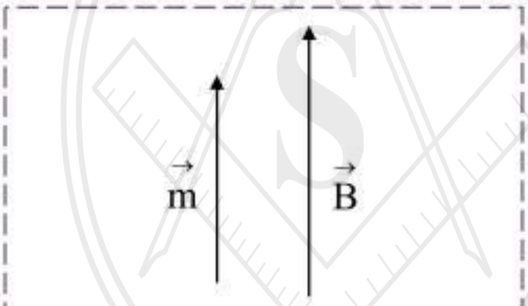
Q.6. State the conditions when magnetic potential energy of a magnetic dipole (current carrying coil) kept in uniform magnetic field be minimum and maximum.

Ans:

i. **Condition for minimum magnetic potential energy:**

In a magnetic field when $\vec{m}$ and $\vec{B}$ are parallel to each other, magnetic potential energy of a magnetic dipole is minimum.

Mathematically, when $\theta = 0$, $U = -\text{m B} \cos 0° = -\text{m B}$

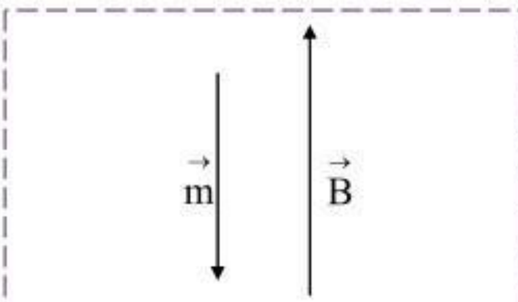


Minimum potential energy

ii. Condition for maximum magnetic potential energy:

When $\vec{m}$ and $\vec{B}$ are antiparallel to each other, magnetic potential energy of a magnetic dipole is maximum.

Mathematically, when $\theta = 180°$, $U = -\text{m B} \cos 180° = \text{m B}$



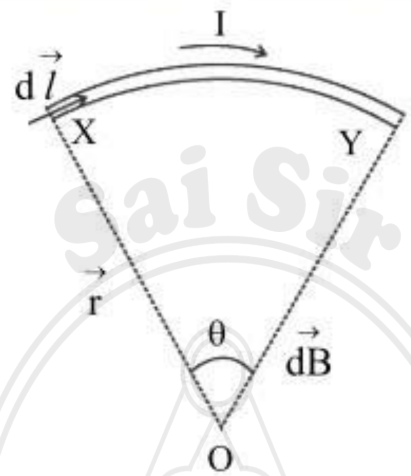
Maximum potential energy

Q.7. Derive the expression for magnetic field produced by a current in a circular arc of wire.

Ans:

i. Consider circular arc of a wire (XY), carrying a current I.

ii. The circular arc XY subtends an angle θ at the centre O of the circle with radius r of which the arc is a part, as shown in figure below.



Current carrying wire of a shape of circular arc.

iii. Consider length element $d\vec{l}$ lying always perpendicular to $\vec{r}$.
Using Biot-Savart law, magnetic field produced at O is:

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l} \times \vec{r}}{r^3}$$

$$dB = \frac{\mu_0}{4\pi} I \frac{dl\, r \sin 90^\circ}{r^3}$$

$$= \frac{\mu_0}{4\pi}\frac{I\,dl}{r^2} \quad \text{....(1)}$$

iii. Equation (1) gives the magnitude of the field. The direction of the field is given by the right-hand rule. Thus, the direction of each of the dB is into the plane of the paper. The total field at O is

$$B = \int dB = \frac{\mu_0}{4\pi} I \int_A^\theta \frac{dl}{r^2}$$

$$= \frac{\mu_0}{4\pi} I \int_A^\theta \frac{r\,d\theta}{r^2} = \frac{\mu_0}{4\pi}\frac{I}{r}\theta \quad \text{....(2)}$$

where, the angle θ is in radians.

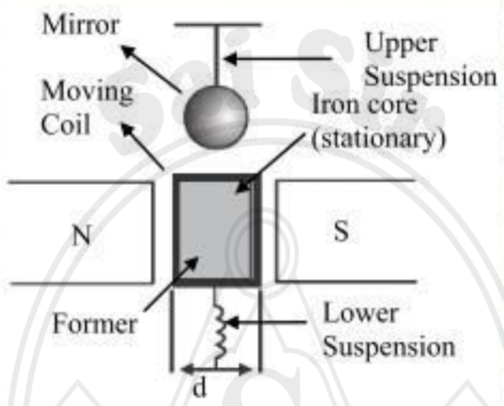
Short Answer II (SA2) (3 Marks Each)

Q.1. Explain construction and working of moving coil galvanometer.

Ans: Construction:

i. M.C.G. consists of a coil of several turns mounted (suspended or pivoted) in such a way that it can freely rotate about a fixed axis, in a radial uniform magnetic field.

ii. A soft iron cylindrical core makes the field radial and strong.



Moving coil galvanometer

Working:

i. The coil rotates due to a torque acting on it as the current flows through it. Torque acting on current carrying coil is $\tau = NIAB \sin\theta$.

Here $\theta = 90°$ as the field is radial.

$\therefore$ $\tau = NIAB$

where A is the area of the coil, B the strength of the magnetic field, N the number of turns of the coil and I the current in the coil.

ii. This torque is counter balanced by a torque due to a spring fitted at the bottom so that a fixed steady current I in the coil produces a steady angular deflection ϕ.

iii. Larger the current is, larger is the deflection and larger is the torque due to the spring. If the

deflection is ϕ, the restoring torque due to the spring is equal to $K\phi$ where K is the torsional constant of the spring.

Thus, $K\phi = NIAB$,

and the deflection $\phi = \left(\frac{NAB}{K}\right) I$

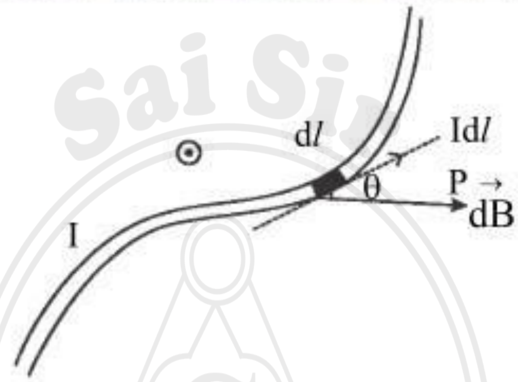
This means, the deflection ϕ is proportional to the current I i.e., $\phi \propto I$.

Q.2. Explain Biot Savart's law.

Ans:

i. Consider an arbitrarily shaped wire carrying a current I.

ii. Let dl be a length element along the wire. The current in this element is in the direction of the length vector $\vec{dl}$ which produces differential magnetic field $\vec{dB}$ directed into the plane of paper as shown in figure below:



Current carrying wire of arbitrary shape

iii. Consider point P at distance r from element dl. Net magnetic field at the point P can be obtained by integrating i.e., summing up of magnetic fields $\vec{dB}$ from these length elements.

iv. Experimentally, the magnetic fields $\vec{dB}$ produced by current I in the length element $\vec{dl}$ is

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2} \quad \text{....(1)}$$

where, θ is the angle between the directions of $\vec{dl}$ and $\vec{r}$,

μ_0 (permeability of free space) = $4\pi \times 10^{-7}$ T m/A ≈ 1.26×10^{-6} T m/A

v. The direction of $\vec{dB}$ is dictated by the cross product $\vec{dl} \times \vec{r}$.

Vectorially, $$\vec{dB} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l} \times \vec{r}}{r^3} \quad \text{....(2)}$$

Equations (1) and (2) are known as the Biot-Savart law.

Q.3. A rectangular coil of 10 turns, each of area 0.05 m^2, is suspended freely in a uniform magnetic field of induction 0.01 T. A current of 30 μA is passed through it.

i. What is the magnetic moment of the coil?

ii. What is the maximum torque experienced by the coil?

Solution:

Given: $N = 10$, $A = 0.05\ m^2$, $I = 30\ \mu A = 30 \times 10^{-6}\ A$

$B = 0.01\ T$

To find: i. Magnetic moment (m) of coil

ii. Maximum torque experienced (τ_{max})

Formulae: i. $m = NIA$

ii. $\tau_{max} = (NIA)B = mB$

Calculation: From formula (i),

$m = 10 \times 30 \times 10^{-6} \times 0.05$

$= 15 \times 10^{-6}\ Am^2$

$= \mathbf{15\ \mu Am^2}$

From formula (ii),

$\tau_{max} = 15 \times 10^{-6} \times 0.01$

$= 15 \times 10^{-8}$

$= \mathbf{1.5 \times 10^{-7}\ Nm}$

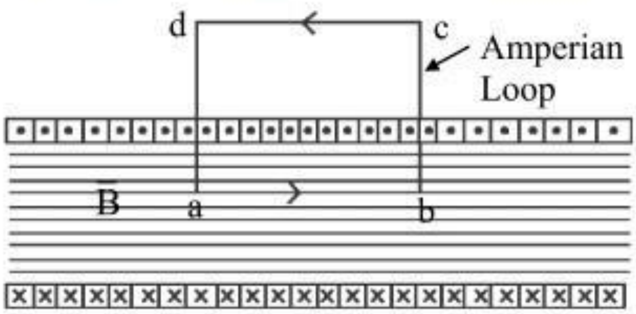
Ans: i. Magnetic moment of coil is **15 μAm^2**.

ii. Maximum torque experienced by coil is **1.5×10^{-7} Nm**.

Q.4. Using Ampere's law, derive an expression for the magnetic induction inside an ideal solenoid carrying a steady current.

Ans:

i. Consider an ideal solenoid as shown in figure below.



Ampere's law applied to a part of a long ideal solenoid

ii. The dots (·) show that the current is coming out of the plane of the paper and the crosses (×) show that the current is going into the plane of the paper, both in the coil of square cross section wire.

iii. For the application of the Ampere's law, an Amperian loop is drawn as shown figure and box.

iv. Using Ampere's law,

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I$$

Over the rectangular loop abcd, the above integral takes the form

$$\int_a^b \vec{B} \cdot \vec{dl} + \int_b^c \vec{B} \cdot \vec{dl} + \int_c^d \vec{B} \cdot \vec{dl} + \int_d^a \vec{B} \cdot \vec{dl} = \mu_0 I$$

where, I is the net current encircled by the loop.

∴ $B L + 0 + 0 + 0 = \mu_0 I$ ....(1)

Here, the second and fourth integrals are zero because $\vec{B}$ and $\vec{dl}$ are perpendicular to each other. The third integral is zero because outside the solenoid, $B = 0$.

v. If the number of turns is n per unit length of the solenoid and the current flowing through the wire is i, then the net current coming out of the plane of the paper is

$I = nLi$

∴ Using equation (1)

$BL = \mu_0 nLi$

∴ $B = \mu_0 ni$ ....(2)

Equation (2) is the required expression.

Q.5. Derive an expression for the net torque on a rectangular current carrying loop placed in a uniform magnetic field with its rotational axis perpendicular to the field.

Ans:

i. Consider rectangular loop abcd placed in a uniform magnetic field $\vec{B}$ such that the sides ab and cd are perpendicular to the magnetic field $\vec{B}$ but the sides bc and da are not, as shown in figure (a) below:

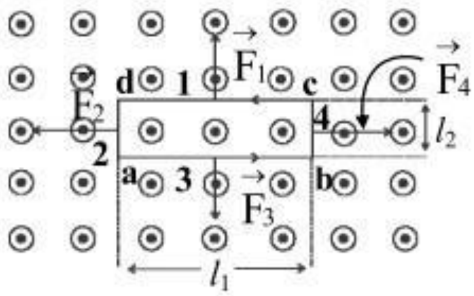


Fig (a): Loop abcd placed in a uniform

ii. The force $\vec{F}_4$ on side 4 (bc) will be

$$\vec{F}_4 = Il_2 \, B \sin (90° - \theta)$$

iii. The force $\vec{F}_2$ on side 2 (da) will be equal and opposite to $\vec{F}_4$ and both act along the same line. Thus, $\vec{F}_2$ and $\vec{F}_4$ will cancel out each other.

iv. The magnitudes of forces $\vec{F}_1$ and $\vec{F}_3$ on sides 1 (cd) and 3 (ab) will be Il_1 B sin 90° i.e., Il_1 B. These two forces do not act along the same line and hence they produce a net torque.

v. This torque results into rotation of the loop so that the loop is perpendicular to the direction of $\vec{B}$, the magnetic field.

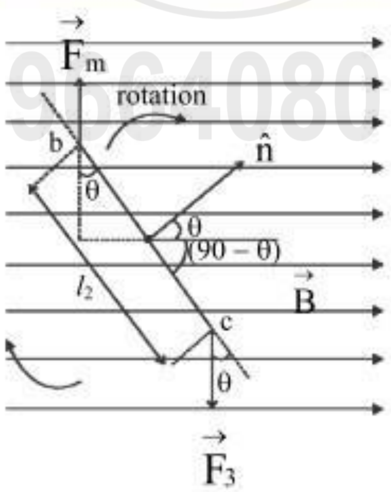


Fig (b): Side view of the loop abcd at an angle θ

vi. Now the moment arm is $\frac{1}{2}(l_2 \sin\theta)$ about the central axis of the loop.

Hence, torque τ due to forces $\vec{F}_1$ and $\vec{F}_3$ will be

$$\tau = \left(Il_1\ B\frac{1}{2}l_2\ \sin\theta\right) + \left(Il_1\ B\frac{1}{2}l_2\ \sin\theta\right)$$

$$= Il_1\ l_2\ B\sin\theta$$

vii. If the current carrying loop is made up of multiple turns N, in the form of a flat coil, the total torque will be

$\tau' = N\tau = N\ I\ l_1\ l_2\ B\ \sin\theta$

$\tau' = (NIA)B\ \sin\theta$; where, A is the are enclosed by the coil $= l_1 l_2$

This is the required expression.

Q.6. A circular loop of radius 9.7 cm carries a current 2.3 A. Obtain the magnitude of the magnetic field (i) at the centre of the loop (ii) at a distance of 9.7 cm from the centre of the loop but on the axis.

Solution:

Given: $R = 9.7\ \text{cm} = 9.7 \times 10^{-2}$ m,
$I = 2.3$ A,
$z = 9.7\ \text{cm} = 9.7 \times 10^{-2}$ m

To find: Magnetic field
i. at the centre of the loop
ii. on the axis at a distance

Formulae: i. $B_c = \frac{\mu_0 I}{2R}$ ii. $B_a = \frac{\mu_0 IR^2}{2(z^2 + R^2)^{3/2}}$

Calculation:

From formula (i),

$$B_c = \frac{4\pi \times 10^{-7} \times 2.3}{2 \times 9.7 \times 10^{-2}}$$

$$= \frac{2 \times 3.142 \times 2.3}{9.7} \times 10^{-7+2}$$

$= \{\text{antilog}\ [\log 2 + \log 3.142 + \log 2.3 - \log 9.7]\} \times 10^{-5}$

$= \{\text{antilog}\ (0.3010 + 0.4972 + 0.3617 - 0.9868)\} \times 10^{-5}$

$= \{\text{antilog}\ (0.1731)\} \times 10^{-5}$

$= 1.489 \times 10^{-5}$

$\approx 1.49 \times 10^{-5}$ T

$\mathbf{= 14.9\ \mu T}$

From formula (ii),

$$B_a = \frac{4\pi \times 10^{-7} \times 2.3 \times \left(9.7 \times 10^{-2}\right)^2}{2\left[\left(9.7 \times 10^{-2}\right)^2 + \left(9.7 \times 10^{-2}\right)^2\right]^{3/2}}$$

$$= \frac{4\pi \times 10^{-7} \times 2.3 \times \left(9.7 \times 10^{-2}\right)^2}{2 \times 2^{3/2} \times \left(9.7 \times 10^{-2}\right)^3}$$

$$= \frac{\pi \times 10^{-7} \times 2.3}{2^{1/2} \times \left(9.7 \times 10^{-2}\right)}$$

$$= 5.268 \times 10^{-6}\ \text{T}$$

$$= \mathbf{5.268\ \mu T}$$

Ans: i. Magnetic field at the centre of **14.9 μT.**

ii. Magnetic field on the axis at a distance of 9.7 cm from the centre is **5.268 μT.**

Q.7. The magnetic field at the centre of a circular loop of radius 12.3 cm is 6.4 × 10^{-6} T. What will be the magnetic moment of the loop?

Solution:

Given: $B = 6.4 \times 10^{-6}$ T,

$R = 12.3$ cm

$= 12.3 \times 10^{-2}$ m

To find: Magnetic moment (m)

Formula: $B = \dfrac{\mu_0 I R^2}{2\left(z^2 + R^2\right)^{3/2}}$

Calculation: From formula,

$$B = \frac{\mu_0 I \pi R^2}{2\pi\left(z^2 + R^2\right)^{3/2}}$$

$$= \frac{\mu_0 m}{2\pi\left(z^2 + R^2\right)^{3/2}} \qquad \text{....}[\because m = I(\pi R^2)]$$

$$B = \frac{\mu_0 m}{2\pi R^3} \qquad \text{....}(\because z = 0)$$

$\therefore \quad m = \dfrac{B \times 2\pi R^3}{\mu_0}$

$= \dfrac{6.4 \times 10^{-6} \times 2 \times \pi \times \left(12.3 \times 10^{-2}\right)^3}{4\pi \times 10^{-7}}$

$= 3.2 \times 10^{-6+7-6} \times (12.3)^3$

$= \{\text{antilog}\ (\log 3.2 + 3 \log 12.3)\} \times 10^{-5}$

$= \{\text{antilog}\ (0.5051 + 3.2697)\} \times 10^{-5}$

$= \{\text{antilog}\ (3.7748)\} \times 10^{-5}$

$= 5.954 \times 10^{3} \times 10^{-5}$

$= \mathbf{5.954 \times 10^{-2}\ Am^2}$

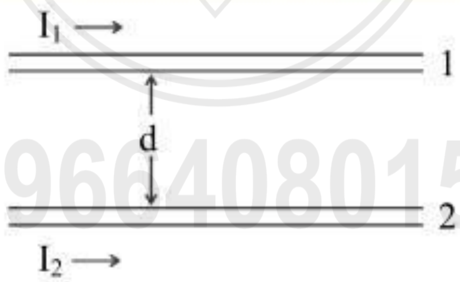
Ans: Magnetic moment of the loop is $\mathbf{5.954 \times 10^{-2}\ Am^2}$.

Long Answer (LA) (4 Marks Each)

Q.1. Show that currents in two long, straight, parallel wires exert forces on each other. Derive the expression for the force per unit length on each conductor.

Ans: Case I: Both wires carry current in same direction.

i. Consider two long parallel wires separated by distance d and carrying current I_1 and I_2 respectively same direction as same as shown in figure below:



Two long parallel wires, distance d apart

ii. The magnetic field at the second wire due to the current I_1 in the first one, according to Biot – Savart's law is

$B = \dfrac{\mu_0 I_1}{2\pi d}$ ....(1)

iii. By the right-hand rule, the direction of this field is into the plane of the paper.

iv. Force on the wire 2, because of the current I_2 and the magnetic field B due to current in wire 1, applying Lorentz force law is,

$$F = I_2\left(\frac{\mu_0 I_1}{2\pi d}\right)\int dl \qquad ....(2)$$

The direction of this force is towards wire 1, i.e., it will be attractive force.

v. Force of attraction per unit length of the wire will be

$$\frac{F}{L} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d} \qquad ....(3)$$

Case II: Two wires carry current in opposite direction.

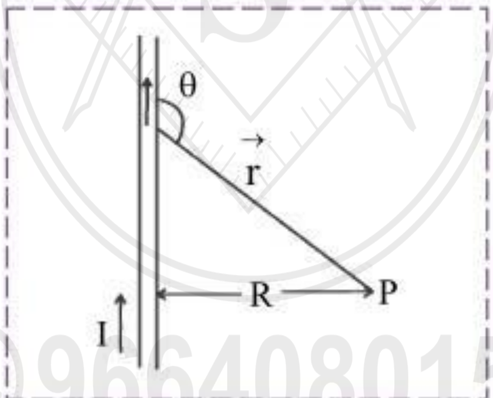
Force is of repulsive nature between antiparallel currents and magnitude of force of repulsion per unit length is, $\left|\frac{F}{L}\right| = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d}$

Q.2. Using Biot Savart's law, obtain the expression for the magnetic induction near a straight infinitely long current carrying wire.

Ans:

i. Consider a straight wire of length l carrying current I.

ii. Let a point P situated at a perpendicular distance R from the wire as shown below.



iii. Consider infinitesimal length $d\vec{l}$ of wire carrying current I, then current element = $I\,\vec{dl}$.

iv. Current element is situated at distance r from point P making an angle θ, as shown in figure above.

v. Using Biot Savart law, magnetic field, produced $\vec{dB}$ at P due to current element $I\,d\vec{l}$ is,

$$\vec{dB} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l}\sin\theta}{r^2} \qquad ....(1)$$

vi. According to properties of cross-product, $\vec{dl} \times \vec{r}$ indicates direction of $\vec{dB}$, in this case, is into the plane of paper.

vii. Summing up all current elements from upper half of infinitely long wire,

$$B_{upper} = \int_0^\infty dB = \frac{\mu_0}{4\pi}\int_0^\infty \frac{I\vec{dl}\sin\theta}{r^2} \quad ....(2)$$

viii. Taking into account symmetry of wire, current elements in lower half of infinitely long wire will also contribute same as upper half.

i.e., $B_{lower} = B_{upper}$ ....(3)

ix. Adding contributions from lower and upper part, total magnetic field point P is

$$B = 2\int_0^\infty dB \quad ....\text{[using equation (2)]}$$

$$= \frac{2\mu_0}{4\pi}\int_0^\infty \frac{I\,dl\sin\theta}{r^2} \quad ....\text{[using equation (1)]}$$

But $r = \sqrt{l^2 + R^2}$ and

$$\sin\theta = \sin(\pi - \theta) = \frac{R}{r} = \frac{R}{\sqrt{l^2 + R^2}}$$

$$\therefore \quad B = \frac{\mu_0 I}{2\pi}\int_0^\infty \frac{R\,dl}{(l^2 + R^2)\sqrt{l^2 + R^2}}$$

$$= \frac{\mu_0 I}{2\pi}R\int_0^\infty \frac{dl}{(l^2 + R^2)^{\frac{3}{2}}}$$

Solving the integration,

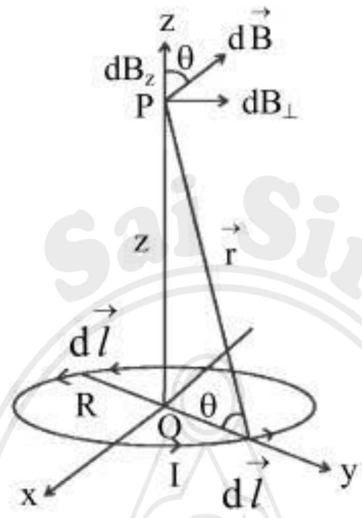
$$B = \frac{\mu_0 I}{2\pi}R \times \frac{1}{R^2} = \frac{\mu_0 I}{2\pi R} \quad ....(4)$$

This is the equation for magnetic field at a point situated at a perpendicular distance R from infinitely long wire carrying current I.

Q.3. Derive an expression for axial magnetic field produced by current in a circular loop.

Ans:

i. Consider loop of radius R carrying current I placed in x–y plane with its centre at origin O as shown in figure below.



Magnetic field on the axis of a circular current loop of radius R

ii. Let point P can be on z–axis at distance $\vec{r}$ from line element $d\vec{l}$ of the loop.

iii. Using Biot-Savart law, the magnitude of the magnetic field dB is given by

$$dB = \frac{\mu_0}{4\pi} I \frac{\left|d\vec{l} \times \vec{r}\right|}{r^3}$$

iv. Any element $d\vec{l}$ will always be perpendicular to the vector $\vec{r}$ from the element to the point P. The element $d\vec{l}$ is in the x-y plane, while the vector $\vec{r}$ is in the y-z plane. Hence

$$d\vec{l} \times \vec{r} = dl\, r$$

$$\therefore \quad dB = \frac{\mu_0}{4\pi} I \frac{dl}{r^2}$$

but $r^2 = R^2 + z^2$

$$\therefore \quad dB = \frac{\mu_0}{4\pi} I \frac{dl}{\left(z^2 + R^2\right)}$$

v. Now, direction of $d\vec{B}$ is perpendicular to the plane formed by $d\vec{l}$ and $\vec{r}$. Its z component is dB_z and the component perpendicular to the z-axis is $dB_\perp$. The components $dB_\perp$ when summed over, yield zero as they cancel out due to symmetry. Hence, only z component remains

vi. The net contribution along the z axis is obtained by integrating $dB_z = dB \cos\theta$ over the entire loop.

From figure,

$$\cos\theta = \frac{R}{r} = \frac{R}{\sqrt{z^2 + R^2}}$$

$$\therefore \quad B_z = \int dB_z = \frac{\mu_0}{4\pi} I \int \frac{dl}{\left(z^2 + R^2\right)} \cos\theta$$

$$= \frac{\mu_0}{4\pi} I \int \frac{R\,dl}{\left(z^2 + R^2\right)^{\frac{3}{2}}}$$

$$= \frac{\mu_0}{4\pi} \times \frac{IR}{\left(z^2 + R^2\right)^{\frac{3}{2}}} \times 2\pi R$$

$$B_z = \frac{\mu_0}{2} \times \frac{IR^2}{\left(z^2 + R^2\right)^{\frac{3}{2}}}$$

This is the magnitude of the magnetic field due to current I in the loop of radius R, on a point at P on the z axis of the loop.

11 Magnetic Materials

Multiple Choice Questions (1 Mark Each)

1. The magnetic susceptibility is given by
   (A) $\chi = \frac{1}{H}$ (B) $\chi = \frac{B}{H}$
   (C) $\chi = \frac{M_{net}}{V}$ **(D) $\chi = \frac{M}{H}$**

2. The relation between relative permeability and magnetic susceptibility is given by
   (A) $\chi = \mu_r + 1$ (B) $\chi = -\mu - 1$
   (C) $\mu_r = 1 - \chi$ **(D) $\mu_r = 1 + \chi$**

3. If an electron of charge (–e) and mass m_e revolves around the nucleus of an atom having orbital magnetic moment m_o,then angular momentum of electron is
   (A) $L = \frac{m_o e}{2m_e}$ (B) $L = \frac{e}{2m_o m_e}$
   (C) $L = \frac{2m_o m_e}{e}$ (D) $L = \frac{2e}{m_o m_e}$

4. If m_o and L denote the orbital angular moment and the angular momentum of the electron due to its orbital motion, then the gyromagnetic ratio is given by
   (A) $\frac{L}{m_o}$ **(B) $\frac{m_o}{L}$**
   (C) Lm_o (D) $\sqrt{\frac{m_o}{L}}$

5. Relative permeability of iron 5500, then its magnetic susceptibility will be
   (A) 5500×10^7 (B) 5501
   (C) 5499 (D) 5500×10^{-7}

Hint: $\chi = \mu_r - 1 = 5500 - 1 = 5499$

6. What is magnetization of a bar magnet having length 6 cm and area of cross section 5 cm^2? (m_{net} = 1)
(A) 1.2×10^{-4} A/m (B) **3.3×10^{4} A/m**
(C) 1.25×10^{-4} A/m (D) 3.3×10^{-4} A/m

Hint: $M = \frac{m_{net}}{V} = \frac{m_{net}}{AL} = \frac{1}{5\times10^{-4}\times6\times10^{-2}} = 3.3 \times 10^{4}$ A/m

7. A magnetic material of susceptibility 3×10^{-4}, and magnetic intensity is 4×10^{-4} Am^{-1}. Then the magnetization in the units of Am^{-1} is
(A) **12×10^{-8}** (B) 1.33×10^{8}
(C) 0.75×10^{-8} (D) 14×10^{-8}

Hint: $M = \chi H = 3 \times 10^{-4} \times 4 \times 10^{-4} = 12 \times 10^{-8}$ Am^{-1}
*[**Note:** The option (A) is modified to remove the ambiguity from question.]*

Very Short Answer (VSA) (1 Mark Each)

Q.1. Give gyromagnetic ratio.
Ans: Gyromagnetic ratio is given by, $\frac{m_{orb}}{L} = \frac{e}{2m_e}$. Its value is given by 8.8×10^{10} C/kg.

Q.2. What is stated in term of Bohr magneton.
Ans: The magnetic moment of an atom is stated in term of Bohr magneton.

Q.3. Define magnetization.
Ans: *The ratio of magnetic moment to the volume of the material is called **magnetization.***

Q.4. What does the ratio of magnetization to magnetic intensity indicates?
Ans: The ratio of magnetization to magnetic intensity indicates magnetic susceptibility (χ).

Q.5. The relative permeability of a medium is 0.075. What is its magnetic susceptibility ?
Ans: Magnetic susceptibility, $\chi = \mu_r - 1 = 0.075 - 1 = -0.925$
*[**Note:** Answer calculated above is in accordance with textual method of calculation.]*

Q.6. The moment of a magnet (15 cm × 2 cm × 1 cm) is 1.2 A-m². What is its intensity of magnetization?

Ans: Magnetization,

$$M = \frac{m_{net}}{V} = \frac{m_{net}}{l \times b \times h} \quad ....[\because V = l \times b \times h]$$

$$= \frac{1.2}{15\times10^{-2}\times2\times10^{-2}\times1\times10^{-2}}$$

$$= 4\times 10^{4} \text{ A/m}$$

Q.7. The electron in hydrogen atom is moving with a speed of 2.5×10^6 m/s in an orbit of radius 0.5 Å. What is the magnetic moment of the revolving electron .

Ans: Magnetic moment,

$$m_0 = \frac{evr}{2} = \frac{1.6\times10^{-19}\times2.5\times10^{6}\times0.5\times10^{-10}}{2} = 10^{-23} \text{ Am}^2$$

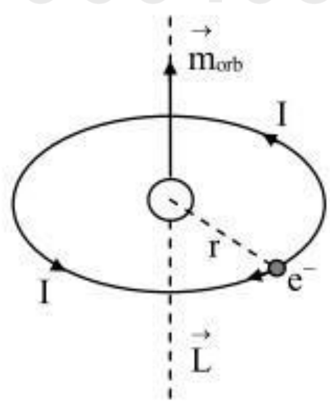
[Note:** Answer calculated above is in accordance with textual method of calculation.**]

Short Answer I (SA1) (2 Marks Each)

1. Show that the orbital magnetic dipole moment of a revolving electron is $\frac{evr}{2}$.

Ans: Expression for magnetic dipole moment:

i. Consider an electron of mass m_e and charge e revolving in a circular orbit of radius r around the positive nucleus in clockwise direction, leading to an anticlockwise current.



U.C.M of electron around nucleus

ii. If the electron travels a distance $2\pi r$ in time T then, its orbital speed $v = 2\pi r/T$

iii. Magnitude of circulating current is given by, $I = e\left(\frac{1}{T}\right)$

But, $T = \frac{2\pi r}{v}$

$\therefore \quad I = e\left(\frac{1}{2\pi r/v}\right) = \frac{ev}{2\pi r}$

iv. The orbital magnetic moment associated with orbital current loop is given by,

$m_{orb} = IA = \frac{ev}{2\pi r} \times \pi r^2$ ....[$\because$ Area of current loop, $A = \pi r^2$]

$\therefore \quad m_{orb} = \frac{evr}{2}$

2. Derive the quantity for Bohr magneton and also state its value.

Ans:

i. According to Bohr's theory, an electron in an atom can revolve only in certain stationary orbits in which angular momentum (L) of electron is an integral multiple (n) of $\frac{h}{2\pi}$, where h is Planck's constant.

$\therefore \quad L = m_e vr = \frac{nh}{2\pi}$ ....(1)

ii. The orbital magnetic momentum of an electron is given as, $m_{orb} = \frac{eL}{2m_e}$ ....(2)

iii. Substituting equation (1) and (2), we have,

$m_{orb} = n\left(\frac{eh}{4\pi m_e}\right)$

iv. For the 1st orbit, n = 1,

$\therefore \quad m_{orb} = \frac{eh}{4\pi m_e}$

v. The quantity $\frac{eh}{4\pi m_e}$ is called Bohr Magneton and its value is 9.274×10^{-24} Am^2.

3. Define magnetization. State its SI unit and dimensions.

Ans:

i. *The ratio of magnetic moment to the volume of the material is called* ***magnetization.***

ii. Unit: Am^{-1} in SI system.

iii. Dimensions: $[M^0L^{-1}T^0I^1]$

4. Calculate the gyromagnetic ratio of electron. (given e = 1.6×10^{-19} C, $m_e = 9.1 \times 10^{-31}$ kg)

Solution:

Given: $e = 1.6 \times 10^{-19}$ C,
$m_e = 9.1 \times 10^{-31}$ kg

To find: Gyromagnetic ratio

Formula: Gyromagnetic ratio $= \dfrac{e}{2\ m_e}$

Calculation: From formula,

$$\text{Gyromagnetic ratio} = \frac{1.6 \times 10^{-19}}{2 \times 9.1 \times 10^{-31}}$$
$$= \frac{8 \times 10^{-20}}{9.1 \times 10^{-31}}$$
$$= 0.88 \times 10^{-20+31}$$
$$= 0.88 \times 10^{-11}$$
$$= \mathbf{8.8 \times 10^{10}\ Ckg^{-1}}$$

Ans: Gyromagnetic ratio of electron is $\mathbf{8.8 \times 10^{10}\ Ckg^{-1}}$.

*[**Note:** The value of mass of electron given in question is modified to remove the ambiguity from question.]*

5. An iron rod of area of cross-section 0.1 m² is subjected to a magnetizing field of 1000 A/m. Calculate the magnetic permeability of the iron rod. (χ for iron = 599, $\mu_0 = 4\pi \times 10^{-7}$ SI unit)

Solution:

Given: H = 1000 A/m, $\chi = 599$,
$\mu_0 = 4\pi \times 10^{-7}$ S.I. unit

To find: Permeability (μ)

Formula: $\mu = \mu_0 (1 + \chi)$

Calculation: From formula,

$\mu = 4\pi \times 10^{-7} (1 + 599)$

$= 4 \times 3.142 \times 10^{-7} \times 600$

$\therefore \quad \mu = \mathbf{7.54 \times 10^{-4}\ Hm^{-1}}$

Ans: The magnetic permeability of the iron rod is $\mathbf{7.654 \times 10^{-4}\ Hm^{-1}}$.

6. A solenoid has core of a material with relative permeability 500 and its windings carry current of 1 A. The number of turns of the solenoid is 500 per metre. Calculate the magnetization of the material.

Solution:

Given: $\mu_r = 500$, I = 1 A, n = 500

To find: Magnetization (M)

Formula: $M = (\mu_r - 1)nI$

Calculation: From formula,

$M = (500 - 1) \times 500 \times 1$

$= \mathbf{2.495 \times 10^5\ Am^{-1}}$

Ans: The magnetization of the material is $\mathbf{2.495 \times 10^5\ Am^{-1}}$.

Short Answer II (SA2) (3 Marks Each)

Q.1. Define magnetic intensity. Explain magnetization of a material.

Ans:

i. *The ratio of the strength of magnetising field to the permeability of free space is called as **magnetic intensity.***

ii.. **Magnetization:**

a. The ratio of magnetic moment to the volume of the material is called magnetization.

b. It is denoted by $\vec{M}$.

c. If magnetic specimen of volume 'V' acquires net magnetic dipole moment 'm_{net}' due to the magnetising field, then $\vec{M} = \frac{m_{net}}{V}$

d. It is a vector quantity.

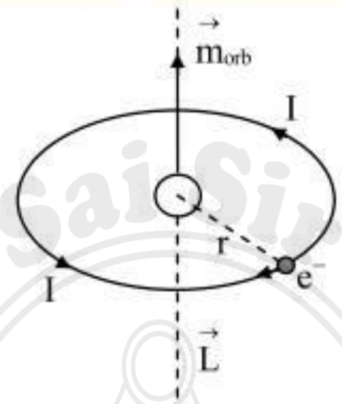
e. Unit: Am^{-1} in SI system.

f. Dimensions: $[M^0L^{-1}T^0I^1]$

Q.2. Obtain an expression for orbital magnetic moment of an electron rotating about the nucleus in an atom.

Ans: Expression for magnetic dipole moment:

i. Consider an electron of mass m_e and charge e revolving in a circular orbit of radius r around the positive nucleus in clockwise direction, leading to an anticlockwise current.



U.C.M of electron around nucleus

ii. If the electron travels a distance $2\pi r$ in time T then, its orbital speed $v = 2\pi r/T$

iii. Magnitude of circulating current is given by,

$$I = e\left(\frac{1}{T}\right)$$

But, $T = \frac{2\pi r}{v}$

$$\therefore \quad I = e\left(\frac{1}{2\pi r / v}\right) = \frac{ev}{2\pi r}$$

iv. The orbital magnetic moment associated with orbital current loop is given by,

$$m_{orb} = IA = \frac{ev}{2\pi r} \times \pi r^2 \qquad [\because \text{Area of current loop, } A = \pi r^2]$$

$$\therefore \quad m_{orb} = \frac{evr}{2} \qquad \ldots(1)$$

v. The angular momentum of an electron due to its orbital motion is given by,

$L = m_e vr$

vi. Multiplying and dividing the R.H.S of equation (1) by m_e,

$$m_{orb} = \frac{e}{2m_e} \times m_e vr$$

$$\therefore \quad m_{orb} = \frac{eL}{2m_e}$$

vii. This equation shows that orbital magnetic moment is proportional to the angular momentum. But as the electron bears negative charge, the orbital magnetic moment and orbital angular momentum are in opposite directions and perpendicular to the plane of the orbit.

Using vector notation, $\vec{m}_{orb} = -\left(\frac{e}{2m_e}\right)\vec{L}$

Q.3. Define gyromagnetic ratio. Find relation for Bohr magneton.

Ans:

i. *The ratio of magnetic dipole moment with angular momentum of revolving electron is called the* ***gyromagnetic ratio.***

Gyromagnetic ratio is given by, $\frac{m_{orb}}{L} = \frac{e}{2m_e}$

ii. Relation for Bohr magneton:

a. According to Bohr's theory, an electron in an atom can revolve only in certain stationary orbits in which angular momentum (L) of electron is an integral multiple (n) of $\frac{h}{2\pi}$, where h is Planck's constant.

$$\therefore \quad L = m_e vr = \frac{nh}{2\pi} \quad \text{....(1)}$$

b. The orbital magnetic momentum of an electron is given as,

$$m_{orb} = \frac{eL}{2m_e} \quad \text{....(2)}$$

c. Substituting equation (1) and (2), we have,

$$m_{orb} = n\left(\frac{eh}{4\pi m_e}\right)$$

d. For the 1st orbit, n = 1,

$$\therefore \quad m_{orb} = \frac{eh}{4\pi m_e}$$

e. The quantity $\frac{eh}{4\pi m_e}$ is called Bohr Magneton and its value is 9.274×10^{-24} Am^2.

f. The magnetic moment of an atom is stated in terms of Bohr magnetons (B.M.).

Q.4. When a plate of magnetic material of size 10 cm × 0.5 cm × 0.2 cm (length, breadth and thickness respectively) is located in magnetizing field of 0.5 × 10^4 Am^{-1} then magnetic moment of 5 Am^2 is induced in it. Find out magnetic induction in the plate.

Solution:

Given:
$l = 10$ cm $= 10^{-1}$ m,
$b = 0.5$ cm $= 5 \times 10^{-3}$ m,
$h = 0.2$ cm $= 2 \times 10^{-3}$ m,
$H = 0.5 \times 10^4$ Am^{-1} $= 5 \times 10^3$ Am^{-1},
$\mu_0 = 4\pi \times 10^{-7}$ TmA^{-1}
$m_{net} = 5$ Am^2

To find: Magnetic induction in the plate (B)

Formulae: i. $M = \frac{m_{net}}{V}$ ii. $B = \mu_0 (M + H)$

Calculation: From formula (i),

$$M = \frac{5}{10^{-1} \times 5 \times 10^{-3} \times 2 \times 10^{-3}}$$
$$= 5 \times 10^6 \text{ Am}^{-1}$$

From formula (ii),

$$B = 4 \times 3.142 \times 10^{-7} [5 \times 10^6 + 5 \times 10^3]$$
$$= 12.568 \times 10^{-7} \times 5 \times 10^3 [10^3 + 1]$$
$$= 6.284 \times 10^{-3} \times 1001$$
$$= 6.29 \times 10^3 \times 10^{-3}$$
$$= \mathbf{6.29\ T}$$

Ans: Magnetic induction in the plate is **6.29 T**.

Q.5. A magnet of magnetic moment 3 Am² weighs 75 g. The density of the material of the magnet is 7500 kg/m³. What is the magnetization?

Solution:

Given:
$m_{net} = 3$ Am^2
mass = 75 g = 75×10^{-3} kg
Density, d = 7500 kg/m^3

To find: Magnetization (M)

Formulae: i. $\text{Density} = \dfrac{\text{Mass}}{\text{Volume}}$

ii. $M = \dfrac{m_{net}}{V}$

Calculation: From formula (i) and (ii),

$$M = m_{net} \times \frac{\text{density}}{\text{mass}}$$

$$= 3 \times \frac{7500}{75 \times 10^{-3}}$$

$$= \mathbf{3 \times 10^5\ A/m}$$

Ans: The magnetization is $\mathbf{3 \times 10^5\ A/m}$.

Q.6. Find the relative permeability, if the permeability of a metal is 0.1256 $\mathbf{TmA^{-1}}$

Solution:

Given: $\mu = 0.1256\ \text{T mA}^{-1}$

To find: Relative permeability (μ_r),

Formula: $\mu_r = \dfrac{\mu}{\mu_0}$

Calculation: From formula,

$$\mu_r = \frac{0.1256}{4\pi \times 10^{-7}} = \mathbf{10^5}$$

Ans: The relative permeability is $\mathbf{10^5}$.

Long Answer (LA) (4 Marks Each)

Q.1. Define magnetization. State its SI unit and dimensions. Derive the relation between magnetic field intensity(H) and magnetization(M) for a magnetic material placed in magnetic field.

Ans:

i. *The ratio of magnetic moment to the volume of the material is called* ***magnetization.***

ii. Unit: Am^{-1} in SI system.

iii. Dimensions: $[M^0L^{-1}T^0I^1]$

iv. **Relation between magnetic field intensity(H) and magnetization(M):**

a. Consider a magnetic material (rod) placed in a magnetising field (solenoid with n turns per unit length and carrying current I).

b. The magnetic field inside the solenoid is given by,

$B_0 = \mu_0 nI$ ....(1)

Where μ_0 = permeability of free space.

c. The magnetic field inside the rod is given as,

$B_m = \mu_0 M$ ....(2)

Where M = magnetisation of the material

d. The net magnetic field inside the rod is expressed as,

$B = B_0 + B_m$ ....(3)

$\therefore$ $B = \mu_0 nI + \mu_0 M$

$\therefore$ $B = \mu_0 H + \mu_0 M$

Where H = nI = Magnetic field intensity

$\therefore$ $B = \mu_0 (H + M)$

$\therefore$ $H = \dfrac{B}{\mu_0} - M$ ....(4)

e. Equation (4) shows that the magnetic field (B) induced in the material depends on magnetic field intensity (H) and magnetization (M).

Q.2. Explain origin of magnetism in material, hence find magnetic moment of electron revolving around the nucleus of an atom.

Ans:

i. Origin of magnetism in material:

a. Magnetism has its origin in the circulating charges in an atom.

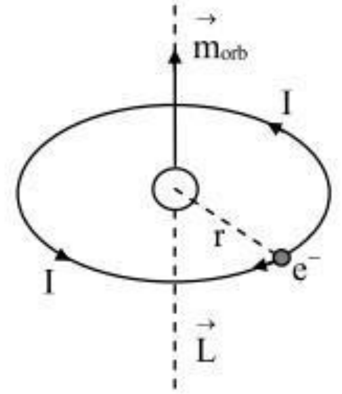
b. Circulating electron is equivalent to a current loop and has a magnetic dipole moment.

c. An atom of any substance consists of a small massive positively charged nucleus surrounded by negatively charged electrons revolving in circular orbit around the nucleus.

d. The magnetic moment is associated with motion of electron in its orbit and is termed as orbital magnetic moment.

ii. Expression for magnetic dipole moment:

a. Consider an electron of mass m_e and charge e revolving in a circular orbit of radius r around the positive nucleus in clockwise direction, leading to an anticlockwise current.



U.C.M of electron around nucleus

b. If the electron travels a distance $2\pi r$ in time T then, its orbital speed $v = 2\pi r/T$

c. Magnitude of circulating current is given by,

$$I = e\left(\frac{1}{T}\right)$$

But, $T = \frac{2\pi r}{v}$

$\therefore \quad I = e\left(\frac{1}{2\pi r / v}\right) = \frac{ev}{2\pi r}$

d. The orbital magnetic moment associated with orbital current loop is given by,

$m_{orb} = IA = \frac{ev}{2\pi r} \times \pi r^2$ [$\because$ Area of current loop, $A = \pi r^2$]

$\therefore \quad m_{orb} = \frac{evr}{2}$ ….(1)

e. The angular momentum of an electron due to its orbital motion is given by,

$L = m_e vr$

f. Multiplying and dividing the R.H.S of equation (1) by m_e,

$$m_{orb} = \frac{e}{2m_e} \times m_e vr$$

$\therefore \quad m_{orb} = \frac{eL}{2m_e}$

g. This equation shows that orbital magnetic moment is proportional to the angular momentum. But as the electron bears negative charge, the orbital magnetic moment and orbital angular momentum are in opposite directions and perpendicular to the plane of the orbit.

Using vector notation, $\vec{m}_{orb} = -\left(\frac{e}{2m_e}\right)\vec{L}$

Q.3. An electron in an atom is revolving round the nucleus in a circular orbit of radius 5.3 × 10^{-11} m, with a speed of 2 × 10^6 ms^{-1}. Find resultant orbital magnetic moment and angular momentum of electron. (e = 1.6 × 10^{-19} C, m = 9.1 × 10^{-31} kg)

Solution:

Given: $r = 5.3 \times 10^{-11}$ m,
$v = 2 \times 10^6$ ms^{-1},
$e = 1.6 \times 10^{-19}$ C,
$m_e = 9.1 \times 10^{-31}$ kg

To find: i. Orbital magnetic moment (m_{orb})
ii. Angular momentum of electron

Formulae: i. $m_{orb} = \frac{evr}{2}$
ii. $L = mvr$

Calculation: From formula (i),

$$m_{orb} = \frac{1.6 \times 10^{-19} \times 2 \times 10^6 \times 5.3 \times 10^{-11}}{2}$$
$$= 1.6 \times 5.3 \times 10^{-24}$$
$$= \mathbf{8.48 \times 10^{-24}\ Am^2}$$

From formula (ii),

$$L = 9.1 \times 10^{-31} \times 2 \times 10^6 \times 5.3 \times 10^{-11}$$
$$= 96.46 \times 10^{-36}$$

∴ $L \approx \mathbf{9.646 \times 10^{-35}\ kgm^2/s}$

Ans: i. Orbital magnetic moment is **8.48×10^{-24} Am^2**.
ii. Angular momentum of electron is **9.646×10^{-35} kgm^2/s**.

*[**Note:** Answer calculated above is in accordance with textual method of calculation.]*

12 Electromagnetic Induction

Multiple Choice Questions (1 Mark Each)

1. In which of the following devices, the eddy current is not used
   (A) Electromagnet (B) Induction furnace
   (C) Electric heater (D) Magnetic breaking in train

2. An ideal transformer has 100 turns in the primary and 250 turns in the secondary. The peak value of the AC is 28 V. The rms secondary voltage is nearest to
   (A) 100 V (B) 70 V
   (C) 50 V (D) 40 V`

Hint: $\frac{e_P}{e_S} = \frac{N_P}{N_S}$

$\therefore \quad e_S = e_P \times \frac{N_S}{N_P} = 28 \times \frac{250}{100} = 70 \text{ V}$

$\therefore \quad e_{rms} = \frac{e_S}{\sqrt{2}} = \frac{70}{\sqrt{2}} \approx 50 \text{ V}$

3. The role of inductance is equivalent to
   (A) inertia (B) force
   (C) energy (D) momentum

4. The energy stored in a 50 mH inductor carrying a current of 4 A is
   (A) 0.4 J (B) 0.1 J
   (C) 0.04 J (D) 0.01 J

Hint: $U = \frac{1}{2}LI^2 = \frac{1}{2} \times 50 \times 10^{-3} \times 4^2 = 0.4 \text{ J}$

5. In the expression $e = -d\phi/dt$, the -ve sign signifies
   (A) The induced emf is produced only when magnetic flux decreases.
   (B) The induced emf opposes the change in the magnetic flux.
   (C) The induced emf is opposite to the direction of the flux.
   (D) The induced emf is independent of change in magnetic flux.

6. Two pure inductors each of self inductance L are connected in series, the net inductance is
   (A) 2L (B) L
   (C) L/2 (D) L/4

7. A magnet is moved towards a coil (i) quickly (ii) slowly, then the induced e.m.f. is
   (A) larger in case (i)
   (B) smaller in case (i)
   (C) equal to both the cases
   (D) larger or smaller depending upon the radius of the coil

Very Short Answer (VSA) (1 Mark Each)

Q.1. Sate Faraday's Law of electromagnetic Induction.

Ans:

i. First law:

Whenever there is a change in the magnetic flux associated with a coil, an e.m.f is induced in the coil.

ii. Second law:

The magnitude of the induced e.m.f is directly proportional to the rate of change of magnetic flux through the coil.

Q.2. State the mathematical relation between number of turns in primary coil to secondary coil in step up transformer.

Ans: In step up transformer, the number of turns in secondary coil is more than that in primary coil ($N_S > N_P$).

Q.3. State the condition at which we say to the two coils kept close to each other are perfectly coupled with each other.

Ans: When coefficient of coupling (K) is 1, then the two coils kept close to each other are said to be perfectly coupled with each other.

Q.4. State Lenz's Law.

Ans: **Statement:** *The direction of induced current in a circuit is such that the magnetic field produced by the induced current opposes the change in the magnetic flux that induces the current. The direction of induced emf is same as that of induced current.*

Q.5. A pair of adjacent coil has a mutual inductance of 1.5 H. If the current in one coil varies from 0 to 20 A in 0.5 s, what is the change of flux linked with the other coil.

Ans: Change of flux linked with the other coil ,

$d\phi = MdI$

$= 1.5 \times 20$

$= 30$ Wb

*[**Note:** Answer calculated above is in accordance with textual method of calculation.]*

Q.6. An aircraft of wing span of 50 m flies horizontally in earth's magnetic field of 6×10^{-5} T at a speed of 400 m/s. Calculate the emf generated between the tips of the wings of the aircraft.

Ans: Induced emf between tips of wings ,

$e = Blv$

$= 6\times10^{-5} \times 50 \times 400$

$= 1.2$ V

Q.7. A coil of self inductance 3 H carries a steady current of 2 A. What is the energy stored in the magnetic field of the coil?

Ans: Energy stored in the magnetic field, $U_B = \frac{1}{2} LI^2 = \frac{1}{2}\times 3\times 2^2 = 6$ J

Short Answer I (SA1) (2 Marks Each)

Q.1. Why and where are eddy currents are undesirable? How are they minimised?

Ans: Undesirable effects of eddy currents:

The soft iron core is used in dynamo transformers, motors, generators etc. When a.c is passed through these instruments the flux changes and eddy currents are set up in the core. Therefore, the core is heated up so the electrical energy is wasted in the form of heat energy.

Minimisation of undesirable effect of eddy current:

i. To minimise undesirable effect of eddy currents, laminated or insulated iron core are used which minimise the magnitude of eddy currents.

ii. If the surface area of the metal plate is reduced, amount of eddy current generated is reduced.

Q.2. Define Self Inductance, Mutual Inductance.

Ans:

i. *The **self-inductance** of a circuit is the ratio of magnetic flux (produced due to current in the circuit) linked with the circuit to the current flowing in it.*

ii. *The **mutual inductance** M of two circuits (or coils) is the magnetic flux (ϕ_s) linked with the secondary circuit per unit current (I_P) of the primary circuit.*

Q.3. Explain why the inductance of two coils connected in parallel is less than the inductance of either coil.

Ans:

i. For parallel combination of two coils, the current through each parallel inductor is a fraction of the total current and the voltage across each parallel inductor is same.

ii. As a result, a change in total current will result in less voltage dropped across the parallel array than for any one of the individual inductors.

iii. There will be less voltage drop across parallel inductors for a given rate of change in current than for any of the individual inductors.

iv. Less voltage for the same rate of change in current results in less inductance.

v. Thus, the total inductance of two coils is less than the inductance of either coil.

Q.4. An emf of 96 mV is induced in the windings of a coil when a current in a nearby coil is increasing at the rate of 1.20 A/s. What is the mutual inductance of the two coil?

Solution:

Given: $e = 96\text{ mV} = 96 \times 10^{-3}\text{ V}$,

$\frac{dI}{dt} = 1.20\text{ A/s}$

To find: Mutual Inductance (M)

Formula: $M = \frac{|e|}{\left|\frac{dI}{dt}\right|}$

Calculation: From formula

$$M = \frac{96 \times 10^{-3}}{1.2} = 80 \times 10^{-3} = \mathbf{80\text{ mH}}$$

Ans: Mutual Inductance of the two coils is **80 mH**.

Q.5. Calculate the induced emf between the ends of an axle of a railway carriage 1.75 m long travelling on level ground with a uniform velocity 50 kmph. The vertical component of Earth's magnetic field (B_v) is 5×10^{-5} T.

Solution:

Given: $l = 1.75$ m, $B_V = 5\times10^{-5}$ T,

$v = 50$ km/h $= 50 \times \frac{5}{18}$ m/s

To find: Induced emf

Formula: $e = Blv$

Calculation: From formula

$$e = 5\times10^{-5} \times 1.75 \times 50 \times \frac{5}{18}$$

$$= 121.5 \times 10^{-5}\ \text{V}$$

$$= \mathbf{1.215\ mV}$$

Ans: Induced emf is **1.215 mV**.

Q.6. The magnetic flux through a loop varies according to the relation $\phi = 8t^2 + 6t + 2$, ϕ is in milliweber and t is in second. What is the magnitude of the induced emf in the loop at t = 2 seconds?

Solution:

Given: $\phi = 8t^2 + 6t + 2$ (in milliweber)

$t = 2$ s

To find: Magnitude of induced e.m.f. (e)

Formula: $e = \frac{d\phi}{dt}$ (in magnitude)

Calculation: Using formula,

$$e = \frac{d}{dt}(8t^2 + 6t + 2)$$

$$= 8 \times 2t + 6$$

$$= 16t + 6$$

At t = 2s

$$|e| = 16 \times 2 + 6$$

$$= \mathbf{38\ mV}$$

Ans: The magnitude of induced e.m.f. is **38 mV.**

Q.7. Distinguish between Step up and Step down Transformer.

Ans:

No.	Step-up transformer	Step-down transformer
i.	The number of turns in its secondary is more than that in its primary ($N_S > N_P$).	The number of turns in primary is greater than secondary ($N_P > N_S$).
ii.	Alternating voltage across the ends of its secondary is more than that across its primary i.e., $e_S > e_P$	Alternating voltage across the ends of the primary is more than that across its secondary i.e., $e_P > e_S$
iii.	Transformer ratio K > 1.	Transformer ratio K < 1.
iv.	Primary coil made of thick wire.	Secondary coil made of thick wire.
v.	Secondary coil is made of thin wire.	Primary coil is made of thin wire.
vi.	Current through secondary is less than primary.	Current through primary is less than secondary.

[Any four differences]

Short Answer II (SA2) (3 Marks Each)

Q.1. What is Transformer? Explain step up and step down transformer?

Ans: Transformer:

i. ***Transformer*** *is an electrical device which converts low alternating voltage at high current to high alternating voltage at low current and vice-versa.*

ii. Step up transformer:

a. A transformer which converts a low voltage at high current into a high voltage at low current is called step-up transformer.

b. In step-up transformer, number of turns in secondary coil N_S is greater than the number of turns in primary coil N_P. In this transformer, $e_S > e_P$ and $I_S < I_P$.

c. The primary coil is made from a thick insulated copper wire so that it can sustain the high current. The secondary coil is made of a thin insulated wire.

d. Current through secondary is less than primary.

iii. Step down transformer:

a. A transformer which converts a high voltage at low current into a low voltage at high current is called step-down transformer.

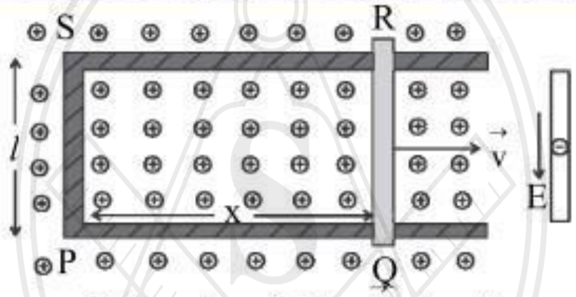
b. In step down transformer, number of turns in secondary coil N_S is less than the number of turns in the primary coil N_P. In this transformer $e_S < e_P$ and $I_S > I_P$.

c. The primary coil is made of a thin insulated wire and the secondary coil is made from thick wire so that it can sustain the high current.

d. Current through primary is less than secondary.

Q.2. Determine the motional emf induced in a straight conductor moving in a uniform magnetic field with constant velocity on the basis of Lorentz force.

Ans:



A frame of wire PQRS in magnetic field $\vec{B}$ and wire BC is moving with velocity $\vec{v}$ along x-axis

i. Consider a rectangular frame of wires PQRS of area (lx) situated in a constant magnetic field $\left(\vec{B}\right)$.

ii. As the wire QR of length l is moved out with velocity $\vec{v}$ to increase x, the area of the loop PQRS increases. Thus the flux of $\vec{B}$ through the loop increases with time.

iii. According to the flux rule, the induced emf will be equal to the rate at which the magnetic flux through a conducting circuit changes.

iv. The induced emf will cause a current in the loop. It is assumed that there is enough resistance in the wire so that the induced currents are very small producing negligible magnetic field.

v. As the flux ϕ through the frame PQRS is Blx, magnitude of the induced emf can be written as

$$|e| = \frac{d\phi}{dt} = \frac{d}{dt}(Blx) = Bl\frac{dx}{dt} = Blv \qquad ....(1)$$

where v is the velocity of wire QR increasing the length x of wires PQ and SR.

vi. Now, a charge q which is carried along by the moving wire QR, experiences Lorentz force $\vec{F} = q(\vec{v} \times \vec{B})$; which is perpendicular to both $\vec{v}$ and $\vec{B}$ and hence is parallel to wire QR.

vii. The force $\vec{F}$ is constant along the length l of the wire QR (as v and B are constant) and zero elsewhere ($\because$ v = 0 for stationary part RSPQ of wire frame).

viii. When the charge q moves a distance l along the wire, the work done by the Lorentz force is $W = F.l = qvB\sin\theta \cdot l$

where θ = angle between $\vec{B}$ and $\vec{v}$.

ix. The emf generated is, $e = \frac{\text{Work}}{\text{charge}} = \frac{W}{q} = vB\sin\theta \cdot l$

x. For maximum induced emf, $\sin\theta = 1$

$e_{max} = Blv \qquad ....(2)$

xi. Thus, from equation (1) and (2), for any circuit whose parts move in a fixed magnetic field, the induced emf is the time derivative of flux (ϕ) regardless of the shape of the circuit.

Q.3. The primary of a transformer has 40 turns and works on 100 V and 100 W. Find a number of turns in the secondary to step up the voltage to 400 V. Also calculate the current in the secondary and primary.

Solution:

Given: $N_P = 40$, $e_P = 100$ V, $P_P = 100$ watt, $e_S = 400$ V

To find:
i. Number of turns in the secondary (N_S)
ii. Current in the secondary (I_S)
iii. Current in the primary (I_P)

Formulae: i. $\frac{e_P}{e_S} = \frac{N_P}{N_S}$ ii. $P = Ie$

Calculation: From formula (i),

$$N_S = N_P \times \frac{e_S}{e_P} = \frac{40 \times 400}{100}$$

$\therefore$ $N_S = \mathbf{160}$

For an ideal transformer, $P_S = P_P$

From formula (ii),

$P_S = I_S e_S$

$\therefore$ $I_P e_P = I_S e_S$

$$\therefore \quad I_S = \frac{I_P e_P}{e_S}$$

$$= \frac{P_P}{e_S} = \frac{100}{400} = \mathbf{0.25\ A}$$

$$I_P = \frac{P_P}{e_P}$$

$$I_P = \frac{100}{100} = 1\text{A}$$

$\therefore$ $I_P = \mathbf{1\ A}$

Ans: i. The number of turns in the secondary is **160.**

ii. The current in the secondary is **0.25 A.**

iii. The current in the primary is **1 A.**

Q.4. The primary and secondary coil of a transformer each have an inductance of 200 × 10^{-6} H. The mutual inductance (M) between the windings is 4 × 10^{-6} H. What percentage of the flux from one coil reaches the other?

Solution:

Self-inductance, L= 200×10^{-6} H

Mutual inductance, M = 4×10^{-6} H

$$\text{Percentage of flux transfer} = \frac{M}{L} \times 100$$

$$= \frac{4 \times 10^{-6}}{200 \times 10^{-6}} \times 100$$

$$= \mathbf{2\ \%}$$

Ans: 2 % of the flux from one coil reaches the other.

Q.5. Obtain an expression for the self inductance of a solenoid.

Ans:

i. Consider a current I established in the windings (turns) of a long solenoid. The current produces a magnetic flux ϕ_B through the central region.

ii. The inductance of the solenoid is given by,

$$L = \frac{N\phi_B}{I},$$

where N = the number of turns,
ϕ_B = magnetic flux linkage.

iii. The flux linkage for a length l near the middle of the solenoid is,

$$N\phi_B = (nl)\left(\vec{B}\cdot\vec{A}\right) = nlBA, \text{ (for } \theta = 0°\text{)},$$

where n = the number of turns per unit length,
B = magnetic field
A= the cross-sectional area of the solenoid.

iv. The magnetic field inside the solenoid is given as, $B = \mu_0 ni$

v. Hence, $L = \frac{N\phi_B}{i}$

$$= \frac{(nl)BA}{i}$$

$$= \frac{nl(\mu_0 ni)A}{i}$$

∴ $L = \mu_0 n^2 lA$
where, Al is the interior volume of solenoid.

Q.6. A plane of coil of 10 turns is tightly wound around a solenoid of diameter 2 cm having 400 turns per centimeter. The relative permeability of the core is 800. Calculate the inductance of solenoid.

Solution:

Given: $N = 10$, $d = 2$ cm $= 0.02$ m
$N = 400$ turns/cm $= 4 \times 10^4$ turns/m
$\mu_r = 800$

To find: Inductance of solenoid (L)

Formulae: i. $\mu = \mu_r\mu_o$ ii. $N = nl$

iii. $L = \mu n^2 l\left(\frac{\pi d^2}{4}\right)$

Calculation: From formula (i), (ii) and (iii),

$$L = \mu_r\mu_o nN\left(\frac{\pi d^2}{4}\right)$$

$$= \frac{800 \times 4\pi\times10^{-7} \times4 \times10^4 \times10\times\pi\times0.02^2}{4}$$

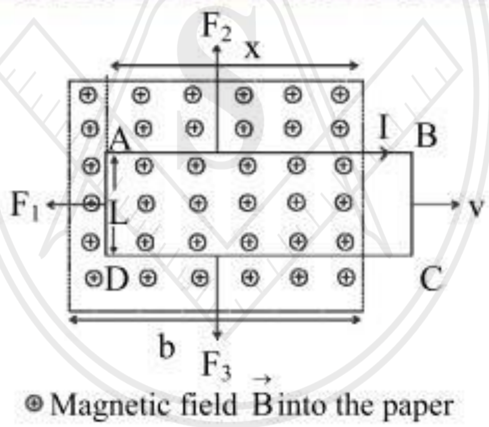
$$= \mathbf{0.1264\ H}$$

Ans: The inductance of solenoid is **0.1264 H**.

Long Answer (LA) (4 Marks Each)

Q.1. Find an expression for the power expended in pulling a conducting loop out of a magnetic field.

Ans:



Figure(a): A loop is moving out of magnetic field with velocity v

i. Consider a loop ABCD moving with constant velocity $\vec{v}$ in a constant magnetic field B.

ii. A current (i) is induced in the loop in clockwise direction and the loop segments, being still in magnetic field, experience forces, F_1, F_2 and F_3.

iii. To pull the loop at a constant velocity $\vec{v}$ towards right, it is required to apply an external force $\vec{F}$ on the loop so as to overcome the magnetic force of equal magnitude but acting in opposite direction.

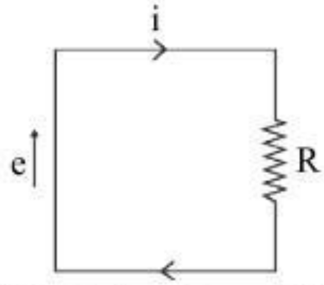


Figure (b): Induced emf (e), Induced current (i) and collective resistance (R) of the loop

iv. The rate of doing work on the loop is,

$$P = \frac{\text{Work}\ (W)}{\text{time}\ (t)} = \frac{\text{Force}\ (F) \times \text{displacement}\ (d)}{\text{time}\ (t)}$$

P = Force (F) × velocity (v)

∴ $\vec{P} = \vec{F} \cdot \vec{v}$ ….(1)

v. Magnitude of magnetic flux through the loop is,

$\phi_B = B.A = B.Lx$ ….(2)

vi. As the loop is moved to the right, the area lying within the magnetic field decreases, thus causing a decrease in the magnetic flux linked with the moving loop. As per Lenz's law, the decreasing magnetic flux induces current in the loop.

vii. According to Faraday's law, the magnitude of induced emf,

$$|e| = \left|\frac{d\phi}{dt}\right| = \frac{d}{dt}(BLx)$$

$$= BL \cdot \frac{dx}{dt} = BLv \quad ….(3)$$

viii. The magnitude of induced current i can be written using equation (3) as

$$i = \frac{|e|}{R} = \frac{BLv}{R} \quad ….(4)$$

ix. The three segments of the current carrying loop experience the deflecting forces $\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_3$ in the magnetic field $\vec{B}$ in accordance with equation, $\vec{F} = i\vec{L} \times \vec{B}$

x. From the symmetry, the forces $\vec{F}_2$ and $\vec{F}_3$ being equal and opposite, cancel each other. The remaining force $\vec{F}_1$ is directed opposite to the external force $\vec{F}$ on the loop, $\vec{F} = -\vec{F}_1$.

xi. The magnitude of $\left|\vec{F}_1\right|$ can be written as

$$\left|\vec{F}_1\right| = \text{iLB} \sin 90 = \text{iLB} = \left|\vec{F}\right| \quad ....(5)$$

xii. From equation (4), and equation (5)

$$\left|\vec{F}\right| = \left|\vec{F}_1\right| = \text{iLB}$$

$$= \frac{\text{BLv}}{\text{R}} \cdot \text{LB} = \frac{\text{B}^2 \text{L}^2 \text{v}}{\text{R}} \quad ....(6)$$

xiii. From equation (1) and (6), the rate of doing mechanical work (power), is given as,

$$\text{P} = \vec{F} \cdot \vec{v} = \frac{\text{B}^2 \text{L}^2 \text{v}}{\text{R}} \cdot \text{v} = \frac{\text{B}^2 \text{L}^2 \text{v}^2}{\text{R}} \quad ....(7)$$

xiv. If current i is flowing in the closed circuit with collective resistance R, the rate of production of heat energy in the loop as we pull it along at constant speed v, can be written as,

$$\text{P} = \text{i}^2 \text{R} \quad ....(8)$$

xv. From equation (4) and equation (8)

$$\text{P} = \left(\frac{\text{BLv}}{\text{R}}\right)^2 \cdot \text{R}$$

$$\text{P} = \frac{\text{B}^2 \text{L}^2 \text{v}^2}{\text{R}} \quad ....(9)$$

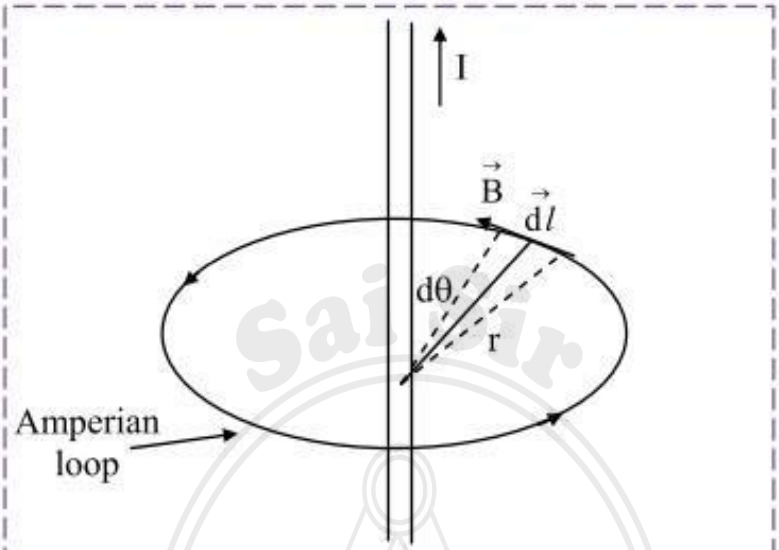
xvi. Comparing equation (7) and equation (9), it can be found that the rate of doing mechanical work is exactly same as the rate of production of heat energy in the circuit/loop.

xvii. Thus the work done in pulling the loop through the magnetic field appears as heat energy in the loop.

Q.2. Using Ampere's law, obtain an expression for the magnetic induction near a current carrying straight infinitely long wire.

Ans:

i. Consider a long straight wire carrying a current I as shown in figure below.



Long straight current carrying wire

$\vec{B}$ and $\vec{dl}$ are tangential to the Amperian loop which in this case is a circle.

$\therefore \quad \vec{B} \cdot \vec{dl} = B\,dl$

$= B\,r\,d\theta$

ii. The field $\vec{B}$ at a distance r from the wire is given by

$B = \frac{\mu_0 I}{2\pi r}$

$\therefore \quad \oint_c \vec{B} \cdot \vec{dl} = \int_0^{2\pi} \frac{\mu_0 I}{2\pi r} r\,d\theta = \mu_0 I$

*[**Note:** The question belongs to chapter 10, Magnetic Fields due to Electric Current]*

Q.3. Describe the construction and working of a transformer with a neat labelled diagram.

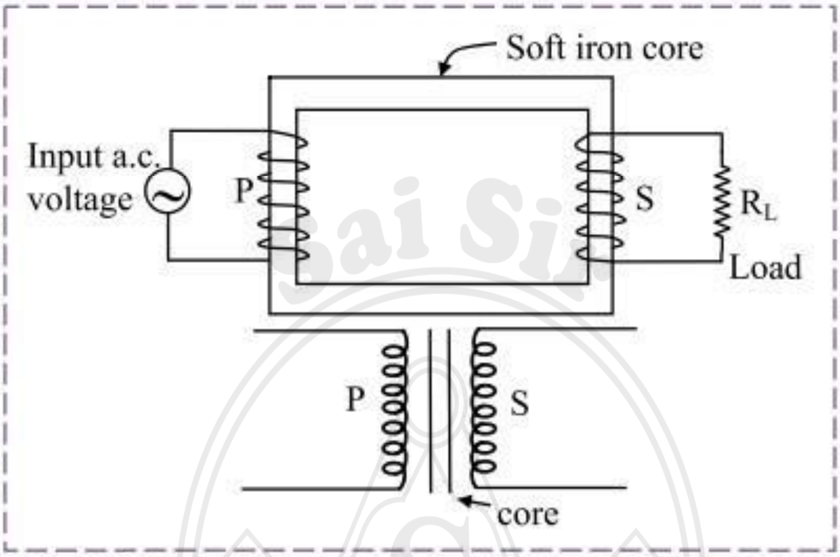
Ans: Principle:

It is based on the principle of mutual induction i.e., whenever the magnetic flux linked with a coil changes, an e.m.f is induced in the neighbouring coil.

i. **Construction:**

a. A transformer consists of two sets of coils primary P and secondary S insulated from each other. The coil P is called the input coil and coil S is called the output coil.

b. The two coils are wound separately on a laminated soft iron core.



ii. **Working:**

a. When an alternating voltage is applied to the primary coil the current through the coil goes on changing. Hence, the magnetic flux through the core also changes.

b. As this changing magnetic flux is linked with both the coils, an e.m.f is induced in each coil.

c. The amount of the magnetic flux linked with the coil depends upon the number of turns of the coil.

d. Let, 'ϕ' be the magnetic flux linked per turn with both the coils at certain instant 't'.

e. Let 'N_P and 'N_S' be the number of turns of primary and secondary coil,
$N_P\phi$ = magnetic flux linked with the primary coil at certain instant 't'
$N_S\phi$ = magnetic flux linked with the secondary coil at certain instant 't'

f. Induced e.m.f produced in the primary and secondary coil is given by,

$$e_P = -\frac{d\phi_P}{dt} = -N_P\frac{d\phi}{dt} \quad \text{....(1)}$$

$$e_S = -\frac{d\phi_S}{dt} = -N_S\frac{d\phi}{dt} \quad \text{....(2)}$$

g. Dividing equation (2) by (1),

∴ $\frac{e_S}{e_P} = \frac{N_S}{N_P}$ ....(3)

Equation (3) represents equation of transformer.

The ratio $\frac{N_S}{N_P}$ is called turns ratio (transformer ratio) of the transformer.

h. For an ideal transformer,

Input power = Output power

∴ $e_P I_P = e_S I_S$

∴ $\frac{e_S}{e_P} = \frac{I_P}{I_S}$ ....(4)

i. From equation (3) and (4), $\frac{e_S}{e_P} = \frac{N_S}{N_P} = \frac{I_P}{I_S}$

13 A.C Circuits

Multiple Choice Questions (1 Mark Each)

1. An electric current in an LC – circuit at resonance is called
   (A) The wattless current
   (B) The displacement current
   (C) The idle current
   (D) The apparent current

2. In a series LCR circuit at resonance, the applied emf and current are
   (A) Out of phase
   (B) Differ in phase by $\pi/4$ radian.
   (C) Differ in phase by $\pi/2$ radian.
   (D) In phase

3. A series LCR resonant circuit is used as
   (A) A potential divider circuit.
   (B) A radio wave transmitter
   (C) A source of displacement current
   (D) A tunning circuit in a television receiver set.

4. If AC voltage is applied to pure capacitor, then voltage across the capacitor
   (A) Leads the current by phase angle π rad.
   (B) Leads the current by phase $\pi/2$ rad.
   (C) Lags the current by phase angle π rad.
   (D) Lags the current by phase angle $\pi/2$ rad.

5. A parallel LC resonant circuit is used as
   (A) a tunning circuit in a television receiver set.
   (B) a transformer
   (C) a rectifier
   (D) a filter circuit.

6. An electric bulb operates 10 V d.c. If this bulb is connected to an a.c. source and gives normal brightness, then peak value of the source is
(A) 141.4 V (B) **14.14 V**
(C) 1.414 V (D) 0.1414 V

Hint: $e_0 = e_{rms}\sqrt{2} = 10 \times \sqrt{2} = 14.14$ V

7. A coil of resistance 300 Ω and inductance 1.0 H is connected across an alternating voltage of frequency $\frac{300}{2\pi}$ Hz, therefore phase difference between the voltage and current in the circuit is
(A) 180° (B) 90°
(C) **45°** (D) 0°

Hint: $\tan \phi = \frac{X_L}{R} = \frac{2\pi fL}{R} = \frac{2\pi \times \frac{300}{2\pi} \times 1}{300} = 1$

∴ $\phi = 45°$

Very Short Answer (VSA) (1 Mark Each)

Q.1. Define capacitive reactance.

Ans: *The capacitive reactance of a capacitor is defined as the ratio of r.m.s voltage (e.m.f) across the capacitor to the corresponding r.m.s current.*

Q.2. A charged 10 micro farad capacitor is connected to a 81 mH inductor. What is the angular frequency of free oscillations of the circuit?

Ans: Angular frequency,

$$\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{81 \times 10^{-3} \times 10 \times 10^{-6}}} = 1.1 \times 10^3 \text{ rad/s}$$

Q.3. State the equation for impedance Z in an A.C. circuit.

Ans: Equation for impedance in an A.C. circuit., $Z = \sqrt{R^2 + (X_L - X_C)^2}$

Q.4. In LCR series circuit, what is the condition for current resonance?

Ans: When the frequency of the applied e.m.f source is adjusted so that at particular frequency $X_L = X_C$ then Z = R. Thus the circuit behaves like a purely resistive circuit. The impedance of the circuit is minimum and current is maximum. The current and e.m.f of the source are in phase.

Q.5. State any one characteristic of a parallel LC AC resonance circuit.

Ans:

i. Resonance occurs when $X_L = X_C$.

ii. Resonant frequency $f_r = \frac{1}{2\pi\sqrt{LC}}$

iii. Impedance is maximum.

iv. Current is minimum.

v. When alternating current of different frequencies are sent through parallel resonant circuit, it offers a very high impedance to the current of the resonant frequency (f_r) and rejects it. However, it allows the current of the other frequencies to pass through it, hence called a rejector circuit.

[Any one characteristic]

Q.6. What is the relation between average current and rms current over half cycle.

Ans: Relation between average current and rms current over half cycle, $i_{av} = \frac{2\sqrt{2}}{\pi} i_{rms}$

Q.7. If the peak value of an alternating emf is 15V, what is its mean value over half cycle?

Ans: Mean value over half cycle, $e_{av} = \frac{2}{\pi} \times e_0 = \frac{2}{\pi} \times 15 = 9.548$ V

Short Answer I (SA1) (2 Marks Each)

Q.1. State the average or mean value of an alternating emf? Obtain the expression for it.

Ans:

i. Average or mean value of A.C. is the average of all values of the voltage (or current) over one half cycle.

ii. Since the average value of sin ωt over a cycle is zero, the average value over a full cycle is always zero.

iii. Hence, the mean value of AC over a cycle has no significance and the mean value of AC is defined as the average over half cycle.

iv. Average value of sinθ in the range 0^c to π^c is given as,

$$\langle \sin\theta \rangle = \frac{\int_0^{\pi} \sin\theta \, d\theta}{\int_0^{\pi} d\theta} = \frac{[-\cos\theta]_0^{\pi}}{[\theta]_0^{\pi}} = \frac{2}{\pi} = 0.637$$

v. Therefore, average value of emf = 0.637 × peak value
i.e., $e_{av} = 0.637\ e_0$

Q.2. Explain term impedance and state the formula for it in the case of an LCR series circuit.

Ans:

i. The ratio of rms voltage to the rms value of current is called impedance. The SI unit of impedance is ohm (Ω).

ii. Formula for impedance in case of an LCR series circuit,
$Z = \sqrt{R^2 + (X_L - X_C)^2}$
Where, R= Resistance,
X_L= Inductive reactance ,
X_C = Capacitive reactance.

Q.3. State any two characteristics of a series LCR AC resonance circuit.

Ans:

i. Resonance occurs when $X_L = X_C$.

ii. Resonant frequency $f_r = \dfrac{1}{2\pi\sqrt{LC}}$

iii. Impedance is minimum and circuit is purely resistive.

iv. Current has a maximum value.

v. When a number of frequencies are fed to it, it accepts only one frequency (f_r) and rejects the other frequencies. The current is maximum for this frequency. Hence it is called acceptor circuit.

[Any two characteristics]

Q.4. In LCR series circuit, what is the (i) impedance and (ii) reactance at current resonance?

Ans: At resonance,

i. the impedance is minimum, i.e, Z = R

ii. net reactance is zero as $|X_L - X_C| = 0$.

Q.5. A series LCR circuit has resistance 10 Ω and reactance is $7\sqrt{2}$ Ω. What is the impedance of the circuit?

Solution:

Given: $R = 10\ \Omega,\ |X_L - X_C| = 7\sqrt{2}\ \Omega$

To find: Impedance (Z)

Formula: $X = \sqrt{R^2 + (X_L - X_C)^2}$

Calculation: From formula,

$$Z = \sqrt{10^2 + \left(7\sqrt{2}\right)^2}$$

$$= \sqrt{100 + 98}$$

$$= \mathbf{14.07\ \Omega}$$

Ans: The impedance of the circuit is **14.07 Ω.**

Q.6. A coil of resistance 10 Ω and inductance 100 mH and a capacitor of variable capacitance are connected across a 20 V, 50 Hz A.C. supply. At what capacitance will resonance occur?

Solution:

Given: $R = 10\ \Omega$, $L = 100\ \text{mH} = 100 \times 10^{-3}\ \text{H} = 10^{-1}\ \text{H}$

$V = 20$ volt, $f_r = 50$ Hz

To find: Capacitance (C)

Formula: $f_r = \dfrac{1}{2\pi\sqrt{LC}}$

Calculation: From formula,

$$C = \frac{1}{4\pi^2 f_r^2 L}$$

$$= \frac{1}{4 \times (3.142)^2 \times 50^2 \times 10^{-1}}$$

$$= 101.3 \times 10^{-6}\ \text{F}$$

$$= \mathbf{101.3\ \mu F}$$

Ans: The resonance will occur when capacitance will have a value of **101.3 μF**.

[Note:** Answer calculated above is in accordance with textual method of calculation.**]

Q.7. Find the current in a circuit consisting of a coil and a capacitor in series with an A.C source of 110 V (r.m.s.), 60 Hz. The inductance of a coil in 0.80 H and its resistance is 50 Ω. The capacitance of a capacitor is 8 μF.

Solution:

Given: $f = 60$ Hz, $e_{rms} = 110$ V, $L = 0.8$ H,
$R = 50\ \Omega$, $C = 8\ \mu F$
$V = 20$ volt

To find: Current in the circuit (i_{rms})

Formulae:
i. $X_L = 2\pi fL$

ii. $X_C = \dfrac{1}{2\pi fC}$

iii. $Z = \sqrt{R^2 + (X_L - X_C)^2}$

iv. $i_{rms} = \dfrac{e_{rms}}{Z}$

Calculation: From formula (i),

$X_L = 2 \times 3.142 \times 60 \times 0.8 = 301.63\ \Omega$

From formula (ii),

$$X_C = \frac{1}{2 \times 3.142 \times 60 \times 8 \times 10^{-6}} = 331.53\ \Omega$$

From formula (iii),

$$Z = \sqrt{50^2 + (301.63 - 331.53)^2} = 58.26\ \Omega$$

From formula (iv),

$$i_{rms} = \frac{110}{58.26} = \mathbf{1.88\ A}$$

Ans: The current in the circuit is **1.88 A**.

Q.8. A 0.5 μF capacitor is discharged through a 10 millihenry inductor. Find the frequency of discharged.

Solution:

Given: $C = 0.5\ \mu F = 0.5 \times 10^{-6}$ F
$L = 10$ mH $= 10 \times 10^{-3}$ H $= 10^{-2}$ H

To find: Frequency (f)

Formula: $f = \dfrac{1}{2\pi\sqrt{LC}}$

Calculation: From formula,

$$f = \frac{1}{2 \times 3.142 \times \sqrt{0.5 \times 10^{-6} \times 10^{-2}}}$$

$$= \mathbf{2.25 \times 10^3\ Hz}$$

Ans: The frequency of discharged is $\mathbf{2.25 \times 10^3\ Hz}$.

Q.9. What is the capacitive reactance of a capacitor of 5 μF at a frequency (i) 50 Hz and (ii) 20 kHZ?

Solution:

Given: $C = 5\ \mu F = 5 \times 10^{-6}\ F$

$f_1 = 50\ Hz$, $f_2 = 20\ kHz$

To find:
i. Capacitive reactance at 50 Hz (X_{C_1})
ii. Capacitive reactance at 20 kHz (X_{C_2})

Formula: $X_C = \frac{1}{2\pi fC}$

Calculation: i. From formula,

$$X_{C_1} = \frac{1}{2\pi f_1 C}$$

$$= \frac{1}{2 \times 3.142 \times 50 \times 5 \times 10^{-6}}$$

$$= \mathbf{636.94\ \Omega}$$

ii. From formula,

$$X_{C_2} = \frac{1}{2\pi f_2 C}$$

$$= \frac{1}{2 \times 3.142 \times 20 \times 10^3 \times 5 \times 10^{-6}}$$

$$= \mathbf{1.59\ \Omega}$$

Ans: i. When frequency is 50 Hz, the capacitive reactance is **636.94 Ω.**

ii. When frequency is 20 kHz, the capacitive reactance is **1.59 Ω.**

Short Answer II (SA2) (3 Marks Each)

Q.1. State the rms value of an alternating current? Write the relation between the rms value and peak value of an alternating current that varies with time.

Ans:

i. The constant current which produces the same amount of heat in a given resistance in a given time as is produced by an alternating current, when flowing through the same resistance for the same time is called root mean square (rms) value of current.

It is given by, $i_{rms} = \frac{I_0}{\sqrt{2}} = 0.707\ i_0$.

ii. The relation between the rms value and peak value of alternating current is given by,

$$i_{rms}^2 = \frac{\int_0^{2\pi} i^2\, d\theta}{2\pi} = \frac{1}{2\pi}\int_0^{2\pi} i_0^2 \sin^2\theta d\theta$$

$$= \frac{i_0^2}{2\pi}\int_0^{2\pi}\frac{(1-\cos 2\theta)}{2}d\theta \qquad ....(\because \cos 2\theta = 1 - \sin^2\theta)$$

$$= \frac{i_0^2}{2\times 2\pi}\left[\left(\theta - \frac{\sin 2\theta}{2}\right)\right]_0^{2\pi}$$

$$= \frac{i_0^2}{2}$$

$\therefore \quad i_{rms} = \frac{i_0}{\sqrt{2}} = 0.707\ i_0$

Q.2. Explain the term inductive reactance. State its unit and dimensions.

Ans:

i. The opposing nature of an inductor to the flow of alternating current is called inductive reactance.

ii. In an inductive circuit,

$$i_0 = \frac{e_0}{\omega L} \qquad ....(1)$$

iii. For a resistive ac circuit, according to Ohm's law,

$$i = \frac{V}{R} \quad \text{....(2)}$$

where R = resistance in the circuit

iv. Comparing equations (1) and (2), we can conclude that ωL plays a similar role in an inductive ac circuit as resistor in a purely resistor circuit.

v. Hence, the effective resistance X_L offered by the inductance L is called inductive reactance and is given as,

$$X_L = \omega L = 2\pi fL. \quad \text{....}(\because \omega = 2\pi/T = 2\pi f)$$

where f = frequency of the AC supply.

vi. X_L is directly proportional to the inductance (L) and the frequency (f) of the alternating current.

vii. In DC circuits, f = 0

∴ $X_L = 0$

It implies that a pure inductor offers zero resistance to DC, i.e., it cannot reduce DC.

Thus, its passes DC and blocks AC of very high frequency.

viii. In an inductive circuit, the self induced emf opposes the growth as well as decay of current.

ix. The dimensions of inductive reactance is $[ML^2T^{-3}I^{-2}]$ and its SI unit is ohm (Ω).

Q.3. Explain the term capacitive reactance. State its unit and dimensions.

Ans:

i. The peak value of alternating current through a capacitor is given by,

$$i_0 = \frac{e_0}{(1/\omega C)} \quad \text{....(1)}$$

ii. For a capacitive a.c. circuit, According to Ohm's law,

$$i = \frac{V}{R} \quad \text{....(2)}$$

where R = resistance of the circuit.

iii. Comparing equation (1) and (2), we conclude that $\left(\frac{1}{\omega C}\right)$ plays a similar role in capacitive AC circuit as resistance in a purely resistive circuit.

iv. Hence, the effective resistance (X_C) offered by the capacitor C is given as,

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC} \qquad \ldots.\left(\because \omega = \frac{2\pi}{T} = 2\pi f\right)$$

Where f = frequency of AC supply

v. The function of capacitive reactance in a purely capacitive circuit is to limit the amplitude of the current similar to the resistance in a purely resistive circuit.

vi. X_C varies inversely as the frequency of AC and also as the capacitance of the condenser.

vii. In a DC circuit, f = 0

∴ $X_C = \infty$

Thus, capacitor blocks DC and acts as open circuit while it passes AC of high frequency.

viii. The dimensions of capacitive reactance is $[ML^2T^{-3}I^{-2}]$ and its SI unit is ohm (Ω).

Q.4. What is the inductive reactance of a coil of inductance 10 mH at a frequency (1) 50 Hz (2) 1000 Hz (3) 20 kHz?

Solution:

Given: $L = 10\text{ mH} = 10 \times 10^{-3}\text{H} = 10^{-2}\text{ H}$, $f_1 = 50$ Hz, $f_2 = 1000$ Hz, $f_3 = 20\text{ kHz} = 20000$ Hz

To find:
i. Reactance of the coil at 50 Hz $(X_L)_1$
ii. Reactance of the coil at 1000 Hz $(X_L)_2$
ii. Reactance of the coil at 20 kHz $(X_L)_3$

Formula: $X_L = 2\pi fL$

Calculation:
i. Using formula,

$(X_L)_1 = 2\pi f_1L = 2 \times 3.14 \times 50 \times 10^{-2}$

∴ $(X_L)_1 = \mathbf{3.14\ \Omega}$

ii. Using formula,

$(X_L)_2 = 2\pi f_2L = 2 \times 3.14 \times 1000 \times 10^{-2}$

$(X_L)_2 = \mathbf{62.8\ \Omega}$

iii. Using formula,

$(X_L)_3 = 2\pi f_3L = 2 \times 3.14 \times 20000 \times 10^{-2}$

$(X_L)_3 = \mathbf{1256\ \Omega}$

Ans: i. The reactance of the coil at 50 Hz is **31.4 Ω.**
ii. The reactance of the coil at 1000 Hz is **628 Ω.**
ii. The reactance of the coil at 20 kHz is **1256 Ω.**

Q.5. An alternating emf of 230 V, 50 Hz is connected across a pure ohmic resistance of 50 Ω. Find

i. the current

ii. equations for instantaneous values of current and voltage.

Solution:

Given: $e_{rms} = 230$ V, f = 50 Hz, R = 50 Ω

i. R.M.S value of current, $i_{rms} = \frac{e_{rms}}{R} = \frac{230}{50} = \mathbf{4.6\ A}$

ii. a. Peak value of current,

$i_0 = i_{rms} \times \sqrt{2}$

$= 4.6 \times \sqrt{2}$

$= 6.5$ A

∴ Equation for instantaneous value of current,

$i = i_0 \sin 2\pi ft$

$= 6.5 \sin (2 \times \pi \times 50 \times t)$

∴ $\mathbf{i = 6.5 \sin 100\ \pi t}$

b. Peak value of voltage,

$e_0 = e_{rms} \times \sqrt{2} = 230 \times \sqrt{2} = 325.27$ V

∴ Equation for instantaneous value of voltage,

$e = e_0 \sin 2\pi ft$

$= 325.27 \sin (2 \times \pi \times 50 \times t)$

∴ $\mathbf{e = 325.27 \sin 100\ \pi t}$

Ans: i. R.M.S value of current is **4.6 A**.

ii. Equations for instantaneous value of current and voltage are **i = 6.5 sin 100 πt** and **e = 325.27 sin 100 πt** respectively.

Q.6. A radio can tune over the frequency range of a portion of MW broadcast band (800 kHz - 1200 kHz). If its LC circuit has an effective inductance of 200 mH, what must be the range of its variable condenser?

Solution:

Given: $f_1 = 800$ kHz $= 0.8 \times 10^6$ Hz

$f_2 = 1200$ kHz $= 1.2 \times 10^6$ Hz

$L = 200$ mH $= 200 \times 10^{-3} = 0.2$ H

To find: Range of condenser

Formula: $f = \frac{1}{2\pi\sqrt{LC}}$

Calculation: i. For $f_1 = 0.8 \times 10^6$ Hz:
From formula,

$$C_1 = \frac{1}{4\pi^2 f^2 L}$$

$$= \frac{1}{4 \times 3.142^2 \times (0.8 \times 10^6)^2 \times 0.2}$$

$$\approx 198 \times 10^{-15} \text{ F}$$

$$\approx \mathbf{198\ pF}$$

ii. For $f_2 = 1.2 \times 10^6$ Hz:
From formula,

$$C_2 = \frac{1}{4\pi^2 f^2 L}$$

$$= \frac{1}{4 \times 3.142^2 \times (1.2 \times 10^6)^2 \times 0.2}$$

$$\approx 88 \times 10^{-15} \text{ F}$$

$$\approx \mathbf{88\ pF}$$

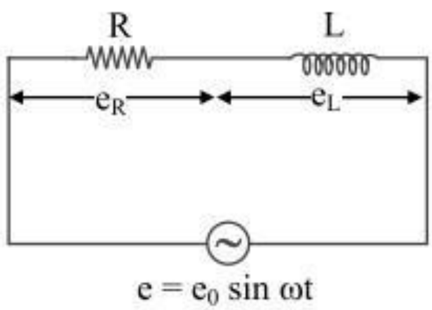
Ans: The range of the variable condenser is **88 pF to 198 pF**.

Long Answer (LA) (4 Marks Each)

Q.1. Obtain the expression for the applied emf and the effective resistance of the circuit when alternating emf is applied to an LR circuit.

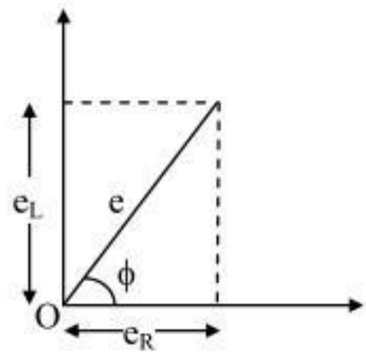
Ans:

i. Consider an alternating e.m.f. applied to a series combination of pure inductor of inductance L and resistor of resistance R.



ii. Let e_R and e_L be the r.m.s. voltage across resistor and inductor respectively. As R and L are in series, the current at any instant is given as, $i = i_0 \sin \omega t$.

iii. Vector diagram,



iv. From the above figure,

$e^2 = e_R^2 + e_L^2$

$\therefore$ $e = \sqrt{e_R^2 + e_L^2} = \sqrt{(iR)^2 + (iX_L)^2} = i\sqrt{R^2 + X_L^2}$

$\therefore$ $e = iZ$

where $Z = \sqrt{R^2 + X_L^2}$ is the impedance of the RL circuit.

v. a. Expression for applied emf in RL circuit ,

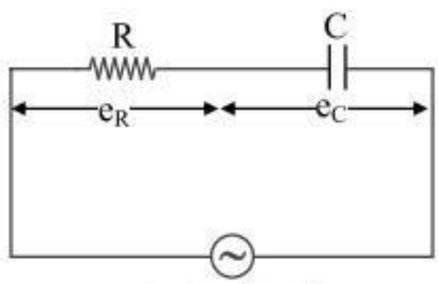
$e = e_0 \sin\omega t = \sqrt{e_R^2 + e_L^2} = iZ$

b. Expression for effective resistance in RL circuit, $Z = \sqrt{R^2 + X_L^2}$

Q.2. Obtain the expression for the applied emf and the effective resistance of the circuit when alternating emf is applied to an CR circuit.

Ans:

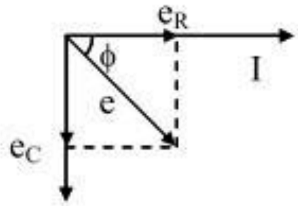
i. Consider an alternating e.m.f. applied to a series combination of pure capacitor of capacitance C and resistor of resistance R.



$e = e_0 \sin \omega t$

ii. Let e_R and e_C be the r.m.s. voltage across resistor and capacitor respectively. As R and C are in series, the current at any instant is given as, $i = i_0 \sin \omega t$.

iii. Vector diagram,



iv. From the above figure,

$e^2 = e_R^2 + e_C^2$

$\therefore$ $e = \sqrt{e_R^2 + e_C^2} = \sqrt{(iR)^2 + (iX_C)^2} = i\sqrt{R^2 + X_C^2}$

$\therefore$ $e = iZ$

where $Z = \sqrt{R^2 + X_C^2}$ is the impedance of the RC circuit.

v. a. Expression for applied emf in RL circuit,

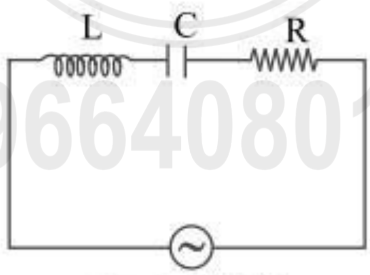
$e = e_0\sin\omega t = \sqrt{e_R^2 + e_C^2} = iZ$

b. Expression for effective resistance in RL circuit, $Z = \sqrt{R^2 + X_C^2}$

Q.3. Obtain the expression for the resonant frequency of the LCR series circuit and explain electrical resonance in an LCR series circuit.

Ans:

i. A circuit in which inductance L, capacitance C and resistance R are connected in series and the circuit admits maximum current corresponding to a given frequency of AC, is called a series resonance circuit.



$e = e_0 \sin \omega t$

ii. The impedance (Z) of an LCR circuit is given by,

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

iii. At very low frequencies, inductive reactance $X_L = \omega L$ is negligible but capacitive reactance $X_C = \dfrac{1}{\omega L}$ is very high.

iv. As we increase the applied frequency then X_L increases and X_C decreases.

v. At some angular frequency (ω_r), $X_L = X_C$

i.e., $\omega_r L = \dfrac{1}{\omega_r C}$

$\therefore \quad (\omega_r)^2 = \dfrac{1}{LC}$ or $(2\pi f_r)^2 = \dfrac{1}{LC}$

$\therefore \quad 2\pi f_r = \dfrac{1}{\sqrt{LC}}$

$\therefore \quad f_r = \dfrac{1}{2\pi\sqrt{LC}}$

Where f_r is called the resonant frequency.

vi. At this particular frequency f_r, since $X_L = X_C$ we get $Z = \sqrt{R^2 + 0} = R$. This is the least value of Z.

vii. Thus, when the impedance of an LCR circuit is minimum, circuit is said to be purely resistive, current and voltage are in phase and hence the current

$i_0 = \dfrac{e_0}{Z} = \dfrac{e_0}{R}$ is maximum. This condition of the LCR circuit is called resonance condition and this frequency is called series resonant frequency.

14 Dual Nature of Radiation and Matter

Multiple Choice Questions (1 Mark Each)

1. The electrons are emitted in the photoelectric effect from a metal surface
   - **(A) only if the frequency of radiation is above a certain threshold value.**
   - (B) only if the temperature of the surface is high.
   - (C) at the that is independent of the nature of metal.
   - (D) with a maximum velocity proportional to the frequency of incident radiation.

2. As the intensity of incident light increases
   - **(A) photoelectric current increases.**
   - (B) photoelectric current decreases.
   - (C) kinetic energy of emitted photoelectrons increases.
   - (D) kinetic energy of emitted photoelectrons decreases.

3. The maximum kinetic energy of the photoelectrons depends only on
   - (A) potential
   - **(B) frequency**
   - (C) incident angle
   - (D) pressure

4. According to De-Broglie, the waves are associated with
   - (A) moving neutral particles only.
   - (B) moving charged particle only.
   - (C) electrons only.
   - **(D) all moving matter particles.**

5. The work function of a metal is 4.2 eV. Its threshold wavelength will be
   - (A) 4000 Å
   - (B) 3500 Å
   - **(C) 2959 Å**
   - (D) 2500 Å

Hint: $\lambda = \frac{hc}{\phi_0} = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{4.2 \times 1.6 \times 10^{-19}}$

$= 2959.8 \times 10^{-10}$ m

≈ 2959 Å

6. Ultraviolet radiation of 6.2 eV falls on an aluminium surface (work function 4.2 eV). The kinetic energy in joules of the fastest electron emitted is

(A) 3.2×10^{-21} **(B) 3.2×10^{-19}**
(C) 3.2×10^{-1} (D) 3.2×10^{-15}

Hint: $K.E._{max} = h\nu - \phi_0$
$= 6.2 - 4.2 = 2$ eV
$= 2 \times 1.6 \times 10^{-19}$ J
$= 3.2 \times 10^{-19}$ J

7. Planck's constant is 6.6×10^{-34} Js. The momentum of each photon in a given radiation is 3.3×10^{-29} kg/s. The λ of radiation is

(A) 2×10^{10} m (B) 2×10^{7} m
(C) 2×10^{5} m **(D) 2×10^{-5} m**

Hint: $\lambda = \dfrac{h}{p}$

$$= \frac{6.6 \times 10^{-34}}{3.3 \times 10^{-29}} = 2 \times 10^{-5} \text{ m}$$

*[**Note:** Since momentum of photon is of the order of* 10^{-28} to 10^{-29}*, the value of momentum of photon in the question is modified considering options.]*

Very Short Answer (VSA) (1 Mark Each)

1. **Define photoelectric effect.**

Ans: *The phenomenon of emission of electrons from a metal surface, when radiation of appropriate frequency is incident on it, is known as **photoelectric effect**.*

2. **Define threshold frequency.**

Ans: *The minimum frequency of incident radiation required to start a photoemission in any photosensitive material is known as **threshold frequency**.*

3. **What is cut off or stopping potential?**

Ans: If increasingly negative potentials were applied to the collector in experiment of photoelectric effect, the photocurrent decreases and for some typical value ($- V_0$), photocurrent becomes zero. This value of V_0 is termed as cut-off or stopping potential.

4. **Define work function of the metal.**

Ans: *The minimum amount of energy required to be provided to an electron to pull it out of the metal from the surface is called the* ***work function of the metal****.*

5. **The minimum frequency for photoelectric effect on a metal is 7×10^{14} Hz, Find the work function of the metal.**

Ans: Work function, $\phi_0 = h\nu_0$

$= 6.63 \times 10^{-34} \times 7 \times 10^{14}$

$= 4.641 \times 10^{-19}$ J

*[**Note:** Answer is calculated considering standard value of Planck's constant.]*

6. **Find the kinetic energy of emitted electron, if in a photoelectric effect energy of incident photon is 4 eV and work function is 2.4 eV.**

Ans: $K.E._{max} = h\nu - \phi_0$

$= 4 - 2.4$

$= 1.6$ eV.

7. **Find energy of photon which have momentum 2×10^{-16} g-cm/s.**

Ans: $E = pc$

$= 2 \times 10^{-16} \times 3 \times 10^{10}$

$= 6 \times 10^{-6}$ erg

Short Answer I (SA1) (2 Marks Each)

1. **Explain the term 'wave particle duality' of matter.**

Ans:

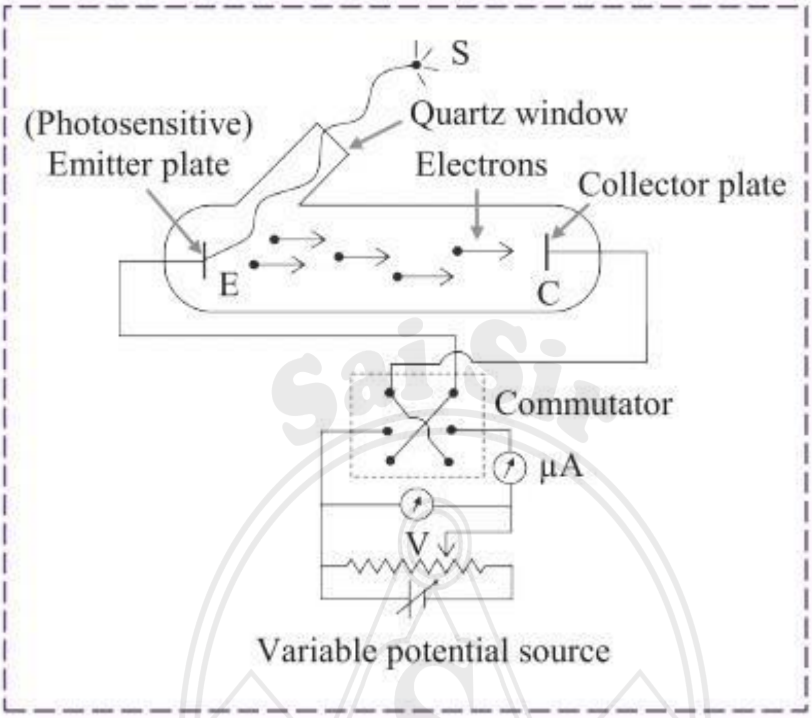
i. De Broglie proposed that if radiant energy (light) has both the wave nature and particle nature, then particle (matter) must have wave associated with its motion.

ii. He believed that energy and matter must have some symmetrical character. This gave rise to concept of wave particle duality of matter.

iii. Material particles show wave-like nature under certain circumstances. This phenomenon is known as wave-particle duality of matter.

2. Draw a neat labelled diagram of schematic of experimental set up for photoelectric effect.

Ans: Schematic of experimental set-up for photoelectric effect:



3. What is meant by dual nature of matter?

Ans:

i. Material particles show wave-like nature under certain circumstances. This phenomenon is known as wave-particle duality of matter or dual nature of matter.

ii. The wave associated with matter is called matter wave.

The wavelength associated with matter of mass m moving with velocity v is given by,

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

4. Explain the concept of photoelectric effect.

Ans:

i. *The phenomenon of emission of electrons from a metal surface, when radiation of appropriate frequency is incident on it, is known as* ***photoelectric effect****.*

ii. The emitted electrons are called photoelectrons and resulting current in the circuit due to them is called photoelectric current.

iii. When ultraviolet radiations fall on the emitter plate, electrons are ejected from it.

iv. They are attracted towards positive collector plate by the electric field. Thus, light falling on the surface of emitter causes current in the external circuit.

5. If the total energy of radiation of frequency 10^{14} Hz is 6.63 J, Calculate the number of photons in the radiation.

Solution:

Given: $E = 6.63$ J, $\nu = 10^{14}$ Hz,
We know, $h = 6.63 \times 10^{-34}$ Js.

To find: Number of photons (n)

Formula: $n = \dfrac{E}{h\nu}$

Calculation: Using formula,

$$n = \frac{6.63}{6.63\times10^{-34}\times10^{14}}$$

$\therefore$ $n = \mathbf{10^{20}}$

Ans: The number of photons emitted in the radiation are $\mathbf{10^{20}}$.

6. An electron is accelerated through a potential of 120 V. Find its de Broglie wavelength.

Solution:

Given: V = 120 V

To find: de Broglie wavelength of electron

Formula: $\lambda \text{ (in nm)} = \dfrac{1.228}{\sqrt{V}}$

Calculation: From formula,

$$\lambda = \frac{1.228}{\sqrt{120}}$$

$= \text{antilog}\{\log(1.228) - 0.5 \times \log(120)\}$

$= \text{antilog}\{0.0892 - 0.5 \times 2.0792\}$

$= \text{antilog}\{\bar{1}.0496\} = \mathbf{0.1121\ nm}$

Ans: The de Broglie wavelength of electron is **0.1121 nm**.

7. Calculate the stopping potential when the metal with the work function 0.6 eV is illuminated with the light of 2 eV.

Solution:

Given: $\phi_0 = 0.6$ eV, E = 2 eV

To find: Stopping potential (V_0)

Formula: $V_0 = \dfrac{E - \phi_0}{e}$

Calculation: $V_0 = \dfrac{2 \times 1.6 \times 10^{-19} - 0.6 \times 1.6 \times 10^{-1}}{1.6 \times 10^{-19}}$

$= 2 - 0.6$

$= \mathbf{1.4\ V}$

Ans: Stopping potential is **1.4 V**.

Short Answer II (SA2) (3 Marks Each)

1. State Einstein photoelectric equation. Explain 2 characteristics of photoelectric effect on the basis of Einstein's photoelectric equation.

Ans: Einstein's photoelectric equation: $K.E._{max} = (h\nu - \phi_0)$

Two characteristics of photoelectric effect:

i. The photoelectric work function ϕ_0 is constant for a given emitter. Hence if the frequency 'ν' of the incident radiation is decreased, the maximum kinetic energy of the emitted photoelectrons decreases, till it becomes zero for a certain frequency ν_0.

Therefore, from Einstein's equation,

$0 = h\nu_0 - \phi_0$

$\therefore \quad \phi_0 = h\nu_0 \qquad ....(1)$

This shows that the threshold frequency is related to the work function of the metal and hence it has different values for different metals.

ii. The photoelectric equation is,

$\frac{1}{2} mv^2_{max} = h\nu - \phi_0 \qquad ....(2)$

where, $h\nu$ = energy of the photon of incident radiation.

$\phi_0 = h\nu_0$ = photoelectric work function of the metal.

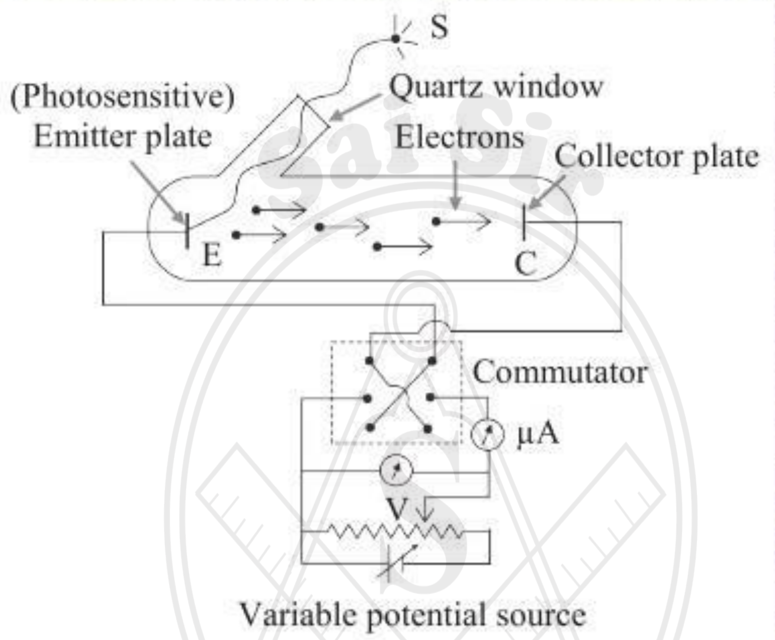
Thus, both the terms on the R.H.S of equation (2) depends on the frequency and not on the intensity of radiation. Hence, the maximum kinetic energy with which photoelectrons are emitted is independent of intensity of radiation. However, since ϕ_0 and h are constants, the maximum kinetic energy of the photoelectrons is directly proportional to the frequency.

2. With the help of circuit diagram describe an experiment to study photoelectric effect.

Ans:

i. A laboratory experimental set-up for the photoelectric effect consists of an evacuated glass tube with a quartz window.

ii. The glass tube contains photosensitive metal plates. One is the emitter E and another plate is the collector C.



Schematic of experimental set-up for photoelectric effect

iii. The emitter and collector are connected to a voltage source whose voltage can be changed and to an ammeter to measure the current in the circuit.

iv. A potential difference of V, as measured by the voltmeter, is maintained between the emitter E and collector C. Generally, C (the anode) is at a positive potential with respect to the emitter E (the cathode). This potential difference can be varied and C can even be at negative potential with respect to E.

v. When the anode potential (V) is positive, it accelerates the electrons. This potential is called accelerating potential. When the anode potential (V) is negative, it retards the flow of electrons. This potential is known as retarding potential.

vi. A source S of monochromatic light of sufficiently high frequency (short wavelength $\leq 10^{-7}$ m) is used.

3. What is photoelectric effect? Define stopping potential and photoelectric work function.

Ans:

i. *The phenomenon of emission of electrons from a metal surface, when radiation of appropriate frequency is incident on it, is known as **photoelectric effect**.*

ii. *If increasingly negative potentials were applied to the collector in experiment of photoelectric effect, the photocurrent decreases and for some typical value (– V_0), photocurrent becomes zero. This value of V_0 is termed as **cut-off** or **stopping potential**.*

iii. *The minimum amount of energy required to be provided to an electron to pull it out of the metal from the surface is called the **work function of the metal**.*

4. Calculate De Broglie wavelength of bullet moving with speed 90 m/s and having a mass 5 g.

Solution:

Given: v = 90 m/s, m = 5 g

To find: De Broglie wavelength (λ)

Formula: $\lambda = \frac{h}{mv}$

Calculation: $\lambda = \frac{6.63 \times 10^{-34}}{5 \times 90} = \mathbf{1.473 \times 10^{-36}}$ **m**

Ans: De Broglie wavelength of given bullet is $\mathbf{1.473 \times 10^{-36}}$ **m.**

[Note:** Answer is calculated considering standard value of Planck's constant.**]

5. The energy of photon is 2 eV. Find its frequency and wavelength.

Solution:

Given: $E = 2 \text{ eV} = 2 \times 1.6 \times 10^{-19} = 3.2 \times 10^{-19}$ J

To find: i. Frequency (ν) ii. Wavelength (λ)

Formulae: i. E = hν ii. $\lambda = \frac{c}{\nu}$

Calculation: Using formula (i),

$$\nu = \frac{3.2 \times 10^{-19}}{6.63 \times 10^{-34}}$$

$$= \frac{3.2 \times 10^{15}}{6.63}$$

$= \text{antilog}\{\log(3.2) + \log(10^{15}) - \log(6.63)\}$
$= \text{antilog}\{0.5051 + 15 - 0.8215\}$
$= \text{antilog}\{14.6836\}$
$= \mathbf{4.826 \times 10^{14}\ Hz}$

Using formula (ii),

$$\lambda = \frac{c}{\nu} = \frac{3\times10^8}{4.826\times10^{14}}$$

$= \text{antilog}\{\log 3 - \log 4.826\} \times 10^{-6}$
$= \text{antilog}\{0.4771 - 0.6836\} \times 10^{-6}$
$= \text{antilog}\{\bar{1}.7935\} \times 10^{-6}$
$= 0.6216 \times 10^{-6}$ m
$= 6216 \times 10^{-10}$ m
$= \mathbf{6216\ Å}$

Ans: i. Frequency of photon is **4.826×10^{14} Hz**
ii. Wavelength of photon is **6216 Å.**

[Note:** Wavelength of photon is calculated considering standard value of speed of light in vacuum.**]

6. The work function of a surface is 3.1 eV. A photon of frequency 1×10^{15} Hz. Is incident on it. Calculate the incident wavelength. Will photoelectric emission occur or not?

Solution:

Given: $\phi_0 = 3.1\ \text{eV} = 3.1 \times 1.6 \times 10^{-19}$ J
$\nu = 1 \times 10^{15}$ Hz

To find: i. Incident wavelength (λ)
ii. Will photoelectric emission occur.

Formulae: i. $\lambda_0 = \frac{hc}{\phi_0}$ ii. $\lambda = \frac{c}{\nu}$

Calculation: Using formula (i),

$$\lambda_0 = \frac{6.63\times10^{-34}\times3\times10^8}{3.1\times1.6\times10^{-19}}$$

$$= \frac{6.63\times3}{3.1\times1.6}\times10^{-7}$$

$= \text{antilog}\{\log 6.63 + \log 3 - \log 3.1 - \log 1.6\} \times 10^{-7}$
$= \text{antilog}\{0.8215 + 0.4771 - 0.4914 - 0.2041\} \times 10^{-7}$
$= \text{antilog}\{0.6031\} \times 10^{-7}$
$= 4.010 \times 10^{-7}$ m
$= 4010$ Å

Using formula (ii),

$$\lambda = \frac{3\times10^{8}}{1\times10^{15}}$$

$= 3 \times 10^{-7}$ m = **3000 Å**

As $\lambda < \lambda_0$, **photoelectric mission will occur.**

Ans: i. Incident wavelength is **3000 Å**

ii. Photoelectric emission **will occur**.

*[**Note:** Threshold wavelength (λ_0) is calculated considering standard value of Planck's constant.]*

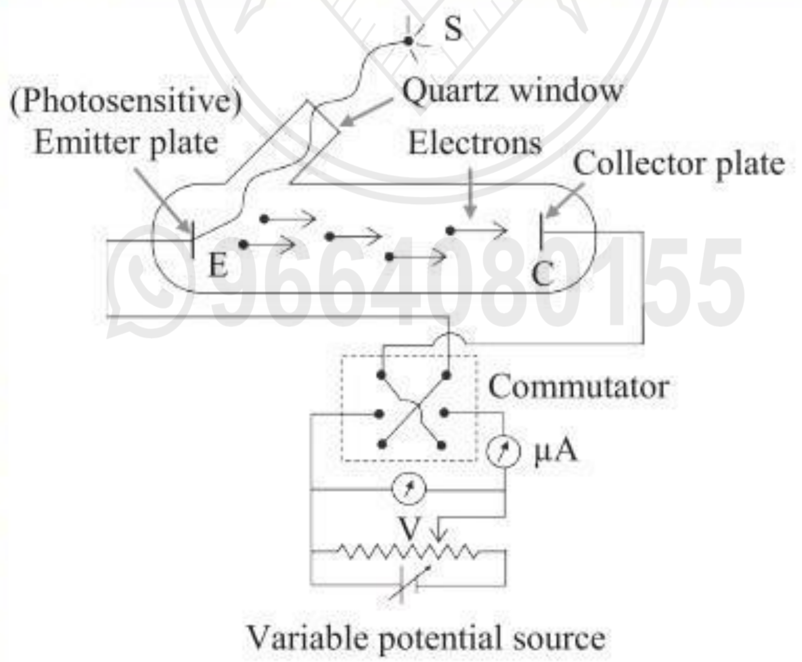
Long Answer (LA) (4 Marks Each)

1. With the help of circuit diagram describe the experiment to study the characteristics of photoelectric effect. Hence discuss any 2 characteristics of photoelectric effect.

Ans:

i. A laboratory experimental set-up for the photoelectric effect consists of an evacuated glass tube with a quartz window.

ii. The glass tube contains photosensitive metal plates. One is the emitter E and another plate is the collector C.



Schematic of experimental set-up for photoelectric effect

iii. The emitter and collector are connected to a voltage source whose voltage can be changed and to an ammeter to measure the current in the circuit.

iv. A potential difference of V, as measured by the voltmeter, is maintained between the emitter E and collector C. Generally, C (the anode) is at a positive potential with respect to the emitter E (the cathode).This potential difference can be varied and C can even be at negative potential with respect to E.

v. When the anode potential (V) is positive, it accelerates the electrons. This potential is called accelerating potential. When the anode potential (V) is negative, it retards the flow of electrons. This potential is known as retarding potential.

vi. A source S of monochromatic light of sufficiently high frequency (short wavelength $\leq 10^{-7}$ m) is used.

Two characteristics of photoelectric effect:

i. The photoelectric work function ϕ_0 is constant for a given emitter. Hence if the frequency 'ν' of the incident radiation is decreased, the maximum kinetic energy of the emitted photoelectrons decreases, till it becomes zero for a certain frequency ν_0.

Therefore, from Einstein's equation,

$0 = h\nu_0 - \phi_0$

$\therefore$ $\phi_0 = h\nu_0$ ....(1)

This shows that the threshold frequency is related to the work function of the metal and hence it has different values for different metals.

ii. The photoelectric equation is,

$$\frac{1}{2}mv^2_{max} = h\nu - \phi_0 \quad ....(2)$$

where, $h\nu$ = energy of the photon of incident radiation.

$\phi_0 = h\nu_0$ = photoelectric work function of the metal.

Thus, both the terms on the R.H.S of equation (2) depends on the frequency and not on the intensity of radiation. Hence the maximum kinetic energy with which photoelectrons are emitted is independent of intensity of radiation. However, since ϕ_0 and h are constants, the maximum kinetic energy of the photoelectrons is directly proportional to the frequency.

2. State Einstein's photoelectric equation. Explain all characteristics of photoelectric effect, on the basis of Einstein's photoelectric equation.

Ans: Einstein's photoelectric equation: $K.E._{max} = (h\nu - \phi_0)$

Characteristics of photoelectric effect:

i. The photoelectric work function ϕ_0 is constant for a given emitter. Hence if the frequency 'ν' of the incident radiation is decreased, the maximum kinetic energy of the emitted photoelectrons decreases, till it becomes zero for a certain frequency ν_0.

Therefore, from Einstein's equation,

$0 = h\nu_0 - \phi_0$

$\therefore \quad \phi_0 = h\nu_0 \qquad ....(1)$

This shows that the threshold frequency is related to the work function of the metal and hence it has different values for different metals.

ii. Using equation (1), Einstein's equation can be written as

$$\frac{1}{2}mv^2_{max} = h\nu - h\nu_0$$

$$\therefore \quad \frac{1}{2}mv^2_{max} = h(\nu - \nu_0)$$

This equation shows that:

a. If $\nu < \nu_0$, then K.E is negative, which is not possible. In this case, photoelectric emission is not possible.

b. If $\nu > \nu_0$, then photoelectrons move with some velocity.

$\therefore$ K.E > 0, which is possible. Hence, photoelectrons are emitted.

c. If $\nu = \nu_0$, the photoelectrons are just emitted. In this case, K.E = 0.

iii. The photoelectric equation is,

$$\frac{1}{2}mv^2_{max} = h\nu - \phi_0 \qquad ....(2)$$

where, $h\nu$ = energy of the photon of incident radiation.

$\phi_0 = h\nu_0$ = photoelectric work function of the metal.

Thus, both the terms on the R.H.S of equation (2) depends on the frequency and not on the intensity of radiation. Hence the maximum kinetic energy with which photoelectrons are emitted is independent of intensity of radiation. However, since ϕ_0 and h are constants, the maximum kinetic energy of the photoelectrons is directly proportional to the frequency.

iv. According to the quantum theory, when intensity of the radiation increases, there is proportional increase in the number of photons incident per second on the surface. One photon can cause emission of one photoelectron. Therefore, with the increase in the intensity of radiation, there will be increase in the photoelectron interactions and rate of emission of electrons.

v. The emission of a photoelectron is the result of collision between a photon and an electron. As soon as the radiation is incident on the photosensitive surface, the entire energy of the photon is absorbed by the electron at once. Therefore, the electrons are emitted at a moment when light is incident on the metal surface. This explains why photoelectric emission is instantaneous.

vi. If the electron, with which the incident photon collides, is situated on the emitting surface, the electron will be ejected with maximum K.E as given by Einstein's equation.

vii. If, however, the electron is situated in the interior of the emitting material, it will lose some energy in coming to the surface. This explains why the photoelectrons are emitted with different kinetic energies.

Thus, all the features of photoelectric effect are explained.

3. Explain De Broglie's hypothesis.

Ans:

i. De Broglie proposed that a moving material particle of total energy E and momentum p has a wave associated with it (analogous to a photon).

ii. He suggested a relation between properties of wave, like frequency and wavelength, with that of a particle, like energy and momentum.

$$p = \frac{E}{c} = \frac{h\nu}{c} = \frac{h}{\lambda}$$

iii. Thus, frequency and wavelength of a wave associated with a material particle, of mass m moving with a velocity v, are given as

$$\nu = \frac{E}{h} \text{ and } \lambda = \frac{h}{p} = \frac{h}{mv} \quad \ldots(1)$$

iv. De Broglie referred to these waves associated with material particles as matter waves. The wavelength of the matter waves, given by equation (1), is now known as de Broglie wavelength and the equation is known as de Broglie relation.

15 Structure of Atoms and Nuclei

Multiple Choice Questions (1 Mark Each)

1. When an electron jumps from higher energy orbit to lower energy orbit, the difference in the energies in the two orbits is radiated as quantum (photon) of
   (A) $E = mc^2$ (B) $E = h/\nu$
   (C) $E = \frac{hc}{\lambda}$ (D) $E = \frac{\lambda}{hc}$

2. The radii of Bohr orbit are directly proportional to….
   (A) Principal quantum number
   (B) Square of principal quantum number
   (C) Cube of principal quantum number
   (D) Independent of principal quantum number

3. According to Bohr second postulate, the angular momentum of electron is the integral multiple of $\frac{h}{2\pi}$. The S.I unit of Planck constant h is same as
   (A) Linear momentum **(B) angular momentum**
   (C) Energy (D) Centripetal force

4. The ionization energy of Hydrogen atom in its ground state is
   (A) 3.4 e V (B) 10.2 eV
   (C) 13.6 eV (D) –13.6 eV

5. For hydrogen atom, the minimum excitation energy (of n = 2) is
   (A) 3.4 e V **(B) 10.2 eV**
   (C) 13.6 eV (D) –10.2 eV

Hint: $E_n = \frac{-E_1 Z^2}{n^2}$

For hydrogen, Z = 1,

∴ For n = 2

$$E_2 = \frac{-13.6}{2^2} = -3.4 \text{ eV}$$

∴ Minimum excitation energy = $E_2 - E_1 = -3.4 - (-13.6) = 10.2$ eV

6. The dimensions of Rydberg's constant are

(A) $[M^0L^1T^0]$ (B) $[M^0L^{-1}T^0]$
(C) $[M^0L^1T^1]$ (D) $[M^0L^{-1}T^{-1}]$

7. In a Hydrogen, electron jumps from fourth orbit to second orbit. The wave number of the radiations emitted by electron is

(A) $\frac{R}{16}$ (B) $\frac{3R}{16}$
(C) $\frac{5R}{16}$ (D) $\frac{7R}{16}$

Hint: Wave number, $\frac{1}{\lambda} = R\left[\frac{1}{n^2} - \frac{1}{m^2}\right] = R\left[\frac{1}{2^2} - \frac{1}{4^2}\right] = \frac{3R}{16}$

8. The speed of electron having de Broglie wavelength of 10^{-10} m is ($m_e = 9.1 \times 10^{-31}$ kg, $h = 6.63 \times 10^{-34}$ J-s)

(A) 7.28×10^6 m/s (B) 4×10^6 m/s
(C) 8×10^5 m/s (D) 5.25×10^5 m/s

Hint: $\lambda = \frac{h}{mv}$

$\therefore \quad v = \frac{h}{m\lambda} = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 10^{-10}} \approx 7.28 \times 10^6$ m/s

*[**Note:** The question belongs to chapter 14, Dual Nature of Radiation and Matter.]*

9. The decay constant λ of a certain radioactive material is 0.2166 per day. The average life τ of the radioactive material is

(A) 5.332 days (B) 4.617 days
(C) 2.166 days (D) 1.083 days

Hint: Average life, $\tau = \frac{1}{\lambda} = \frac{1}{0.2166} = 4.617$ days

10. The ratio of areas of the circular orbit of an electron in the ground state to that of first excited state of an electron in hydrogen atom is…

(A) 16 : 1 (B) 4 :1
(C) 1 :4 (D) 1 :16

Hint: As, $r_n \propto n^2$

$$\frac{a_1}{a_2} = \frac{r_1^2}{r_2^2} = \left(\frac{n_1}{n_2}\right)^4 = \left(\frac{1}{2}\right)^4 = \frac{1}{16}$$

Very Short Answer (VSA) (1 Mark Each)

1. **What is the angular momentum of an electron in first exited state for hydrogen atom?**

Ans: For 1st excited state, n = 2

∴ Angular momentum = $\frac{nh}{2\pi} = \frac{h}{\pi}$

2. **If a_0 is the Bohr radius and n is the principal quantum number then, state the relation for the radius of n^{th} orbit of electron in terms of Bohr radius and principal quantum number.**

Ans: The required relation is $r_n = a_0n^2$

3. **In which region of electromagnetic spectrum for Hydrogen, does the Lyman series lies?**

Ans: The Lyman series lies in ultra-violet (UV) region.

4. **How much energy must be supplied to hydrogen atom, to free (remove) the electron in the ground state?**

Ans: Energy that needs be supplied to hydrogen atom, to free the electron in the ground state is 13.6 eV

5. **State the value of minimum excitation energy for Hydrogen atom.**

Ans: Minimum excitation energy = $E_2 - E_1 = -3.4 - (-13.6) = 10.2$ eV

6. **What is the energy of electron in hydrogen atom for n = ∞?**

Ans: The energy of electron in hydrogen atom for n = ∞ is zero.

7. **The radius of the smallest orbit of the electron (a_0) in hydrogen atom is 0.053 nm. What is the radius of the 4^{th} orbit of the electron in hydrogen atom?**

Ans: Radius of the 4^{th} orbit of the electron in hydrogen atom,

$r_4 = a_0 n^2 = 0.053 \times (4)^2 = 0.848$ nm

8. **The half life of a certain radioactive species is 6.93×10^5 seconds. What is the decay constant?**

Ans: Decay constant $\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{6.93\times10^5} = 10^{-6}$ s

9. The linear momentum of the particle is 6.63 kg m/s. Calculate the de Broglie wavelength.

Ans: $\lambda = \frac{h}{p}$

$= \frac{6.63 \times 10^{-34}}{6.63} = 10^{-34}$ m

*[**Note:** The question belongs to chapter 14, Dual Nature of Radiation and Matter.]*

Short Answer I (SA1) (2 Marks Each)

1. Starting with $r = \frac{\varepsilon_0 h^2 n^2}{\pi m Z e^2}$, Show that the speed of electron in n^{th} orbit varies inversely to principal quantum number.

Ans: According to Bohr's second postulate,

$$m r_n v_n = \frac{nh}{2\pi}$$

$$\therefore \quad m^2 v_n^2 r_n^2 = \frac{n^2 h^2}{4\pi^2}$$

$$\therefore \quad v_n^2 = \frac{n^2 h^2}{4\pi^2 m^2 r_n^2}$$

Substituting, $r_n = \frac{\varepsilon_0 h^2 n^2}{\pi m Z e^2}$ in above relation,

$$v_n^2 = \frac{n^2 h^2}{4\pi^2 m^2} \times \left(\frac{\pi m Z e^2}{\varepsilon_0 h^2 n^2}\right)^2$$

$$= \frac{n^2 h^2}{4\pi^2 m^2} \times \frac{\pi^2 m^2 Z^2 e^4}{\varepsilon_0^2 h^4 n^4}$$

$$= \frac{Z^2 e^4}{4\varepsilon_0^2 h^2 n^2}$$

$$\therefore \quad v_n^2 \propto \frac{1}{n^2}$$

$$\Rightarrow v_n \propto \frac{1}{n}$$

2. State Bohr second postulate for atomic model. Express it in its mathematical form.

Ans: Bohr's second postulate:

The radius of the orbit of an electron can only take certain fixed values such that the angular momentum of the electron in these orbits is an integral multiple of $\frac{h}{2\pi}$, *h being the Planck's constant.*

Mathematical form: $m_e r_n v_n = \frac{nh}{2\pi}$

where, m_e = mass of electron, r_n = radius of n^{th} Bohr's orbit, v_n = linear velocity of electron in n^{th} orbit, n = principal quantum number.

3. State any two limitations of Bohr's model for hydrogen atom.

Ans: Two limitations of Bohr's model for hydrogen atom:

i. Bohr's model for hydrogen atom could not explain the line spectra of atoms other than hydrogen. Even for hydrogen, more accurate study of the observed spectra showed multiple components in some lines which could not be explained on the basis of this model.

ii. It could not explain varying intensity of emission lines.

4. Using de Broglie's hypothesis, obtain the mathematical form of Bohr's second postulate.

Ans:

i. De Broglie suggested that instead of considering the orbiting electrons inside atoms as particles, they should be viewed as standing waves. Also, the length of the orbit of an electron should be an integral multiple of its wavelength.

ii. Now, the distance travelled by electron in one complete revolution in n^{th} orbit of radius r_n is $2\pi r_n$ and it should be integral multiple of wavelength.

$\therefore$ $2\pi r_n = n\lambda$ ....(1)

where, n = 1, 2, 3, 4….

iii. By de Broglie hypothesis,

$$\lambda = \frac{h}{p_n} = \frac{h}{m_e v_n}$$

iv. Substituting this value of 'λ' in equation (1),

momentum of electron, $p_n = \dfrac{nh}{2\pi r_n}$

∴ Angular momentum of electron $L_n = p_n r_n = \dfrac{nh}{2\pi}$.

Thus, mathematical form of Bohr's second postulate is obtained.

5. Show that half life period of radioactive material varies inversely to decay constant λ.

Ans: From law of radioactive decay,

$N = N_0 e^{-\lambda t}$

at $t = T_{1/2}$, $N = \dfrac{N_0}{2}$

∴ $\dfrac{N_0}{2} = N_0 e^{-\lambda T_{1/2}}$

∴ $\dfrac{1}{2} = e^{-\lambda T_{1/2}}$

∴ $e^{\lambda T_{1/2}} = 2$

∴ $\lambda T_{1/2} = \log_e 2 = 0.693$

∴ $T_{1/2} = \dfrac{0.693}{\lambda}$

$\Rightarrow T_{1/2} \propto \dfrac{1}{\lambda}$

6. Define (i) Excitation energy (ii) Ionization energy

Ans:

i. *The energy required to take an electron from the ground state to an excited state is called the* ***excitation energy*** *of the electron in that state.*

ii. *The* ***ionization energy*** *of an atom is the minimum amount of energy required to be given to an electron in the ground state of that atom to set the electron free.*

7. Calculate the longest wavelength in Paschen series.
(Given $R_H = 1.097 \times 10^7\ m^{-1}$)

Solution:

Given: $n = 3$, $m = 4$

To find: Longest wavelength in Paschen series (λ_L)

Formula: $\frac{1}{\lambda_L} = R\left[\frac{1}{n^2} - \frac{1}{m^2}\right]$

Calculation: From formula,

$$\frac{1}{\lambda_L} = R\left[\frac{1}{3^2} - \frac{1}{4^2}\right]$$

$$\therefore \quad \frac{1}{\lambda_L} = R\left[\frac{1}{9} - \frac{1}{16}\right]$$

$$= R\left[\frac{16-9}{9\times16}\right] = \frac{1.097\times10^7\times7}{9\times16}$$

$$\therefore \quad \lambda_L = \frac{9\times16}{1.097\times7}\times10^{-7}$$

$$= \text{antilog }\{\log(9) + \log(16) - \log(1.097) - \log(7)\} \times 10^{-7}$$

$$= \text{antilog}\{0.9542 + 1.2041 - 0.0402 - 0.8451\} \times 10^{-7}$$

$$= \text{antilog}\{1.2730\} \times 10^{-7}$$

$$= 18.75 \times 10^{-7}\text{ m}$$

$$\therefore \quad \lambda_L = \mathbf{18750\ Å}$$

Ans: The longest wavelength in Paschen series is **18750 Å.**

8. The angular momentum of electron in 3rd Bohr orbit of Hydrogen atom is 3.165×10^{-34} kg m²/s. Calculate Planck's constant h.

Solution:

Given: $L_3 = 3.165 \times 10^{-34}$ kg m^2/s, n = 3

To find: Planck's constant (h)

Formula: $L_n = n\frac{h}{2\pi}$

Calculation: From formula,

$$h = \frac{2\pi L_n}{n}$$

$$= \frac{2\times3.142\times3.165\times10^{-34}}{3}$$

$$= 6.284 \times 1.055 \times 10^{-34}$$

$$= \text{antilog }\{\log(6.284) + \log(1.055)\} \times 10^{-34}$$

$$= \text{antilog }\{0.7982 + 0.0232\} \times 10^{-34}$$

$$= \text{antilog}\{0.8214\} \times 10^{-34}$$

$$= \mathbf{6.628 \times 10^{-34}\ Js}$$

Ans: Value of Planck's constant (h) is **6.628×10^{-34} Js.**

9. The half-life of a certain radioactive nucleus is 3.2 days. Calculate (i) decay constant (ii) average life of radioactive nucleus.

Solution:

Given: $T_{1/2} = 3.2$ days

To find: i. decay constant (λ)

ii. average life (τ)

Formulae: i. $T_{1/2} = \dfrac{0.693}{\lambda}$

ii. $\tau = \dfrac{1}{\lambda}$

Calculation: From formula (i),

$$3.2 = \frac{0.693}{\lambda}$$

$$\therefore \quad \lambda = \frac{0.693}{3.2}$$

$$\therefore \quad \lambda = \text{antilog}\,\{\log(0.693) - \log(3.2)\}$$

$$= \text{antilog}\,\{\bar{1}.8407 - 0.5051\}$$

$$= \text{antilog}\,\{\bar{1}.3356\}$$

$$= \mathbf{0.2166\ /day}$$

From formula (ii),

$$\tau = \frac{1}{0.2166}$$

Using reciprocal table,

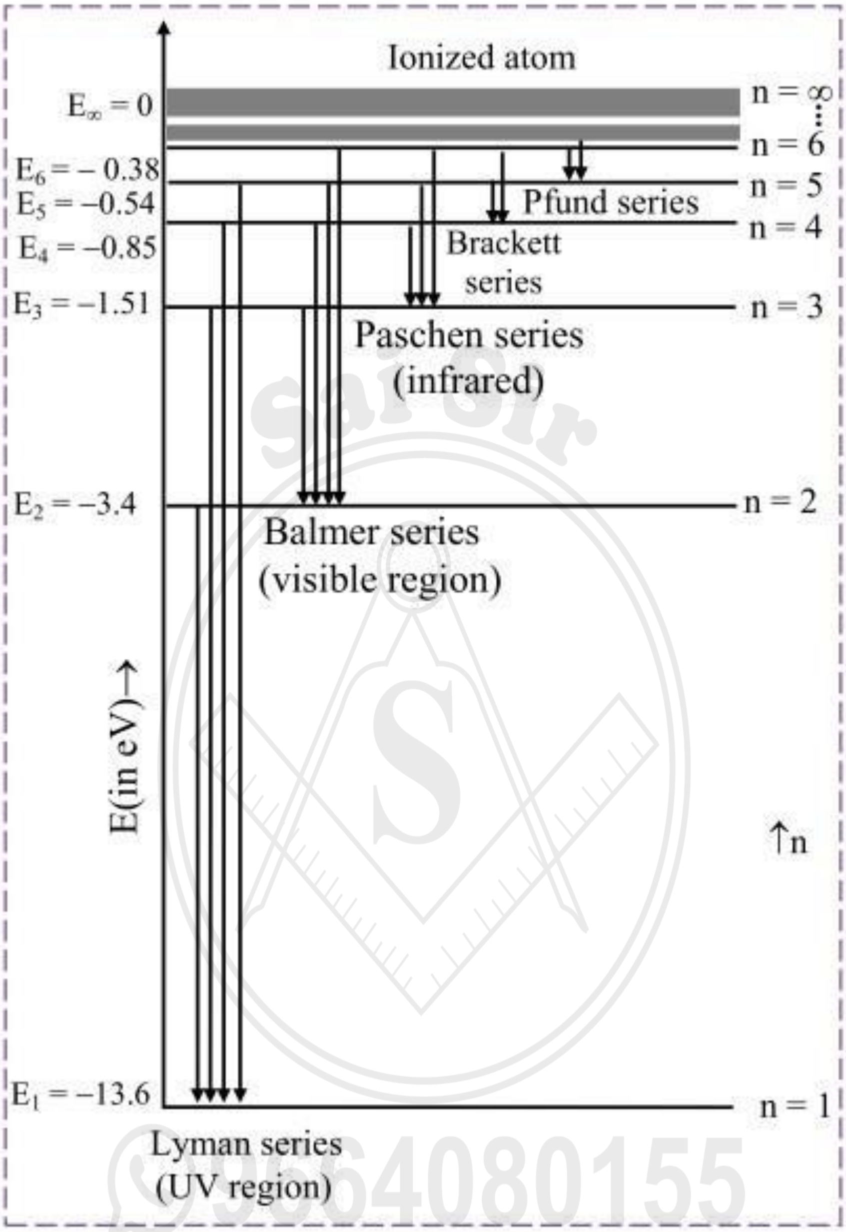
$\tau =$ **4.617 days**

Ans: i. The decay constant of reaction is **0.2166 /day**

ii. The mean life of the species is **4.617 days**.

10. Draw a neat labelled diagram showing energy levels and transition between them for hydrogen atom.

Ans: Energy levels and transition between them for hydrogen atom:



Short Answer II (SA2) (3 Marks Each)

1. Derive an expression for the radius of the n^{th} Bohr orbit for hydrogen atom.

Ans: Expression for radius of Bohr orbit in atom:

i. Let, m_e = mass of electron,

$-e$ = charge on electron,

r_n = radius of n^{th} Bohr's orbit,

$+e$ = charge on nucleus,

v_n = linear velocity of electron in n^{th} orbit,
Z = number of electrons in an atom,
n = principal quantum number.

ii. From Bohr's first postulate,
Coulomb's force F_e = Centripetal force F_{cp}

$$\therefore \quad \frac{Ze^2}{4\pi\varepsilon_0 r_n^2} = \frac{m_e v_n^2}{r_n}$$

$$\therefore \quad v_n^2 = \frac{Ze^2}{4\pi\varepsilon_0 r_n m_e} \qquad ....(1)$$

iii. According to Bohr's second postulate,

$$m_e r_n v_n = \frac{nh}{2\pi}$$

$$\therefore \quad m_e^2 v_n^2 r_n^2 = \frac{n^2h^2}{4\pi^2}$$

$$\therefore \quad v_n^2 = \frac{n^2h^2}{4\pi^2 m_e^2 r_n^2} \qquad ....(2)$$

iv. From equations (1) and (2),

$$\frac{n^2h^2}{4\pi^2 m_e^2 r_n^2} = \frac{Ze^2}{4\pi\varepsilon_0 r_n m_e}$$

$$\therefore \quad r_n = \frac{n^2h^2\varepsilon_0}{\pi m_e Ze^2}$$

$$\therefore \quad r_n = \left(\frac{\varepsilon_0 h^2}{\pi m_e Ze^2}\right) n^2 \qquad ....(3)$$

This is the required expression for radius of n^{th} orbit.

2. Using the expression for energy of electron in the n^{th} orbit, show that $\frac{1}{\lambda} = R_H\left(\frac{1}{n^2} - \frac{1}{m^2}\right)$, where symbols have their usual meaning.

Ans:

i. Let, E_m = Energy of electron in m^{th} higher orbit
E_n = Energy of electron in n^{th} lower orbit

ii. According to Bohr's third postulate,
$E_m - E_n = h\nu$

$$\therefore \quad \nu = \frac{E_m - E_n}{h} \qquad ....(1)$$

iii. But $E_m = -\frac{Z^2 m_e e^4}{8\varepsilon_0^2 h^2 m^2}$ ....(2)

$$E_n = -\frac{Z^2 m_e e^4}{8\varepsilon_0^2 h^2 n^2} \quad ....(3)$$

iv. From equations (1), (2) and (3),

$$\nu = \frac{-\frac{Z^2 m_e e^4}{8\varepsilon_0^2 h^2 m^2} - \left(-\frac{Z^2 m_e e^4}{8\varepsilon_0^2 h^2 n^2}\right)}{h}$$

$\therefore$ $\nu = \frac{Z^2 m_e e^4}{8\varepsilon_0^2 h^3}\left[-\frac{1}{m^2} + \frac{1}{n^2}\right]$

$\therefore$ $\frac{c}{\lambda} = \frac{Z^2 m_e e^4}{8\varepsilon_0^2 h^3}\left[\frac{1}{n^2} - \frac{1}{m^2}\right]$ ....[$\because$ $\nu = \frac{c}{\lambda}$]

where, c = speed of electromagnetic radiation

$\therefore$ $\frac{1}{\lambda} = \frac{Z^2 m_e e^4}{8\varepsilon_0^2 h^3 c}\left[\frac{1}{n^2} - \frac{1}{m^2}\right]$

v. But, $\frac{m_e e^4}{8\varepsilon_0^2 h^3 c} = R_H$ = Rydberg's constant

$= 1.097 \times 10^7 \text{ m}^{-1}$

$\therefore$ $\frac{1}{\lambda} = R_H Z^2 \left[\frac{1}{n^2} - \frac{1}{m^2}\right]$ ....(4)

For hydrogen Z = 1,

$\therefore$ $\frac{1}{\lambda} = R_H \left[\frac{1}{n^2} - \frac{1}{m^2}\right]$ ....(5)

3. Show that for radioactive decay $N(t) = N_0 e^{-\lambda t}$, where symbols have their usual meaning.

Ans: If 'N(t)' is the number of parent nuclei present at any instant 't', 'dN' is the number of nuclei disintegrated in short interval of time 'dt', then,

$dN \propto -N(t)\, dt$

$dN = -\lambda\, N(t)\, dt$

where, λ is known as decay constant or disintegration constant.

The negative sign indicates disintegration of atoms.

Integrating both sides of equation,

$$\int_{N_0}^{N(t)} \frac{dN}{N(t)} = \int_0^t -\lambda \, dt$$

where, N_0 is number of parent atoms at time $t = 0$.

$\therefore \quad \log_e \frac{N(t)}{N_0} = -\lambda t$

$\therefore \quad N(t) = N_0 \, e^{-\lambda t}$

This is the required relation.

4. Obtain an expression for half life time of radioactive material. Hence state the relation between average life and half life time of radioactive material.

Ans: Expression for half life period ($T_{1/2}$):

From law of radioactive decay, $N(t) = N_0 \, e^{-\lambda t}$

at $t = T_{1/2}$, $N(t) = \frac{N_0}{2}$

$\therefore \quad \frac{N_0}{2} = N_0 \, e^{-\lambda T(1/2)}$

$\therefore \quad e^{\lambda T(1/2)} = 2$

$\therefore \quad \lambda T_{1/2} = \log_e 2 = 0.693$

$\therefore \quad T_{1/2} = \frac{0.693}{\lambda}$

Relation between average life (τ) and half life time of radioactive material:

$T_{1/2} = \tau \, ln \, 2 = 0.693 \, \tau$

5. Calculate the wavelength for the first three lines in Paschen series. (Given $R_H = 1.097 \times 10^7 \text{ m}^{-1}$)

Solution:

Given: $R_H = 1.097 \times 10^7 \text{ m}^{-1}$,
For Paschen series, $n = 3$,

To find: Wavelength of first three lines of Paschen series

Formula: For Paschen series, $\frac{1}{\lambda} = R_H \left(\frac{1}{3^2} - \frac{1}{m^2}\right)$

Calculation: For first line of Paschen series,

From formula,

$$\frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1}{3^2} - \frac{1}{4^2}\right)$$

$$= 1.097 \times 10^7 \times \left(\frac{7}{9 \times 16}\right)$$

$$= 0.05333 \times 10^7 \text{ m}^{-1}$$

Using reciprocal table,

$\lambda_1 = \mathbf{1.876 \times 10^{-6}\ m}$

For second line of Paschen series,

From formula,

$$\frac{1}{\lambda_2} = 1.097 \times 10^7 \left(\frac{1}{3^2} - \frac{1}{5^2}\right)$$

$$= 1.097 \times 10^7 \times \left(\frac{16}{9 \times 25}\right)$$

$$= 0.075 \times 10^7 \text{ m}^{-1}$$

Using reciprocal table,

$\lambda_2 = \mathbf{1.282 \times 10^{-6}\ m}$

For third line of Paschen series,

From formula,

$$\frac{1}{\lambda_3} = 1.097 \times 10^7 \left(\frac{1}{3^2} - \frac{1}{6^2}\right)$$

$$= 1.097 \times 10^7 \times \left(\frac{27}{9 \times 36}\right)$$

$$= 0.0914 \times 10^7 \text{ m}^{-1}$$

Using reciprocal table,

$\lambda_3 = \mathbf{1.094 \times 10^{-6}\ m}$

Ans: Wavelength of first three lines of Paschen series are $\mathbf{1.875 \times 10^{-6}\ m}$, $\mathbf{1.282 \times 10^{-6}\ m}$, and $\mathbf{1.094 \times 10^{-6}\ m}$, respectively.

6. Calculate the shortest wavelength in Paschen series if the longest wavelength in Balmar series is 6563 Å.

Solution:

Given: $(\lambda_B) = 6563 \text{ Å} = 6563 \times 10^{-10}$ m

$= 6.563 \times 10^{-7}$ m

To find: Shortest wavelength (λ_p)

Formula: $\frac{1}{\lambda} = R\left[\frac{1}{n^2} - \frac{1}{m^2}\right]$

Calculation: For (λ_B), m = 3, n = 2

From formula,

$$\frac{1}{\lambda_B} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right]$$

$$\frac{1}{\lambda_B} = \frac{5R}{36}$$

$$\therefore \quad \lambda_B = \frac{36}{5R} \quad \text{....(1)}$$

For Paschen series shortest wavelength (λ_p),

n = 3, m = ∞

$$\therefore \quad \frac{1}{\lambda_p} = R\left[\frac{1}{3^2} - \frac{1}{\infty}\right]$$

$$\therefore \quad \frac{1}{\lambda_p} = R\left[\frac{1}{9}\right]$$

$$\therefore \quad \frac{1}{\lambda_p} = \frac{R}{9}$$

$$\therefore \quad \lambda_p = \frac{9}{R} \quad \text{....(2)}$$

From equations (1) and (2),

$$\frac{\lambda_p}{\lambda_B} = \frac{9/R}{36/5R}$$

$$\therefore \quad \frac{\lambda_p}{\lambda_B} = \frac{9}{R} \times \frac{5R}{36}$$

$$= \frac{5}{4}$$

$$\therefore \quad \lambda_p = \frac{5}{4} \times \lambda_B$$

$$= \frac{5}{4} \times 6563$$

$$\therefore \quad \lambda_p = \mathbf{8203.75\ Å}$$

Ans: The shortest wavelength in Paschen series is **8203.75 Å**.

7. A radioactive substance decays to $(1/10)^{th}$ of its original value in 56 days. Calculate its decay constant.

Ans: Here, $\frac{N(t)}{N} = \frac{1}{10}$ and t = 56 days

We have, $\frac{N(t)}{N_0} = e^{-\lambda t}$

$\therefore \quad \frac{1}{10} = e^{-\lambda t}$

$\therefore \quad e^{\lambda t} = 10$

$\therefore \quad \lambda t = \log_e 10$

$\therefore \quad \lambda = \frac{\log_e 10}{t}$

$= \frac{2.303 \times \log 10}{56}$

$= \frac{2.303}{56}$

= antilog {log(2.303) – log (56)}

= antilog {0.3623 – 1.7481}

= antilog $\{\bar{2}.6142\}$

= **4.113×10^{-2} per day**

Ans: Decay constant of is **4.113×10^{-2} per day**.

Long Answer (LA) (4 Marks Each)

1. State the postulates of Bohr's atomic model. Hence show energy of electron varies inversely to the square of principal quantum number.

Ans: Bohr's three postulates are:

i. *In a hydrogen atom, the electron revolves round the nucleus in a fixed circular orbit with constant speed.*

ii. *The radius of the orbit of an electron can only take certain fixed values such that the angular momentum of the electron in these orbits is an integral multiple of $\frac{h}{2\pi}$, h being the Planck's constant.*

iii. *An electron can make a transition from one of its orbits to another orbit having lower energy. In doing so, it emits a photon of energy equal to the difference in its energies in the two orbits.*

Expression for energy of electron in n^{th} orbit of Bohr's hydrogen atom:

i. **Kinetic energy:**

Let, m_e = mass of electron
r_n = radius of n^{th} orbit of Bohr's hydrogen atom
v_n = velocity of electron
$-e$ = charge of electron
$+e$ = charge on the nucleus
Z = number of electrons in an atom.

According to Bohr's first postulate,

$$\frac{m_e v_n^2}{r_n} = \frac{1}{4\pi\varepsilon_0} \times \frac{Ze^2}{r_n^2}$$

where, ε_0 is permittivity of free space.

$$\therefore \quad m_e v_n^2 = \frac{Z}{4\pi\varepsilon_0} \times \frac{e^2}{r_n} \qquad ....(1)$$

The revolving electron in the circular orbit has linear speed and hence it possesses kinetic energy.

It is given by, $K.E = \frac{1}{2} m_e v_n^2$

$$\therefore \quad K.E = \frac{1}{2} \times \left(\frac{Z}{4\pi\varepsilon_0} \times \frac{e^2}{r_n} \right) \qquad ....[\text{From equation (1)}]$$

$$\therefore \quad K.E = \frac{Ze^2}{8\pi\varepsilon_0 r_n} \qquad ....(2)$$

ii. **Potential energy:**

Potential energy of electron is given by, P.E = V(−e)

where,

V = electric potential at any point due to charge on nucleus

− e = charge on electron.

In this case,

$$\therefore \quad P.E = \frac{1}{4\pi\varepsilon_0} \times \frac{e}{r_n} \times (-Ze)$$

$$\therefore \quad P.E = \frac{1}{4\pi\varepsilon_0} \times \frac{-Ze^2}{r_n}$$

$$\therefore \quad P.E = -\frac{Ze^2}{4\pi\varepsilon_0 r_n} \qquad ....(3)$$

iii. **Total energy:**

The total energy of the electron in any orbit is its sum of P.E and K.E.

$\therefore$ T.E = K.E + P.E

$$= \left(\frac{Ze^2}{8\pi\varepsilon_0 r_n}\right) + \left(-\frac{Ze^2}{4\pi\varepsilon_0 r_n}\right) \quad \text{....[From equations (2) and (3)]}$$

$$\therefore \quad \text{T.E} = -\frac{Ze^2}{8\pi\varepsilon_0 r_n} \quad \text{....(4)}$$

iv. But, $r_n = \left(\frac{\varepsilon_0 h^2}{\pi m_e Ze^2}\right) \times n^2$

Substituting for r_n in equation (4),

$$\therefore \quad \text{T.E} = -\frac{1}{8\pi\varepsilon_0} \times \frac{Ze^2}{\left(\frac{\varepsilon_0 h^2}{\pi m_e Ze^2}\right) n^2}$$

$$= -\frac{1}{8\pi\varepsilon_0} \times \frac{Z^2 e^2 \pi m_e e^2}{\varepsilon_0 h^2 n^2}$$

$$\therefore \quad \text{T.E} = -\left(\frac{m_e Z^2 e^4}{8\varepsilon_0^2 h^2}\right) \times \frac{1}{n^2} \quad \text{....(5)}$$

$$\Rightarrow \text{T.E.} \propto \frac{1}{n^2}$$

2. Obtain an expression for wavenumber, when electron jumps from higher energy orbit to lower energy orbit. Hence show that the shortest wavelength for Balmar series is $4/R_H$.

Ans: Expression for wavenumber:

i. Let, E_m = Energy of electron in m^{th} higher orbit

E_n = Energy of electron in n^{th} lower orbit

ii. According to Bohr's third postulate,

$E_m - E_n = h\nu$

$$\therefore \quad \nu = \frac{E_m - E_n}{h} \quad \text{....(1)}$$

iii. But $E_m = -\frac{Z^2 m_e e^4}{8\varepsilon_0^2 h^2 m^2}$ ....(2)

$$E_n = -\frac{Z^2 m_e e^4}{8\varepsilon_0^2 h^2 n^2} \quad \text{....(3)}$$

iv. From equations (1), (2) and (3),

$$\nu = \frac{-\frac{Z^2 m_e e^4}{8\varepsilon_0^2 h^2 m^2} - \left(-\frac{Z^2 m_e e^4}{8\varepsilon_0^2 h^2 n^2}\right)}{h}$$

$$\therefore \quad \nu = \frac{Z^2 m_e e^4}{8\varepsilon_0^2 h^3}\left[-\frac{1}{m^2} + \frac{1}{n^2}\right]$$

$$\therefore \quad \frac{c}{\lambda} = \frac{Z^2 m_e e^4}{8\varepsilon_0^2 h^3}\left[\frac{1}{n^2} - \frac{1}{m^2}\right] \qquad \ldots .[\because \nu = \frac{c}{\lambda}]$$

where, c = speed of electromagnetic radiation

$$\therefore \quad \frac{1}{\lambda} = \frac{Z^2 m_e e^4}{8\varepsilon_0^2 h^3 c}\left[\frac{1}{n^2} - \frac{1}{m^2}\right]$$

v. But, $\frac{m_e e^4}{8\varepsilon_0^2 h^3 c} = R_H$ = Rydberg's constant

$$= 1.097 \times 10^7 \text{ m}^{-1}$$

$$\therefore \quad \frac{1}{\lambda} = R_H Z^2 \left[\frac{1}{n^2} - \frac{1}{m^2}\right] \qquad \ldots .(4)$$

This is the required expression.

vi. For shortest wavelength for Balmer series:

$n = 2$ and $m = \infty$

$$\frac{1}{\lambda} = R_H \left[\frac{1}{2^2} - \frac{1}{\infty}\right]$$

$$= \frac{R_H}{4}$$

$$\therefore \quad \lambda = \frac{4}{R_H}$$

3. Obtain an expression for decay law of radioactivity. Hence show that the activity $A(t) = \lambda N_0 e^{-\lambda t}$.

Ans: Expression for decay law of radioactivity:

If 'N(t)' is the number of parent nuclei present at any instant 't', 'dN' is the number of nuclei disintegrated in short interval of time 'dt', then,

$dN \propto -N(t)\, dt$

$dN = -\lambda N(t)\, dt$

where, λ is known as decay constant or disintegration constant.

The negative sign indicates disintegration of atoms.

Integrating both sides of equation,

$$\int_{N_0}^{N(t)} \frac{dN}{N(t)} = \int_0^t -\lambda \, dt$$

where, N_0 is number of parent atoms at time t = 0.

$\therefore \quad \log_e \frac{N(t)}{N_0} = -\lambda t$

$\therefore \quad N(t) = N_0 \, e^{-\lambda t}$

Expression for activity:

i. The rate of decay, i.e., the number of decays per unit time $-\frac{dN(t)}{dt}$, is called as activity A(t).

ii. It is given as,

$$A(t) = -\frac{dN(t)}{dt}$$

$$= \lambda N(t)$$

$$= \lambda N_0 e^{-\lambda t}$$

At t = 0, the activity is $A_0 = \lambda N_0$.

$\therefore \quad A(t) = A_0 e^{-\lambda t}$.

4. Using the expression for the radius of orbit for Hydrogen atom, show that the linear speed varies inversely to principal quantum number n the angular speed varies inversely to the cube of principal quantum number n.

Ans: According to Bohr's second postulate,

$$mr_nv_n = \frac{nh}{2\pi}$$

$\therefore \quad m^2v_n^2r_n^2 = \frac{n^2h^2}{4\pi^2}$

$\therefore \quad v_n^2 = \frac{n^2h^2}{4\pi^2m^2r_n^2}$

Substituting, $r_n = \dfrac{\varepsilon_0 h^2 n^2}{\pi m Z e^2}$ in above relation,

$$v_n^2 = \frac{n^2h^2}{4\pi^2m^2} \times \left(\frac{\pi m Z e^2}{\varepsilon_0 h^2 n^2}\right)^2$$

$$= \frac{n^2h^2}{4\pi^2m^2} \times \frac{\pi^2 m^2 Z^2 e^4}{\varepsilon_0^2 h^4 n^4}$$

$$= \frac{Z^2e^4}{4\varepsilon_0^2 h^2 n^2}$$

$$\therefore \quad v_n^2 \propto \frac{1}{n^2}$$

$$\Rightarrow v_n \propto \frac{1}{n}$$

Expression for angular speed:

Since, $v_n = r_n\,\omega$ and $r_n = \dfrac{\varepsilon_0 h^2 n^2}{\pi m_e e^2}$

$$\therefore \quad \omega = \frac{v_n}{r_n} = \left(\frac{e^2}{2\varepsilon_0 h}\right)\frac{1}{n} \Big/ \frac{\varepsilon_0 h^2 n^2}{\pi m_e e^2}$$

$$\therefore \quad \omega = \frac{e^2}{2\varepsilon_0 h n} \times \frac{\pi m_e e^2}{\varepsilon_0 h^2 n^2} = \left(\frac{\pi m_e e^4}{2\varepsilon_0^2 h^3}\right)\frac{1}{n^3}$$

$$\Rightarrow \omega \propto \frac{1}{n^3}$$

16 Semiconductor Devices

Multiple Choice Questions (1 Mark Each)

1. In a Bipolar junction transistor, the largest current flows though
   (A) in the emitter.
   (B) in the collector.
   (C) in the base.
   (D) through CB junction.

2. A LED emits visible white light when its
   (A) junction is reversed biased.
   (B) depletion layer widens.
   (C) holes and electrons recombine.
   (D) junction becomes hot.

3. Solar cell operates on a principle of
   (A) diffusion (B) recombination
   (C) photovoltaic action (D) carrier flow

4. The Boolean expression for Exclusive OR gate (X-OR gate) is
   (A) A + B **(B) $A \oplus B$**
   (C) $\overline{A + B}$ (D) A·B

5. In a common base configuration, the transistor has emitter current of 10 mA and collector current of 9.8 mA. The value of base current is
   (A) 0.1 mA **(B) 0.2 mA**
   (C) 0.3 mA (D) 0.4 mA

Hint: Base current, $I_B = I_E - I_C = 10 - 9.8 = 0.2$ mA

6. For a transistor β =75 and I_E = 7.5 mA. The value of α is….
   (A) 0.1 (B) 0.66
   (C) 0.75 **(D) 0.98**

Hint: $\alpha = \dfrac{\beta}{1+\beta} = \dfrac{75}{1+75} = 0.987$

7. In a transistor amplifier, $I_C = 5.5$ mA, $I_E = 5.6$ mA. The current amplification factor β is

(A) 45 (B) 50
(C) 55 (D) 60

Hint: $I_B = I_E - I_C = 5.6 - 5.5 = 0.1$ mA

$$\beta = \frac{I_C}{I_B} = \frac{5.5}{0.1} = 55$$

8. For which logic gate the following statement is true?
The output is low, if and only if all inputs are low

(A) AND (B) NOR
(C) NAND (D) OR

Very Short Answer (VSA) (1 Mark Each)

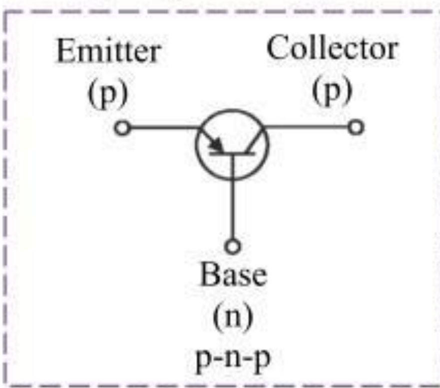
Q.1. State any two special purpose diodes.
Ans: Photodiode, solar cell.

Q.2. What is the purpose of capacitor filter in regulated power supply?
Ans: Capacitor filter removes the AC component or the ripple from a rectifier output and allows only the DC component.

Q.3. State the logical expression for NAND gate.
Ans: Logical expression: $Y = \overline{A \cdot B}$

Q.4. Which method of biasing is used for operating transistor as amplifier?
Ans: For transistor operating as an amplifier, the emitter-base junction is forward biased while collector-base junction is reverse biased.

Q.5. Draw the circuit symbol of PNP transistor.
Ans: Circuit symbol of p-n-p transistor



Q.6. For a transistor I_C = 15 mA, I_B = 0.5 mA. What is the current amplification factor?

Ans: Current amplification factor, $\beta = \frac{I_C}{I_B} = \frac{15}{0.5} = 30$

Q.7. Give the truth table for NOR gate.

Ans: Truth table for NOR gate:

Input A	Input B	Output Y
0	0	1
0	1	0
1	0	0
1	1	0

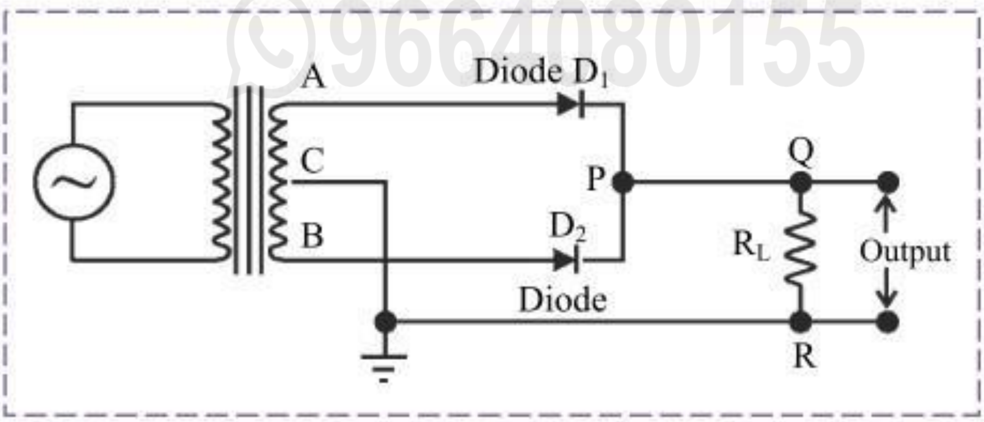
Q.8. What is the need of rectification in regulated power supply?

Ans: In regulated power supply, rectification is needed to convert AC voltage into a DC voltage.

Short Answer I (SA1) (2 Marks Each)

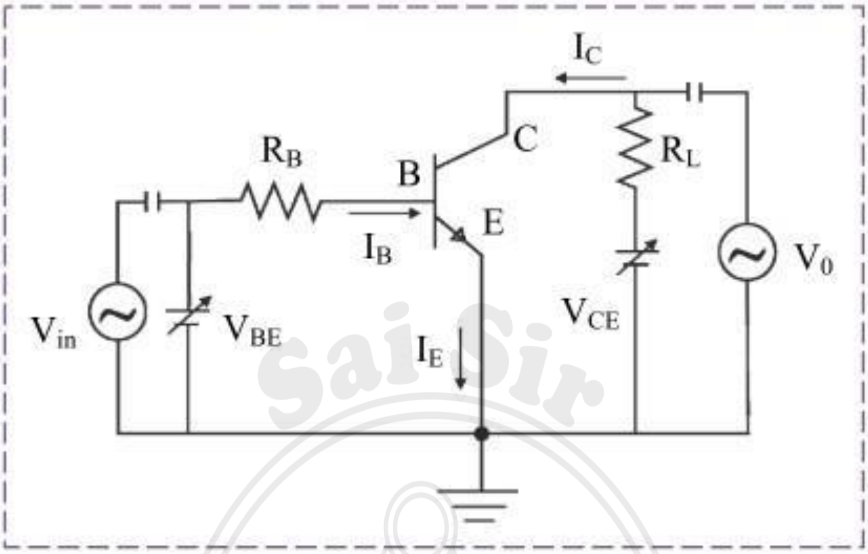
Q.1. Draw a neat labelled circuit diagram of full wave rectifier using semiconductor diode.

Ans: Full wave rectifier:



Q.2. Draw a neat labelled circuit diagram for transistor as common emitter amplifier.

Ans: Common emitter amplifier:



Q.3. State any two advantages and disadvantages of a photodiode.

Ans: Advantages of photodiode:

i. Quick response when exposed to light.

ii. The reverse current is linearly proportional to intensity of incident light. (Linear response)

iii. High speed of operations.

iv. Light weight and compact size.

v. Wide spectral response. E.g., photodiodes made from silicon respond to radiation of wavelengths from 190 nm (UV) to 1100 nm (IR).

vi. Relatively low cost.

[Any two points]

Disadvantages of photodiode:

i. Its properties are temperature dependent, similar to many other semiconductor devices.

ii. Low reverse current for low illumination levels.

Q.4. State the advantages of full wave rectifier.

Ans: Advantages of full wave rectifier:

i. Rectification takes place in both the cycles of the AC input.

ii. Efficiency of a full wave rectifier is higher than that of a half wave rectifier.

iii. The ripple in a full wave rectifier is less than that in a half wave rectifier.

Q.5. Define current amplification factor α_{DC} and β_{DC} Obtain the relation between them.

Ans:

i. In case of common emitter configuration, common base current gain or the current amplification factor (α_{DC}) is the ratio of the collector current to the emitter current.

$$\alpha_{DC} = \frac{I_C}{I_E} \quad \text{....(1)}$$

ii. Similarly, the common emitter current gain or the current amplification factor (β_{DC}) is defined as the ratio of the collector current to the base current.

$$\beta_{DC} = \frac{I_C}{I_B} \quad \text{....(2)}$$

iii. Since, $I_E = I_B + I_C$

Dividing throughout by I_C,

$$\frac{I_E}{I_C} = \frac{I_B}{I_C} + 1$$

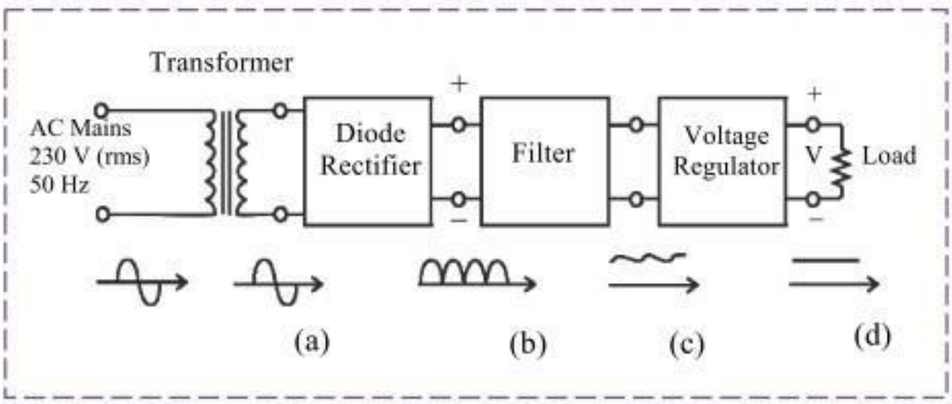
$\therefore$ $$\frac{1}{\alpha_{DC}} = \frac{1}{\beta_{DC}} + 1 \quad \text{....[From (1) and (2)]}$$

$\therefore$ $$\alpha_{DC} = \frac{\beta_{DC}}{1+\beta_{DC}}$$

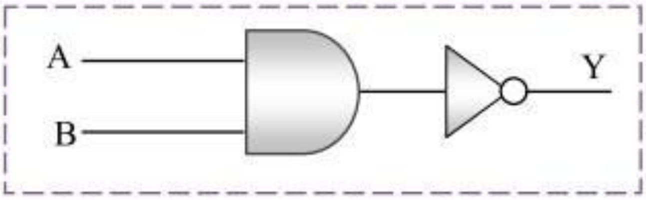
$\therefore$ $$\beta_{DC} = \frac{\alpha_{DC}}{1-\alpha_{DC}}$$

Q.6. Draw the block diagram of a simple rectifier circuit with respective output waveform.

Ans: Simple rectifier circuit with respective output waveform:



Q.7. Give the truth table and Boolean expression for



Ans: Boolean expression for the given gate (NAND):

$Y = \overline{A \cdot B}$

Truth table:

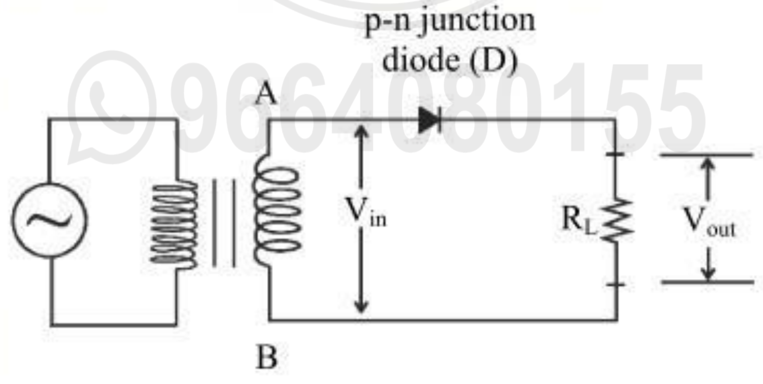
Input A	Input B	Output Y
0	0	1
0	1	1
1	0	1
1	1	0

Short Answer II (SA2) (3 Marks Each)

Q.1. Draw the circuit diagram of a half wave rectifier. Hence explain its working.

Ans: Working of half wave rectifier:

i. The given figure shows the circuit of a half wave rectifier.



Half wave rectifier

ii. The secondary coil AB of a transformer is connected in series with a diode D and the load resistance R_L. The AC voltage across the secondary coil AB changes its polarities after every half cycle.

iii. When the positive half cycle begins, the voltage at the point A is at higher potential with respect to that at the point B, therefore, the diode (D) is forward biased. It conducts and current flows through the circuit.

iv. When the negative half cycle begins, the potential at the point A is lower with respect to that at the point B and the diode is reverse biased, therefore, it does not conduct and no current passes through the circuit.

v. Hence, the diode conducts only in the positive half cycles of the AC input. It blocks the current during the negative half cycles. In this way, current always flows through the load R_L in the same direction for alternate positive half cycles and DC output voltage obtained across R_L in the form of alternate pulses.

vi. In half wave rectifier, the frequency of the ripple in output is same as that of the frequency of input.

Q.2. Explain the construction and working of solar cell.

Ans: Construction:

i. It consists of a p-n junction. The n-side of the junction faces the solar radiation. The p-side is relatively thick and is at the back of the solar cell.

ii. Both the p-side and the n-side are coated with a conducting material. The n-side is coated with antireflection coating which allows visible light to pass through it. The main function of this coating is to reflect the IR (heat) radiations and protect the solar cell from heat.

iii. This coating works as the electrical contact of the solar cell. The contact on the n-side is called the front contact and that at the p-side is called the back contact or the rear contact.

iv. The n-side of a solar cell is thin so that the light incident on it reaches the depletion region where the electron-hole pairs are generated.

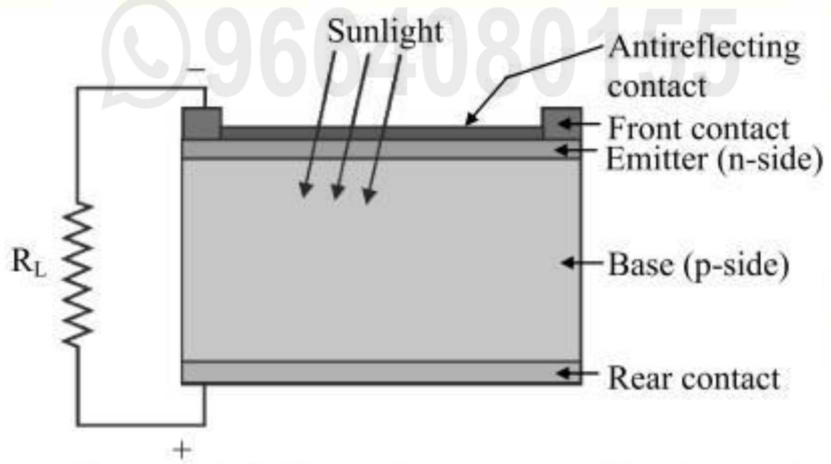


Figure (a): Schematic structure of a solar cell

Working:

i. When a light with photon energy greater than the band gap energy is incident on a solar cell, electron-hole pairs are formed in the depletion region of the diode.

ii. The electrons and holes thus formed get recombined and are not available for conduction.

iii. However, the photo-generated electrons in the p-type material, and the photo-generated holes in the n-type material are spatially separated and prevented from recombination in a solar cell.

iv. This separation of carriers is possible due to the intrinsic electric field of the depletion region. Figure (b) shows this schematically.

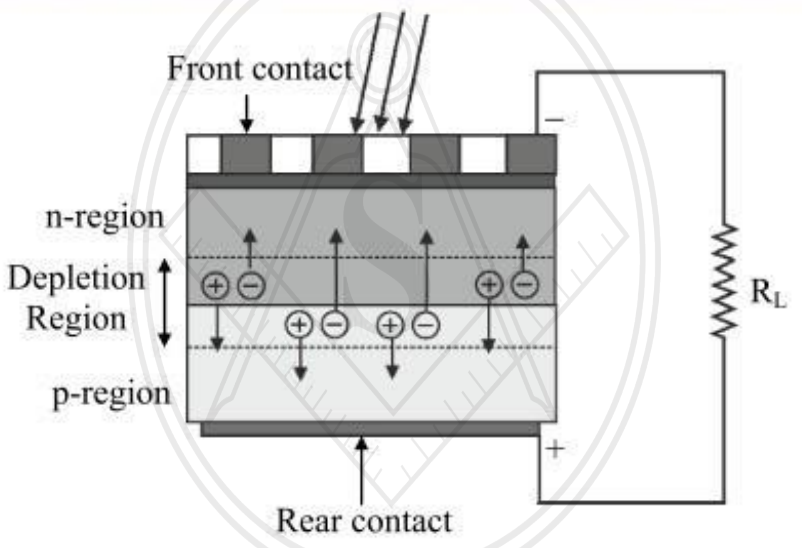


Figure (b)

v. When the light-generated electron in the p-type region reaches the junction, it crosses the junction due to the electric field at the junction. It reaches the n-type region where it is now a majority carrier.

vi. Similarly, the light generated hole reaches the p-type region and becomes a majority carrier in it.

vii. The positive and negative charges are thus accumulated on the p-region and the n-region of the solar cell which can be used as a voltage source.

viii. When the solar cell is connected to an external circuit, the light-generated carriers flow through the external circuit.

Q.3. Explain the working of LED.

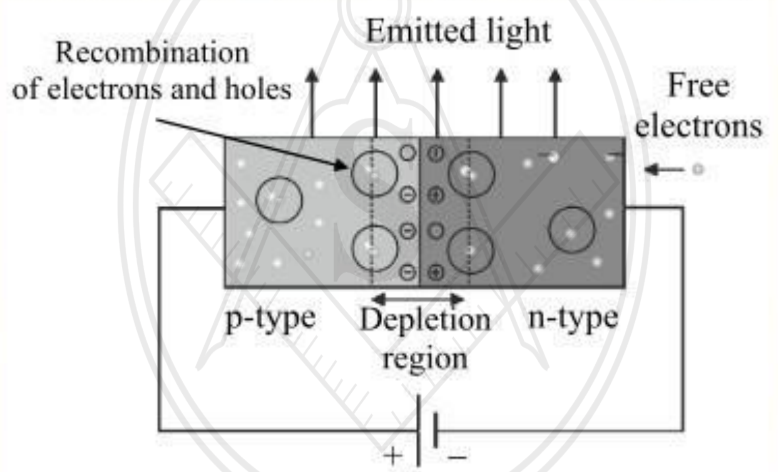
Ans: Working of LED:

i. When the LED is forward biased, electrons from the semiconductor's conduction band recombine with holes from the valence band releasing sufficient energy to produce photons which emit a monochromatic light.

ii. Because of the thin layer, a reasonable number of these photons can leave the junction and emit coloured light.

iii. The amount of light output is directly proportional to the forward current. Thus, higher the forward current, higher is the light output.

iv. The given figure schematically shows the emission of light when electron-hole pair combines.



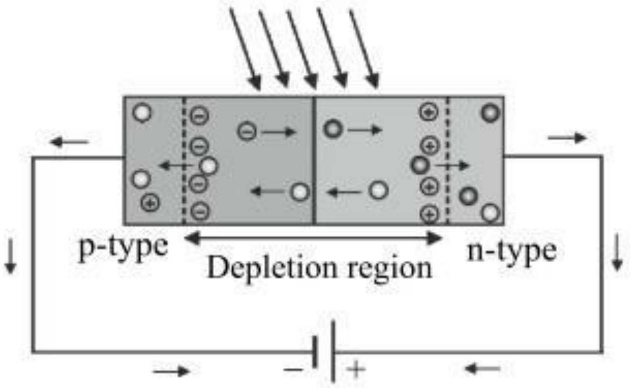
Working of LED

Q.4. Explain the principle of operation of a photodiode.

Ans: Working principle of photodiode:

i. When a p-n junction diode is reverse biased, a reverse saturation current flows through the junction.

ii. The magnitude of this current is constant for a certain range of reverse bias voltages. This current is due to the minority carriers on either side of the junction. The figure shows a schematic representation of working of a photodiode.



Working of a photodiode

iii. The reverse current depends only on the concentration of the minority carriers and not on the applied voltage. This reverse current is called dark current because it flows even when the photodiode is not illuminated.

iv. When the p-n junction is illuminated with photons of energy greater than the band gap energy of semiconductor electron-hole pairs are generated in the depletion region.

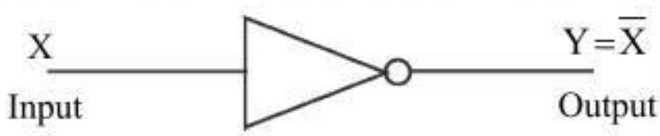
v. The electrons and the holes are separated due to the intrinsic electric field present in the depletion region. The electrons are attracted towards the anode and the holes are attracted towards the cathode. More carriers are available for conduction and the reverse current is increased.

vi. The reverse current of a photodiode depends on the intensity of the incident light. Thus, the reverse current can be controlled by controlling the concentration of the minority carriers in the junction.

Q.5. What is a logic gate? Draw the symbol and give the truth table for NOT gate. Why NOT gate is called inverter?

Ans: Logic gate: *A digital circuit with one or more input signals but only one output signal is called a* ***logic gate.*** It is a switching circuit that follows curtain logical relationship between the input and output voltages.

Schematic symbol:



NOT gate symbol

Truth table for NOT gate:

Input	Output
X	**Y**
0	1
1	0

Reason:

i. NOT gate has one input and one output.

ii. It produces a 'high' output or output '1' if the input is '0'. When the input is 'high' or '1', its output is 'low' or '0'.

iii. That is, it produces a negated version of the input at its output.

Hence, NOT gate is called as an inverter.

Q.6. Explain the working of PNP transistor.

Ans: Working of p-n-p transistor:

i. Majority charge carriers in the emitter of p-n-p transistor are holes.

ii. A typical biasing of a transistor is shown in figure (a). In this, emitter-base junction is forward biased while collector-base junction is reverse biased.

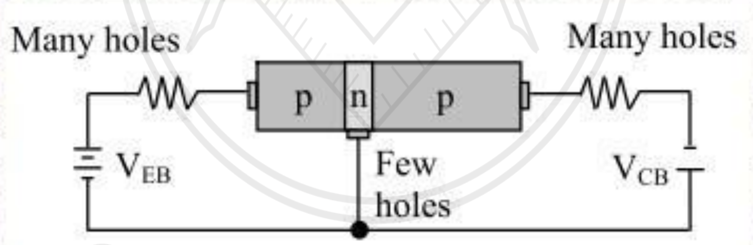


Figure (a)

iii. At the instant when the EB junction is forward biased, holes in the emitter region have not entered the base region as shown in figure (b).

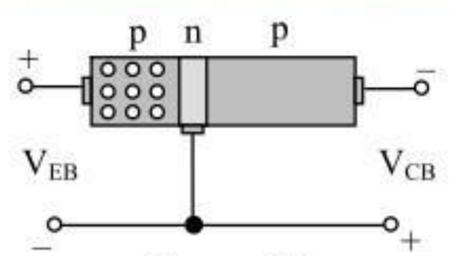


Figure (b)

iv. When the biasing voltage V_{BE} is greater than the barrier potential (0.6 – 0.7 V for Si transistors), many holes enter the base region and form the emitter current I_E as shown in figure (c).

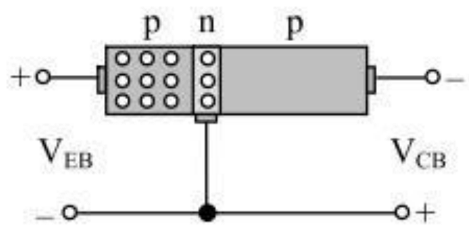


Figure (c)

v. These holes can either flow through the base circuit and constitute the base current (I_B), or they can also flow through the collector circuit and contribute towards the collector current (I_C).

vi. The base being thin and lightly doped, base current is only 5% of I_E.

vii. Holes injected from the emitter into the base diffuse into the collector-base depletion region due to the thin base region. When the holes enter the collector-base depletion region, they are pushed into the collector region by the electric field at the collector-base depletion region. The collector current (I_C) flows through the external circuit as shown in figure (d). The collector current I_C is about 95% of I_E.

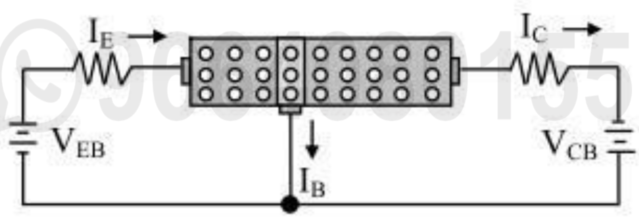


Figure (d)

From the figure we can conclude that, $I_E = I_B + I_C$

Since the base current I_B is very small we can write $I_C \approx I_E$.

Q.7. Draw the circuit symbol for NPN and PNP transistor. What is the difference in emitter, base and collector regions of a transistor?

Ans: The circuit symbols of the two types of transistors:

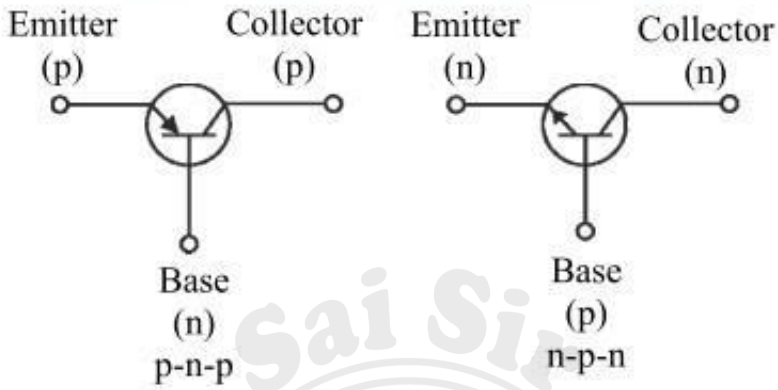


Figure (a) **Figure (b)**

Difference in the Emitter (E), the Base (b) and the Collector (C) is as follows:

i. **Emitter:** It is a thick heavily doped layer. This supplies a large number of majority carriers for the current flow through the transistor

ii. **Base:** It is the thin, lightly doped central layer.

iii. **Collector:** It is a thick and moderately doped layer. Its area is larger than that of the emitter and the base. This layer collects a major portion of the majority carriers supplied by the emitter. The collector also helps dissipation of any small amount of heat generated.

Long Answer (LA) (4 Marks Each)

Q.1. With the help of neat diagram, explain the working of npn transistor.

Ans: Working of n-p-n transistor:

i. Majority charge carriers in the emitter of n-p-n transistor are electrons.

ii. A typical biasing of a transistor is shown in figure (a). In this, emitter-base junction is forward biased while collector-base junction is reverse biased.

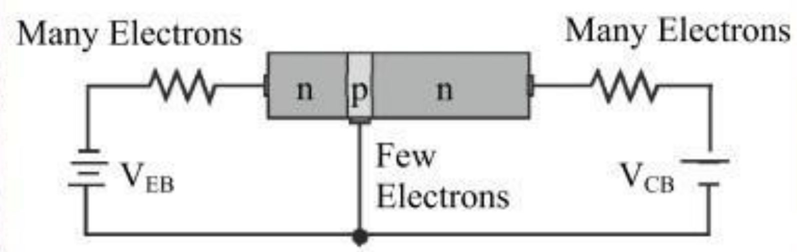


Figure (a)

iii. At the instant when the EB junction is forward biased, electrons in the emitter region have not entered the base region as shown in figure (b).

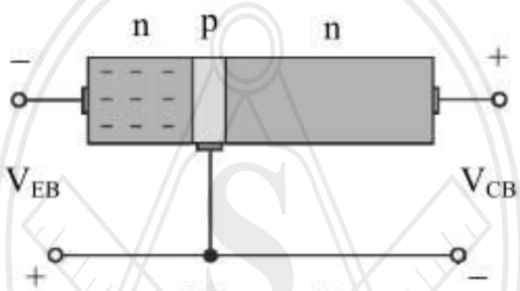


Figure (b)

iv. When the biasing voltage V_{BE} is greater than the barrier potential (0.6 – 0.7 V for Si transistors), many electrons enter the base region and form the emitter current I_E as shown in figure (c).

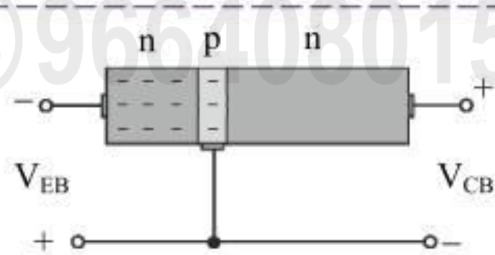


Figure (c)

v. These electrons can either flow through the base circuit and constitute the base current (I_B), or they can also flow through the collector circuit and contribute towards the collector current (I_C).

vi. The base being thin and lightly doped, base current is only 5% of I_E.

vii. Electrons injected from the emitter into the base diffuse into the collector-base depletion region due to the thin base region. When the electrons enter the collector-base depletion region, they are pushed into the collector region by the electric field at the collector-base depletion region. The collector current (I_C) flows through the external circuit as shown in figure (d). The collector current I_C is about 95% of I_E.

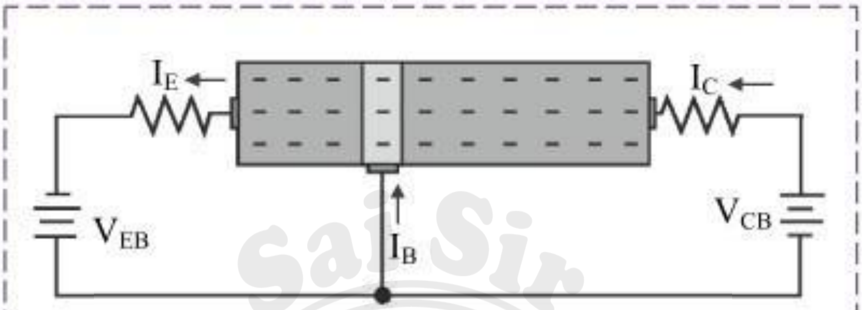


Figure (d)

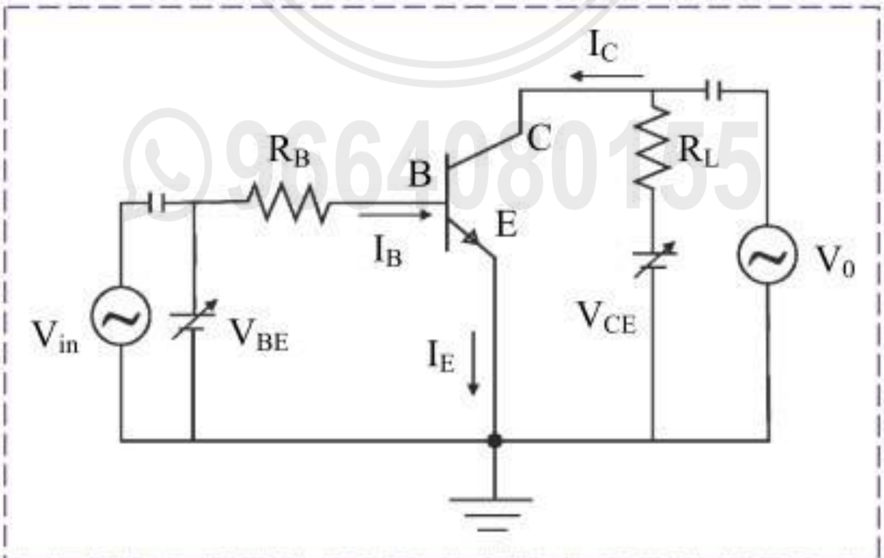
From the figure we can conclude that, $I_E = I_B + I_C$

Since the base current I_B is very small we can write $I_C \approx I_E$.

Q.2. With the help of neat circuit diagram, explain transistor as an amplifier.

Ans: Working of an amplifier:

i. The circuit of an amplifier using a n-p-n transistor in CE configuration is shown in the figure.



ii. When the input voltage V_{in} is not applied, applying the Kirchhoff's law to the output loop, we can write,

$V_{CC} = V_{CE} + I_C R_L$

iii. Similarly, for input loop,

$V_{BB} = V_{BE} + I_B R_B$

iv. When input AC signal is applied, V_{in} is not zero. Thus, the voltage drop across the input loop will now be,

$V_{BB} + V_{in} = V_{BE} + I_B R_B + \Delta I_B (R_B + r_i)$ ....(1)

v. The AC signal applied adds the current of ΔI_B to the original current flowing through the circuit. Therefore, the additional voltage drop in the input loop will be across resistor R_B (= $\Delta I_B R_B$) and across the input dynamic resistance of the transistor (= $\Delta I_B r_i$).

vi. From equation (1),

$V_{in} = \Delta I_B (R_B + r_i)$

As, R_B is very small, we can consider,

$V_{in} = \Delta I_B r_i$

vii. The changes in the base current I_B cause changes in the collector current I_C. This changes the voltage drop across the load resistance because V_{CC} is constant. We can write,

$\Delta V_{CC} = \Delta V_{CE} + R_L I_C = 0$

$\therefore$ $\Delta V_{CE} = - R_L I_C$

viii. The change in the output voltage ΔV_{CE} is the output voltage V_o hence we can write,

$V_o = \Delta V_{CE} = \beta_{AC} R_L \Delta I_B$

Q.3. Define dark current of photodiode. What are the advantages and disadvantages of photodiode?

Ans: When a photodiode is reverse biased, a reverse saturation current flows through the junction which depends only on the concentration of the minority carriers and not on the applied voltage. This reverse current that flows even when the photodiode is not illuminated is called **dark current**.

Advantages of photodiode:

i. Quick response when exposed to light.

ii. The reverse current is linearly proportional to intensity of incident light. (Linear response)

iii. High speed of operations.

iv. Light weight and compact size.

v. Wide spectral response. E.g., photodiodes made from silicon respond to radiation of wavelengths from 190 nm (UV) to 1100 nm (IR).

vi. Relatively low cost.

Disadvantages of photodiode:

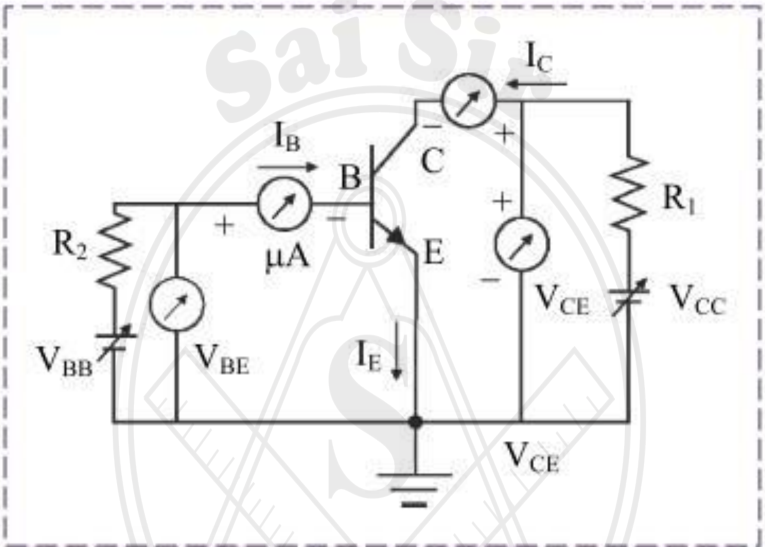
i. Its properties are temperature dependent, similar to many other semiconductor devices.

ii. Low reverse current for low illumination levels.

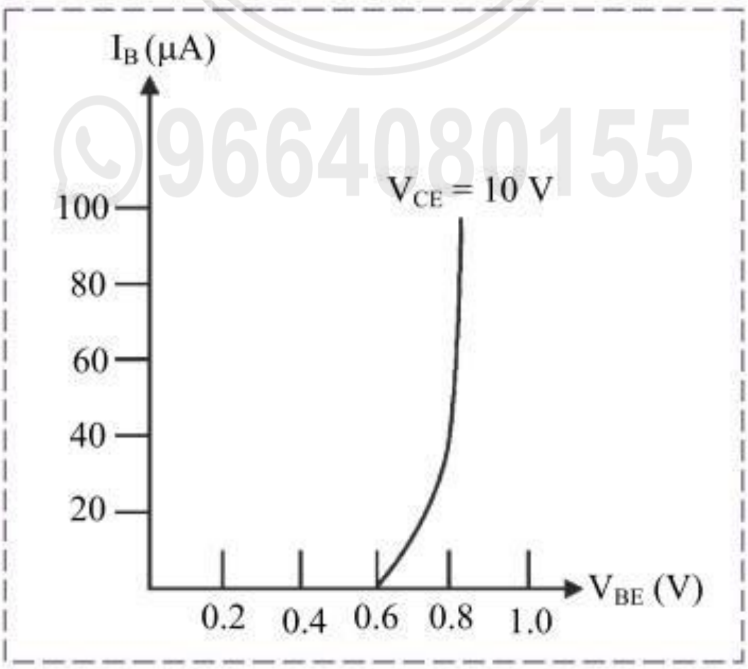
Q.4. Draw the circuit diagram to study the characteristic of transistor in common emitter mode. Draw the input and output characteristic.

Ans:

i. Circuit to study Common Emitter (CE) characteristic:



ii. The Input characteristics:



iii. The output characteristics: