

3

Arithmetic Progression

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**Points to Remember:**

- Sequence:**

Set of natural numbers are 1, 2, 3, 4, ...

Here number 4 is in the 4th position and number 13 is in the 13th position. So we can predict the position of any number in this pattern.

1, 4, 9, 16, 25, 36, 49, ...

In this pattern, $25 = 5^2$ and $49 = 7^2$.

Hence in this pattern, 25 and 49 are in 5th and 7th position respectively.

If set of numbers are arranged in a definite order and according to some precise rule, then it is called a **sequence**.

Each number in the sequence is called a **term of a sequence**.

The first term of the sequence is denoted by t_1 or a , the second term is denoted by t_2 , the third term is denoted by t_3 and so on.

In general, the n^{th} term, is denoted by t_n .

A sequence is usually denoted by $\{t_n\}$

Activity I: Some sequences are given below. Show the position of the terms by $t_1, t_2, t_3, \dots$

[Textbook Page No. 56]

(1) 9, 15, 21, 27, ...

Here, $t_1 = 9$, $t_2 = 15$, $t_3 = 21$

(2) 7, 7, 7, 7, ...

Here, $t_1 = 7$, $t_2 = 7$, $t_3 = 7$...

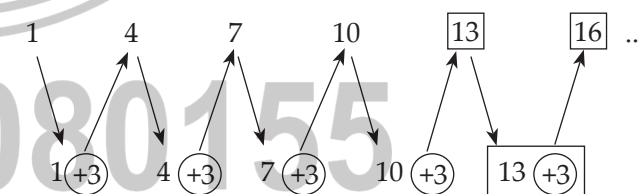
(3) -2, -6, -10, -16, ...

Here, $t_1 = -2$, $t_2 = -6$, $t_3 = -10$, ...

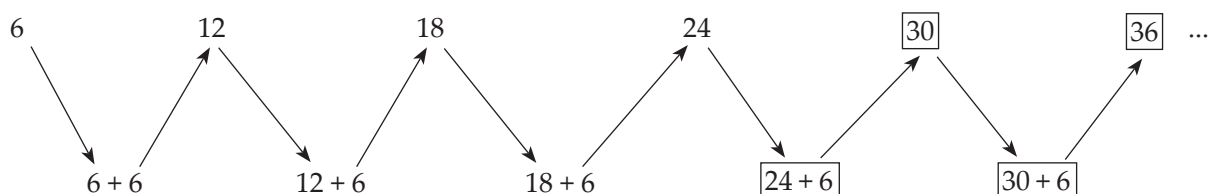
Activity II: some sequences are given below. check whether there is any rule among the terms. Find the similarity between two sequences.

(Textbook Page No. 56)

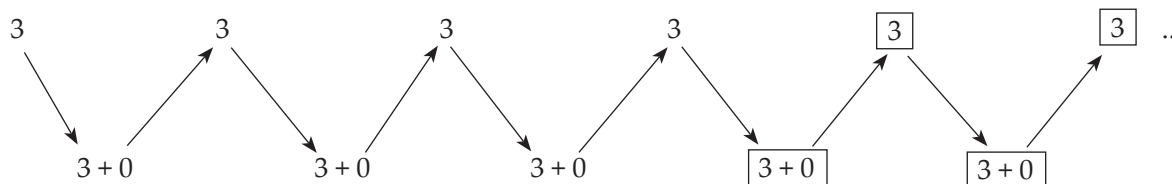
(1) 1, 4, 7, 10, 13, ...



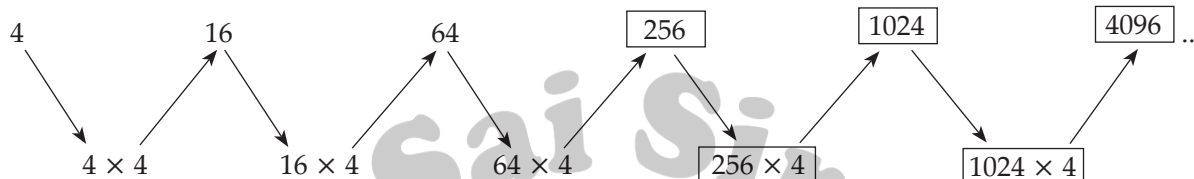
(2) 6, 12, 18, 24, ...



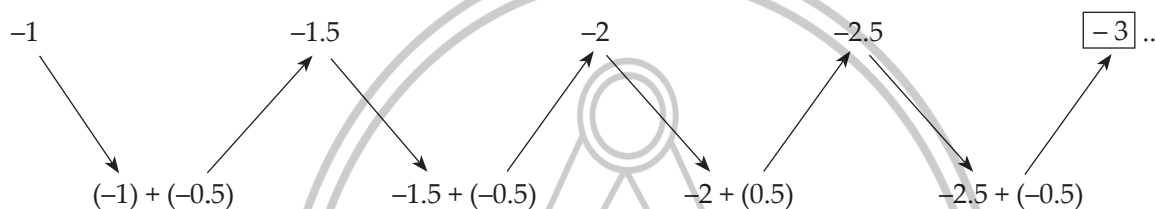
(3) 3, 3, 3, 3, ...



(4) 4, 16, 64, ...



(5) -1, -1.5, -2, -2.5, ...



(6) $1^3, 2^3, 3^3, 4^3, \dots$

In sequence (1), (2), (3), (5), we add a fixed number to previous term to get the next term. Such type of sequence is called **Arithmetic Progression**. Sequence (4) is not an arithmetic progression. Here, we multiply previous term by a fixed number to get the next term. Such progression is called **Geometric Progression**. Sequence (6) is neither arithmetic progression nor geometric progression.

• Arithmetic Progression (A. P.):

A sequence in which the difference between any two consecutive terms ($t_{n+1} - t_n$) is a fixed number (constant) is called '**Arithmetic Progression**' (A.P.).

The fixed number is called the **common difference** of the A.P. and is denoted by ' d '.

The value of ' d ' may be positive, negative or zero. In an A.P., if first term is a and common difference is d then,

$$t_1 = a, t_2 = a + d, t_3 = (a + d) + d = a + 2d.$$

Thus, an A.P. can be expressed as $a, a + d, a + 2d, a + 3d, \dots$ where, a is the first term and d is common difference.

Activity:

(Textbook Page No. 59)

Write one example of finite and infinite A. P. each.

Ans. Finite A. P. - The sequence of three digits natural numbers divisible by 5.

$\therefore 100, 105, 110, \dots, 995.$

Infinite A.P.- The sequence of all even numbers.

$\therefore 2, 4, 6, 8, \dots$

MASTER KEY QUESTION SET - 3

Practice Set - 3.1 (Textbook Page No. 61)

***(1)** Which of the following sequences are A.P.? If they are A.P. find the common difference.

(i) 2, 4, 6, 8, ... (1 mark)

Solution:

Here $t_1 = 2, t_2 = 4, t_3 = 6, t_4 = 8$

$$t_2 - t_1 = 4 - 2 = 2$$

$$t_3 - t_2 = 6 - 4 = 2$$

$$t_4 - t_3 = 8 - 6 = 2$$

This shows that the difference between any two consecutive terms is constant.

$\therefore$ **Hence, the given sequence is an A.P. and common difference (d) is 2.**

(ii) $2, \frac{5}{2}, 3, \frac{7}{3}, \dots$ (1 mark)

Solution:

Here, $t_1 = 2, t_2 = \frac{5}{2}, t_3 = 3, t_4 = \frac{7}{3}$

$$t_2 - t_1 = \frac{5}{2} - 2 = \frac{5-4}{2} = \frac{1}{2}$$

$$t_3 - t_2 = 3 - \frac{5}{2} = \frac{6-5}{2} = \frac{1}{2}$$

$$t_4 - t_3 = \frac{7}{3} - 3 = \frac{7-9}{3} = -\frac{2}{3}$$

This shows that the difference between any two consecutive terms is not constant.

∴ **Hence, the given sequence is not an A.P.**

(iii) $-10, -6, -2, 2, \dots$ (1 mark)

Solution:

Here, $t_1 = -10, t_2 = -6, t_3 = -2, t_4 = 2$

$$t_2 - t_1 = -6 - (-10) = -6 + 10 = 4$$

$$t_3 - t_2 = -2 - (-6) = -2 + 6 = 4$$

$$t_4 - t_3 = 2 - (-2) = 2 + 2 = 4$$

This shows that the difference between any two consecutive terms is constant.

∴ **Hence, the given sequence is an A.P. and common difference (d) is 4.**

(iv) $0.3, 0.33, 0.333, \dots$ (1 mark)

Solution:

Here, $t_1 = 0.3, t_2 = 0.33, t_3 = 0.333$

$$t_2 - t_1 = 0.33 - 0.3 = 0.03$$

$$t_3 - t_2 = 0.333 - 0.33 = 0.003$$

$$\therefore t_3 - t_2 \neq t_2 - t_1$$

This shows that the difference between any two consecutive terms is not constant.

∴ **Hence, the given sequence is not an A.P.**

(v) $0, -4, -8, -12$ (1 mark)

Solution:

Here, $t_1 = 0, t_2 = -4, t_3 = -8, t_4 = -12$

$$t_2 - t_1 = -4 - 0 = -4$$

$$t_3 - t_2 = -8 - (-4) = -8 + 4 = -4$$

$$t_4 - t_3 = -12 - (-8) = -12 + 8 = -4$$

This shows that difference between any two consecutive terms is constant.

∴ **Hence, the given sequence is an A.P. and common difference (d) is -4 .**

(vi) $-\frac{1}{5}, -\frac{1}{5}, -\frac{1}{5}, \dots$ (1 mark)

Solution:

Here $t_1 = -\frac{1}{5}, t_2 = -\frac{1}{5}, t_3 = -\frac{1}{5}$

$$t_2 - t_1 = -\frac{1}{5} - \left(-\frac{1}{5}\right) = -\frac{1}{5} + \frac{1}{5} = 0$$

$$t_3 - t_2 = -\frac{1}{5} - \left(-\frac{1}{5}\right) = -\frac{1}{5} + \frac{1}{5} = 0$$

This shows that the difference between any two consecutive terms is constant.

∴ **Hence, the given sequence is an A.P. and common difference (d) is zero.**

(vii) $3, 3 + \sqrt{2}, 3 + 2\sqrt{2}, 3 + 3\sqrt{2}, \dots$ (2 marks)

Solution:

Here $t_1 = 3, t_2 = 3 + \sqrt{2}, t_3 = 3 + 2\sqrt{2}$,

$$t_4 = 3 + 3\sqrt{2}$$

$$t_2 - t_1 = 3 + \sqrt{2} - 3 = \sqrt{2}$$

$$\begin{aligned} t_3 - t_2 &= 3 + 2\sqrt{2} - (3 + \sqrt{2}) \\ &= 3 + 2\sqrt{2} - 3 - \sqrt{2} = \sqrt{2} \end{aligned}$$

$$\begin{aligned} t_4 - t_3 &= 3 + 3\sqrt{2} - (3 + 2\sqrt{2}) \\ &= 3 + 3\sqrt{2} - 3 - 2\sqrt{2} = \sqrt{2} \end{aligned}$$

This shows that the difference between any two consecutive terms is constant.

∴ **Hence, the given sequence is an A.P. and common difference (d) is $\sqrt{2}$.**

(viii) $127, 132, 137, \dots$ (1 mark)

Solution:

Here $t_1 = 127, t_2 = 132, t_3 = 137$

$$t_2 - t_1 = 132 - 127 = 5$$

$$t_3 - t_2 = 137 - 132 = 5$$

This shows that the difference between any two consecutive terms is constant.

∴ **Hence, the given sequence is an A.P. with common difference (d) is 5.**

(2) Write an A.P. whose first term is a and common difference d in each of the following.

(i) $a = 10, d = 5$ (1 mark)

Solution:

$$a = t_1 = 10, d = 5$$

$$t_2 = t_1 + d = 10 + 5 = 15$$

$$t_3 = t_2 + d = 15 + 5 = 20$$

$$t_4 = t_3 + d = 20 + 5 = 25$$

∴ **Arithmetic progression is $10, 15, 20, 25, \dots$**

(ii) $a = -3, d = 0$ (1 mark)

Solution:

$$a = t_1 = -3, d = 0$$

$$t_2 = t_1 + d = -3 + 0 = -3$$

$$t_3 = t_2 + d = -3 + 0 = -3$$

$$t_4 = t_3 + d = -3 + 0 = -3$$

∴ **Arithmetic progression is $-3, -3, -3, -3, \dots$**

(iii) $a = -7, d = \frac{1}{2}$ (1 mark)

Solution:

$$a = t_1 = -7, d = \frac{1}{2}$$

$$t_2 = t_1 + d = -7 + \frac{1}{2} = -7 + 0.5 = -6.5$$

$$t_3 = t_2 + d = -6.5 + \frac{1}{2} = -6.5 + 0.5 = -6$$

$$t_4 = t_3 + d = -6 + \frac{1}{2} = -6 + 0.5 = -5.5$$

$\therefore$ Arithmetic progression is $-7, -6.5, -6, -5.5, \dots$

(iv) $a = -1.25, d = 3$ (1 mark)

Solution:

$$a = t_1 = -1.25, d = 3$$

$$t_2 = t_1 + d = -1.25 + 3 = 1.75$$

$$t_3 = t_2 + d = 1.75 + 3 = 4.75$$

$$t_4 = t_3 + d = 4.75 + 3 = 7.75$$

$\therefore$ Arithmetic progression is $-1.25, 1.75, 4.75, 7.75, \dots$

(v) $a = 6, d = -3$ (1 mark)

Solution:

$$a = t_1 = 6, d = -3$$

$$t_2 = t_1 + d = 6 + (-3) = 3$$

$$t_3 = t_2 + d = 3 + (-3) = 0$$

$$t_4 = t_3 + d = 0 + (-3) = -3$$

$\therefore$ Arithmetic progression is $6, 3, 0, -3, \dots$

(vi) $a = -19, d = -4$ (1 mark)

Solution:

$$a = t_1 = -19, d = -4$$

$$t_2 = t_1 + d = -19 + (-4) = -23$$

$$t_3 = t_2 + d = -23 + (-4) = -27$$

$$t_4 = t_3 + d = -27 + (-4) = -31$$

$\therefore$ Arithmetic progression is $-19, -23, -27, -31, \dots$

(3) Find the first term and common difference for each of the A. P.

(i) $5, 1, -3, -7$ (1 mark)

Solution:

$$\text{Here } t_1 = a = 5,$$

$$t_2 = 1, t_3 = -3, t_4 = -7$$

$$\text{For an A.P., } d = t_{n+1} - t_n$$

$$d = t_2 - t_1 = 1 - 5 = -4$$

$$d = t_3 - t_2 = -3 - 1 = -4$$

$\therefore$ First term (a) is 5 and Common difference (d) is -4 .

(ii) $0.6, 0.9, 1.2, 1.5, \dots$ (1 mark)

Solution:

$$\text{Here, } t_1 = a = 0.6,$$

$$t_2 = 0.9, t_3 = 1.2, t_4 = 1.5$$

$$\text{For an A.P. } d = t_{n+1} - t_n$$

$$\therefore d = t_2 - t_1 = 0.9 - 0.6 = 0.3$$

$$d = t_3 - t_2 = 1.2 - 0.9 = 0.3$$

$\therefore$ First term (a) is 0.6 and Common difference (d) is 0.3.

(iii) $127, 135, 143, 151, \dots$ (1 mark)

Solution:

$$\text{Here } t_1 = a = 127,$$

$$t_2 = 135, t_3 = 143, t_4 = 151$$

$$\text{For an A.P., } d = t_{n+1} - t_n$$

$$d = t_2 - t_1 = 135 - 127 = 8$$

$$d = t_3 - t_2 = 143 - 135 = 8$$

$\therefore$ First term (a) is 127 and Common difference (d) is 8.

(iv) $\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{7}{4}, \dots$ (2 mark)

Solution:

$$\text{Here } t_1 = a = \frac{1}{4},$$

$$t_2 = \frac{3}{4}, t_3 = \frac{5}{4}, t_4 = \frac{7}{4}$$

$$\text{For an A. P. } d = t_{n+1} - t_n$$

$$d = t_2 - t_1 = \frac{3}{4} - \frac{1}{4} = \frac{3-1}{4} = \frac{2}{4} = \frac{1}{2}$$

$$d = t_3 - t_2 = \frac{5}{4} - \frac{3}{4} = \frac{5-3}{4} = \frac{2}{4} = \frac{1}{2}$$

$\therefore$ First term (a) is $\frac{1}{4}$ and Common difference (d) is $\frac{1}{2}$.



Points to Remember:

• n^{th} term of an A.P.:

In the given A.P. $t_1, t_2, t_3, \dots$ if the first term is a and the common difference is d then,

$$t_1 = a$$

$$t_2 = t_1 + d = a + d = a + (2 - 1)d,$$

$$t_3 = t_2 + d = a + d + d = a + 2d = a + (3 - 1)d$$

$$t_4 = t_3 + d = a + 2d + d = a + 3d = a + (4 - 1)d$$

$$\therefore t_n = a + (n - 1)d$$

Thus the general term i.e. the n^{th} term of an A.P. with the first term as 'a' and the common difference 'd' is given as

$$t_n = a + (n - 1)d$$

Let's think

(Textbook Page No. 62)

- Is 5, 8, 11, 14..... an A.P.? If so then what will be the 100th term? Check whether 92 is in this A.P.? Is number 61 in this A.P.?

Solution:

5, 8, 11, 14.....

$$\text{Here } t_1 = a = 5, t_2 = 8, t_3 = 11$$

$$t_2 - t_1 = 8 - 5 = 3,$$

$$t_3 - t_2 = 11 - 8 = 3$$

Hence the given sequence forms an A.P. with $a = 5$ and $d = 3$.

For $n = 100$

$$t_n = a + (n - 1)d$$

$$\begin{aligned} \therefore t_{100} &= 5 + (100 - 1)3 \\ &= 5 + 99 \times 3 \\ &= 5 + 297 \end{aligned}$$

$$t_{100} = 302$$

$$\therefore \text{100}^{\text{th}} \text{ term} = 302$$

Let $t_p = 92$

$$\therefore t_p = a + (p - 1)d$$

$$\therefore 92 = 5 + (p - 1)3$$

$$\therefore 92 - 5 = (p - 1)3$$

$$\therefore 87 = (p - 1)3$$

$$\therefore (p - 1) = \frac{87}{3} = 29$$

$$\therefore p = 29 + 1 = 30$$

$$\therefore \text{92 is the 30th term of the A.P.}$$

Let $t_q = 61$

$$\therefore t_q = a + (q - 1)d$$

$$\therefore 61 = 5 + (q - 1)3$$

$$\therefore 61 - 5 = (q - 1)3$$

$$\therefore 56 = 3q - 3$$

$$\therefore 3q = 59$$

$$\therefore q = \frac{59}{3} = 19.66$$

Since q should be positive integer

$\therefore$ 61 is not in this A.P.

Practice Set - 3.2 (Textbook Page No. 66)

- (1) Write the correct number in the given boxes from the following A.P.

- (i) 1, 8, 15, 22, ... (2 marks)

Solution:

$$\text{Here, } a = 1, t_1 = 1, t_2 = 8, t_3 = 15, \dots$$

$$t_2 - t_1 = 8 - 1 = 7$$

$$t_3 - t_2 = 15 - 8 = 7$$

$$\therefore d = 7$$

- (ii) 3, 6, 9, 12, ... (2 marks)

Solution:

$$\text{Here, } t_1 = 3, t_2 = 6, t_3 = 9, t_4 = 12, \dots$$

$$t_2 - t_1 = 3, t_3 - t_2 = 3$$

$$\therefore d = 3$$

- (iii) -3, -8, -13, -18 (2 marks)

Solution:

$$\text{Here, } t_1 = -3, t_2 = -8, t_3 = -13, t_4 = -18, \dots$$

$$t_2 - t_1 = -5, t_3 - t_2 = -5$$

$$\therefore a = -3, d = -5$$

- (iv) 70, 60, 50, 40, ... (2 marks)

Solution:

$$\text{Here, } t_1 = 70, t_2 = 60, t_3 = 50$$

$$a = 70, d = -10$$

$$\text{working } d = t_2 - t_1$$

$$= 60 - 70$$

$$= -10$$

- (2) Decide whether following sequence is an A.P., if so find 20th term of the progression. (3 marks)
-12, -5, 2, 9, 16, 23, 30, ...

Solution:

$$\text{Here } t_1 = -12, t_2 = -5, t_3 = 2, t_4 = 9, \dots$$

$$t_2 - t_1 = -5 - (-12) = -5 + 12 = 7$$

$$t_3 - t_2 = 2 - (-5) = 2 + 5 = 7$$

$$t_4 - t_3 = 9 - 2 = 7$$

This shows that the difference between any two consecutive terms is constant.

$\therefore$ The given sequence is an A.P.

Here $a = -12$, $d = 7$, $t_{20} = ?$

$$t_n = a + (n-1)d$$

$$t_n = -12 + (n-1)d$$

$$\begin{aligned}\therefore t_{20} &= -12 + (20-1)7 & [\because n=20] \\ &= -12 + 19 \times 7 \\ &= -12 + 133\end{aligned}$$

$$\therefore t_{20} = 121$$

Thus 20th term of the A.P. is 121.

- (3) Given Arithmetic Progression 12, 16, 20, 24, ...
Find the 24th term of this progression. (2 marks)

Solution:

The A.P. 12, 16, 20, 24, ...

Here $a=12$, $d = 16 - 12 = 4$, $t_{24} = ?$

$$t_n = a + (n-1)d$$

$$\begin{aligned}\therefore t_{24} &= 12 + (24-1)4 & (\because n=24) \\ &= 12 + 23 \times 4 \\ &= 12 + 92\end{aligned}$$

$$\therefore t_{24} = 104$$

Thus 24th term of the A.P. is 104.

- (4) Find the 19th term of the following A.P.
7, 13, 19, 25, ... (2 marks)

Solution:

The A.P. is 7, 13, 19, 25, ...

Here $a = 7$, $d = 13 - 7 = 6$, $t_{19} = ?$

$$t_n = a + (n-1)d$$

$$\begin{aligned}t_{19} &= 7 + (19-1)6 & (\because n=19) \\ &= 7 + 18 \times 6 \\ &= 7 + 108\end{aligned}$$

$$t_{19} = 115$$

Thus 19th term of the A.P. is 115

- (5) Find the 27th term of the following A.P.
9, 4, -1, -6, -11, ... (2 marks)

Solution:

The A.P. is 9, 4, -1, -6, -11, ...

Here, $a = 9$, $d = 4 - 9 = -5$, $t_{27} = ?$

$$t_n = a + (n-1)d$$

$$\begin{aligned}\therefore t_{27} &= 9 + (27-1)(-5) & (\because n=27) \\ &= 9 + 26 \times (-5) \\ &= 9 - 130\end{aligned}$$

$$\therefore t_{27} = -121$$

Thus 27th term of the A.P. is -121

Problem Set - 3 (Textbook Page No. 79)

- (2) Find the fourth term from the end in an A.P. -11, -8, -5, ..., 49 (3 marks)

Solution:

Method - I

For the reverse A.P. 49, ..., -5, -8, -11

$$a = 49, d = -8 - (-5) = -8 + 5 = -3$$

$\therefore$ Fourth term from the end of given A.P. is t_4 in the reverse A.P.

We know,

$$t_n = a + (n-1)d$$

$$\therefore t_4 = 49 + (4-1)(-3)$$

$$\therefore t_4 = 49 + 3(-3)$$

$$\therefore t_4 = 49 - 9$$

$$\therefore t_4 = 40$$

Fourth term from the end of given A.P. is 40

Method - II

For an A.P., First term is a and Common difference is d . Let l be the last term of this A.P. Then n^{th} term from the end $= l - (n-1)d$

The given A.P. is -11, -8, -5, ..., 49.

Here $a = -11$, $d = -8 - (-11) = -8 + 11 = 3$ and last term (l) = 49

$$\begin{aligned}\therefore \text{4th term from the end} &= l - (n-1)d \\ &= 49 - (4-1)3 \\ &= 49 - 3 \times 3 \\ &= 49 - 9 \\ &= 40\end{aligned}$$

Fourth term from the end of given A.P. is 40

Practice Set - 3.2 (Textbook Page No. 66)

- (7) The 11th term and the 21st term of an A.P. are 16 and 29 respectively, then find the 41st term of that A.P. (3 marks)

Solution:

Given For an A.P. : $t_{11} = 16$ and $t_{21} = 29$

To find : $t_{41} = ?$

$$t_n = a + (n-1)d$$

$$\therefore t_{11} = a + (11-1)d$$

$$\therefore 16 = a + 10d$$

$$\therefore a + 10d = 16 \quad \dots(i)$$

$$\text{Also } t_{21} = a + (21-1)d$$

$$\begin{aligned}\therefore 29 &= a + 20d \\ \therefore a + 20d &= 29 \quad \dots(ii)\end{aligned}$$

Subtracting (i) from (ii),

$$\begin{array}{r} a + 20d = 29 \\ a + 10d = 16 \\ \hline (-) \quad (-) \quad (-) \\ 10d = 13 \end{array}$$

$$\therefore d = \frac{13}{10}$$

Substituting $d = \frac{13}{10}$ in equation (i)

$$a + 10d = 16$$

$$\therefore a + 10 \times \frac{13}{10} = 16$$

$$\therefore a + 13 = 16$$

$$\therefore a = 16 - 13$$

$$\therefore a = 3$$

Here, $a = 3$; $d = \frac{13}{10}$, $t_{41} = ?$

$$t_n = a + (n - 1)d$$

$$\therefore t_{41} = 3 + (41 - 1) \frac{13}{10} \quad [\because n = 41]$$

$$= 3 + 40 \times \frac{13}{10}$$

$$= 3 + 4 \times 13$$

$$= 3 + 52$$

$$\therefore t_{41} = 55$$

Thus 41st term of the A.P. is 55.**Problem Set - 3 (Textbook Page No. 79)**

- (3) In an A.P. 10th term is 46, sum of 5th and 7th term is 52. Find the A.P. (3 marks)

Solution:For an A.P., $t_{10} = 46$ and $t_5 + t_7 = 52$

$$t_n = a + (n - 1)d$$

$$\therefore t_{10} = a + (10 - 1)d$$

$$\therefore 46 = a + 9d$$

$$\therefore a + 9d = 46 \quad \dots(i)$$

$$\text{Now, } t_5 = a + (5 - 1)d$$

$$\therefore t_5 = a + 4d \text{ and } \dots(ii)$$

$$t_7 = a + (7 - 1)d$$

$$\therefore t_7 = a + 6d \quad \dots(iii)$$

$$\text{We have } t_5 + t_7 = 52 \quad \dots(\text{given})$$

$$\therefore a + 4d + a + 6d = 52 \quad \dots[\text{from (ii) and (iii)}]$$

$$\therefore 2a + 10d = 52$$

$$\therefore a + 5d = 26 \quad \dots (iv) \text{ (dividing both sides by 2)}$$

Subtracting equation (iv) from equation (i)

$$\begin{array}{r} a + 9d = 46 \\ a + 5d = 26 \\ \hline (-) \quad (-) \quad (-) \\ 4d = 20 \end{array}$$

$$\therefore d = \frac{20}{4}$$

$$\therefore d = 5$$

Substituting $d = 5$ in equation (i) we get,

$$a + 9d = 46$$

$$\therefore a + 9 \times 5 = 46$$

$$\therefore a + 45 = 46$$

$$\therefore a = 46 - 45$$

$$\therefore a = 1$$

$$\text{Now } t_1 = a = 1$$

$$t_2 = t_1 + d = 1 + 5 = 6$$

$$t_3 = t_2 + d = 6 + 5 = 11$$

$$t_4 = t_3 + d = 11 + 5 = 16$$

Hence, the given A.P. is 1, 6, 11, 16...

- (6) If sum of 3rd and 8th terms of an A.P. is 7 and sum of 7th and 14th terms is -3 then find 10th term.

(3 marks)

Given: For an A.P. $t_3 + t_8 = 7$, $t_7 + t_{14} = -3$,To find : t_{10} **Solution:**

$$t_n = a + (n - 1)d$$

$$\therefore t_3 = a + (3 - 1)d$$

$$\therefore t_3 = a + 2d \quad \dots(i)$$

$$\text{Also } t_8 = a + (8 - 1)d$$

$$\therefore t_8 = a + 7d \quad \dots(ii)$$

$$\text{Now, } t_3 + t_8 = 7 \quad \dots(\text{given})$$

$$\therefore a + 2d + a + 7d = 7 \quad \dots[\text{from (i) and (ii)}]$$

$$\therefore 2a + 9d = 7 \quad \dots(iii)$$

$$t_n = a + (n - 1)d$$

$$\therefore t_7 = a + (7 - 1)d$$

$$\therefore t_7 = a + 6d \quad \dots (iv)$$

$$\text{Also } t_{14} = a + (14 - 1)d$$

$$\therefore t_{14} = a + 13d \quad \dots(v)$$

$$\text{Now, } t_7 + t_{14} = -3 \quad \dots(\text{given})$$

$$\therefore a + 6d + a + 13d = -3 \quad \dots(\text{from (iv) and (v)})$$

$$\therefore 2a + 19d = -3 \quad \dots(vi)$$

Subtracting (vi) from (iii) we get,

$$\begin{array}{r} 2a + 9d = 7 \\ 2a + 19d = -3 \\ \hline (-) \quad (-) \quad (+) \\ -10d = 10 \end{array}$$

$$\therefore d = \frac{10}{-10}$$

$$\therefore d = -1$$

Substituting $d = -1$ in (iii), we get,

$$2a + 9d = 7$$

$$\therefore 2a + 9 \times (-1) = 7$$

$$\therefore 2a - 9 = 7$$

$$\therefore 2a = 7 + 9$$

$$\therefore 2a = 16$$

$$\therefore a = \frac{16}{2}$$

$$\therefore a = 8$$

$$t_n = a + (n-1)d$$

$$\therefore t_{10} = 8 + (10-1)(-1)$$

$$\therefore = 8 + (9)(-1)$$

$$\therefore = 8 - 9$$

$$\therefore t_{10} = -1$$

Practice set - 3.2 (Textbook Page No. 66)

- (10) In an A.P. 17th term is 7 more than 10th term. Find the common difference? (3 marks)

Given: For an A.P. $t_{17} = t_{10} + 7$

To find : $d = ?$

Solution:

$$t_n = a + (n-1)d$$

$$\therefore t_{17} = a + (17-1)d$$

$$\therefore t_{17} = a + 16d$$

...(I)

$$\text{Also, } t_{10} = a + (10-1)d$$

$$t_{10} = a + 9d$$

...(II)

$$\text{Now, } t_{17} = t_{10} + 7$$

...(Given)

$$\therefore a + 16d = a + 9d + 7 \quad \dots[\text{From (i) and (ii)}]$$

$$\therefore 7d = 7$$

$$\therefore d = \frac{7}{7}$$

$$\therefore d = 1$$

$\therefore$ The common difference is 1.

- (6) Find how many three digit natural numbers are divisible by 5. (3 marks)

Solution:

The three digit natural numbers divisible by 5 are 100, 105, 110, ..., 995

Here, $t_1 = 100$, $t_2 = 105$, $t_3 = 110$, ...

$$t_2 - t_1 = 105 - 100 = 5$$

$$t_3 - t_2 = 110 - 105 = 5$$

Since the difference between any two consecutive terms is constant.

$\therefore$ The given sequence is an A.P., where $a = 100$, $d = 5$

let $t_n = 995$, $n = ?$

$$t_n = a + (n-1)d$$

$$\therefore 995 = 100 + (n-1)5$$

$$\therefore 895 = 5n - 5$$

$$\therefore 895 + 5 = 5n$$

$$\therefore 5n = 900$$

$$\therefore n = \frac{900}{5}$$

$$\therefore n = 180$$

There are 180 three digit natural numbers which are divisible by 5.

- (9) In the natural numbers from 10 to 250, how many are divisible by 4? (3 marks)

Solution:

The natural numbers from 10 to 250 divisible by 4 are: 12, 16, 20, ..., 248

Here $t_1 = 12$, $t_2 = 16$, $t_3 = 20$, ...

$$t_2 - t_1 = 16 - 12 = 4, t_3 - t_2 = 20 - 16 = 4$$

This shows that the difference between any two consecutive terms is constant.

$\therefore$ The given sequence is an A.P., where $a = 12$, $d = 4$,

let $t_n = 248$, $n = ?$

$$t_n = a + (n-1)d$$

$$\therefore 248 = 12 + (n-1)4$$

$$\therefore 248 - 12 = 4n - 4$$

$$\therefore 236 = 4n - 4$$

$$\therefore 236 + 4 = 4n$$

$$\therefore 4n = 240$$

$$\therefore n = \frac{240}{4}$$

$$\therefore n = 60$$

Thus, there are 60 natural numbers from 10 to 250 which are divisible by 4.

- (8) 11, 8, 5, 2, ... In this A.P. which term is number -151? (3 marks)

Solution:

The A.P. is 11, 8, 5, 2, ...

Here, $a = 11$, $d = t_2 - t_1 = 8 - 11 = -3$, $t_n = -151$, $n = ?$

$$t_n = a + (n-1)d$$

$$\therefore -151 = 11 + (n-1)(-3)$$

$$\therefore -151 = 11 - 3n + 3$$

$$\begin{aligned}\therefore -151 &= 14 - 3n \\ \therefore 3n &= 14 + 151 \\ \therefore 3n &= 165 \\ \therefore n &= \frac{165}{3} \\ \therefore n &= 55\end{aligned}$$

Hence, 55th term of the A.P. is -151.

Problem Set - 3 (Textbook Page No. 79)

- (5) Two A.P.'s are given 9, 7, 5... and 24, 21, 18, ... If n^{th} term of both the progressions are equal then find the value of n and n^{th} term (3 marks)

Solution:

The first A.P. is 9, 7, 5, ...

Here, $a = 9$, $d = 7 - 9 = -2$

$$t_n = a + (n - 1)d$$

$$\therefore t_n = a + (n - 1)d$$

$$\therefore t_n = 9 + (n - 1)(-2)$$

$$\therefore t_n = 9 - 2n + 2$$

$$\therefore t_n = 11 - 2n \quad \dots(i)$$

The second A.P. is 24, 21, 18, ...

Here, $a = 24$, $d = 21 - 24 = -3$

$$t_n = a + (n - 1)d$$

$$\therefore t_n = 24 + (n - 1)(-3)$$

$$\therefore t_n = 24 - 3n + 3$$

$$\therefore t_n = 27 - 3n \quad \dots(ii)$$

Now, n^{th} term of first A.P. = n^{th} term of second A.P.

$$\therefore 11 - 2n = 27 - 3n \quad \dots[\text{from (i) and (ii)}]$$

$$\therefore -2n + 3n = 27 - 11$$

$$\therefore n = 16$$

$$\text{Now, } t_n = 11 - 2n \quad \dots[\text{from (i)}]$$

$$\therefore t_n = 11 - 2 \times 16$$

$$\therefore t_n = 11 - 32$$

$$\therefore t_n = -21$$

Practice Set - 3.3 (Textbook Page No. 73)

- *(9) If the 9th term of an A.P. is zero then show that the 29th term is twice the 19th term. (4 marks)

Solution:

Given: For an A.P. $t_9 = 0$

To prove: $t_{29} = 2t_{19}$ i.e. $t_{29} - 2t_{19} = 0$

Proof:

$$t_n = a + (n - 1)d$$

$$\therefore t_9 = a + (9 - 1)d$$

$$0 = a + 8d$$

$$a + 8d = 0 \quad \dots(i)$$

$$t_{29} = a + (29 - 1)d$$

$$= a + 28d$$

$$t_{19} = a + (19 - 1)d$$

$$= a + 18d$$

$$\therefore t_{29} - 2t_{19} = a + 28d - 2(a + 18d)$$

$$= a + 28d - 2a - 36d$$

$$= -a - 8d$$

$$= -(a + 8d)$$

$$= 0$$

...(from (i))

$$\therefore t_{29} - 2t_{19} = 0$$

$$\therefore t_{29} = 2t_{19}$$

Problem Set - 3 (Textbook Page No. 80)

- *(13) If m times the m^{th} term of an A.P. is equal to n times its n^{th} term then show that $(m + n)^{\text{th}}$ term of the A.P. is zero. (4 marks)

Solution:

$$\text{Given: } mt_m = nt_n$$

$$\text{Prove: } t_{m+n} = 0$$

Let a be the first term and d be the common difference of the A.P.

Then m^{th} term of the A.P.

$$t_m = a + (m - 1)d \quad \dots(i)$$

and n^{th} term of the A.P.

$$t_n = a + (n - 1)d \quad \dots(ii)$$

$$m(t_m) = n(t_n) \quad \dots(\text{given})$$

$$\therefore m[a + (m - 1)d] = n[a + (n - 1)d] \dots[\text{from (i), (ii)}]$$

$$\therefore m[a + md - d] = n[a + nd - d]$$

$$\therefore ma + m^2d - md = na + n^2d - nd$$

$$\therefore ma - na + m^2d - n^2d - md + nd = 0$$

$$\therefore (m - n)a + (m^2 - n^2)d - d(m - n) = 0$$

$$\therefore (m - n)a + (m + n)(m - n)d - d(m - n) = 0$$

$$\therefore m \neq n$$

$$\therefore m - n \neq 0$$

Dividing throughout by $(m - n)$ we get,

$$a + (m + n)d - d = 0$$

$$a + (m + n - 1)d = 0 \quad \dots(iii)$$

$$\text{Now } t_{m+n} = a + (m + n - 1)d$$

$$t_{m+n} = 0 \quad \dots[\text{from (iii)}]$$

$$\therefore (m + n)^{\text{th}} \text{ term of the A.P. is zero.}$$



Points to Remember:

Sum of first n terms of an A.P.:

Let us consider an A.P. $a, a + d, a + 2d, a + 3d, \dots$
 $a + (n - 1)d$

Here a is the first term, d is the common difference and S_n be the sum of n terms of this A.P.

$$S_n = a + (a + d) + \dots + [a + (n - 2)d] + [a + (n - 1)d]$$

Re-writing it in reverse order, we get

$$S_n = [a + (n - 1)d] + [a + (n - 2)d] + \dots + (a + d) + a$$

Adding termwise in pairs, we get,

$$2S_n = [a + a + (n - 1)d] + [a + d + a + (n - 2)d] + \dots + [a + (n - 2)d + a] + [a + (n - 1)d + a]$$

$$\therefore 2S_n = [2a + (n - 1)d] + [2a + (n - 1)d] + \dots + [2a + (n - 1)d] \text{ } n \text{ times}$$

$$\therefore 2S_n = n [2a + (n - 1)d]$$

$$\therefore S_n = \frac{n}{2} [2a + (n - 1)d] \text{ or}$$

$$S_n = \frac{n}{2} [a + a + (n - 1)d]$$

$$S_n = \frac{n}{2} (a + t_n) \quad (\because t_n = a + (n - 1)d)$$

$$\therefore S_n = \frac{n}{2} [t_1 + t_n]$$

Consecutive terms of an A.P.

3 Consecutive terms : $a - d, a, a + d$

4 Consecutive terms : $a - d, a, a + d, a + 2d$

Note : While making assumption for any number of terms in A.P. start with first term as ' $a - d$ ' and then increase each term by ' d '

Example : Find the sum of all odd natural numbers from 1 to 150. (Textbook page no. 71)

Solution:

The odd natural numbers from 1 to 150 are 1, 3, 5, ..., 149. which is an A.P.

Here $a = 1, d = 2$

First let's find how many odd numbers are there from 1 to 150, so find the value of n . if $t_n = 149$

$$t_n = a + (n - 1)d$$

$$\therefore 149 = 1 + (n - 1)2$$

$$\therefore 149 = 1 + 2n - 2$$

$$\therefore n = 75$$

Now let's find the sum of these 75 numbers

$$1 + 3 + 5 + \dots + 149$$

$$a = 1 \text{ and } d = 2, n = 75$$

Method - I :

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

$$S_n = \frac{75}{2} [2(1) + (75 - 1)2]$$

$$S_n = 75 \times 75$$

$$S_n = 5625$$

Method : II

$$S_n = \frac{n}{2} [t_1 + t_n]$$

$$= \frac{75}{2} [1 + 149]$$

$$= S_n = \frac{75}{2} \times 150$$

$$\therefore S_n = 5625$$

Practice Set - 3.3 (Textbook Page No. 73)

- (1) First term and common difference of an A.P. are 6 and 3 respectively: Find S_{27} (2 marks)

Solution:

$$a = 6, d = 3, S_{27} = ?$$

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

$$\therefore S_{27} = \frac{27}{2} [12 + (27 - 1)3]$$

$$= \frac{27}{2} \times 90$$

$$= 27 \times 45$$

$$\therefore S_{27} = 1215$$

- (2) Find the sum of first 123 even natural numbers. (3 marks)

Solution:

The first n even natural numbers are: 2, 4, 6, 8, ...

They form an A.P. with $a = 2$,

$$d = t_2 - t_1 = 4 - 2 = 2$$

$$\text{We know, } S_n = \frac{n}{2} [2a + (n - 1)d]$$

$$\therefore S_{123} = \frac{123}{2} [2(2) + (123 - 1)2]$$

$$= \frac{123}{2} [4 + 122(2)]$$

$$= \frac{123}{2} [4 + 244]$$

$$= \frac{123}{2} [248]$$

$$= 123 [124]$$

$$= 15252$$

∴ The sum of first 123 even natural numbers in 15252.

Problem Set - 3 (Textbook Page No. 79)

- (4) The A.P. in which 4th term is -15 and 9th term is -30. Find the sum of first 10 numbers. (4 marks)

Solution:

Here, $t_4 = -15$, $t_9 = -30$, $S_{10} = ?$

$$t_n = a + (n-1)d$$

$$\therefore t_4 = a + (4-1)d$$

$$\therefore -15 = a + 3d$$

$$\therefore a + 3d = -15 \quad \dots(i)$$

Also, $t_9 = a + (9-1)d$

$$\therefore -30 = a + 8d$$

$$\therefore a + 8d = -30 \quad \dots(ii)$$

Subtracting (i) from (ii) we get,

$$\begin{array}{r} a + 8d = -30 \\ a + 3d = -15 \\ \hline (-) \quad (-) \quad (+) \end{array}$$

$$5d = -15$$

$$d = \frac{-15}{5}$$

$$\therefore d = -3$$

Substituting $d = -3$ in equation (I) we get,

$$a + 3d = -15$$

$$\therefore a + 3(-3) = -15$$

$$\therefore a - 9 = -15$$

$$\therefore a = -15 + 9$$

$$\therefore a = -6$$

$$\text{Now, } S_n = \frac{n}{2} [2a + (n-1)d]$$

$$S_{10} = \frac{10}{2} [2 \times (-6) + (10-1) \times (-3)]$$

$$= 5 [-12 + 9 \times -3]$$

$$= 5 [-12 - 27]$$

$$= 5 \times -39$$

$$\therefore S_{10} = -195$$

Thus, the sum of the first 10 terms of the A.P. is -195.

- (8) Sum of 1 to n natural numbers is 36, then find the value of n . (4 marks)

Solution:

The A.P. is 1, 2, 3, ..., n .

They form an A.P. with $t_1 = 1$, $d = 1$, $t_n = n$ and $S_n = 36$

$$\text{Now } S_n = \frac{n}{2} [t_1 + t_n]$$

$$\therefore 36 = \frac{n}{2} [1 + n]$$

$$\therefore 72 = n [1 + n]$$

$$\therefore 72 = n + n^2$$

$$\therefore n^2 + n - 72 = 0$$

$$\therefore n^2 + 9n - 8n - 72 = 0$$

$$\therefore n(n+9) - 8(n+9) = 0$$

$$\therefore n+9 = 0 \text{ or } n-8 = 0$$

$$\therefore n = -9 \text{ or } n = 8$$

$$\therefore n \neq -9 \text{ as } n \text{ is a natural number.}$$

$$\therefore n = 8$$

Thus, the value of n is 8.

Practice Set - 3.3 (Textbook Page No. 72)

- *(3) Find the sum of all even numbers between 1 to 350. (4 marks)

Solution:

The even natural numbers from 1 to 350 are 2, 4, 6, 8, ..., 348.

They form an A.P. where $a = 2$, $d = 4 - 2 = 2$

$$\text{Let } t_n = 348$$

$$\text{Now, } t_n = a + (n-1)d$$

$$\therefore 348 = 2 + (n-1)2$$

$$\therefore 348 - 2 = 2n - 2$$

$$\therefore 346 = 2n - 2$$

$$\therefore 346 + 2 = 2n$$

$$\therefore 2n = 348$$

$$\therefore n = \frac{348}{2}$$

$$\therefore n = 174$$

$$\text{Now } S_n = \frac{n}{2} [t_1 + t_n]$$

$$= \frac{174}{2} [2 + 348]$$

$$= \frac{174 \times 350}{2}$$

$$\therefore S_n = 30450$$

Thus, the sum of even natural numbers between 1 to 350 is 30,450.

- (5) Complete the following activity to find the sum of natural numbers from 1 to 140 which are divisible by 4. (3 marks)

Solution:

Between 1 to 140. natural number divisible by 4

4, 8, ..., 136

How many numbers? $\therefore n = 34$

$n = 34, a = 4, d = 4$

$$t_n = a + (n-1)d$$

$$136 = 4 + (n-1) \times 4$$

$$\rightarrow n = 34 \rightarrow S_n = \frac{n}{2} [2a + (n-1)d]$$

$$S_{34} = \frac{34}{2} [2 \times 4 + (34-1)4] = 2380$$

Working:

$$a = 4, d = 4, t_n = 136$$

$$\therefore t_n = a + (n-1)d$$

$$136 = 4 + (n-1)4$$

$$136 = 4 + 4n - 4$$

$$\therefore n = \frac{136}{4}$$

$$n = 34$$

$$S_{34} = \frac{34}{2} [2 \times 4 + (34-1)4]$$

$$= 17(8 + 33 \times 4)$$

$$= 17(140)$$

$$\therefore S_{34} = 2380$$

$\therefore$ The sum of numbers between 1 to 140, which are divisible by 4 = 2380

***(6)** Sum of first 55 terms in an A.P. is 3300. Find its 28th term. (3 marks)

Solution:

$$\text{Here } S_{55} = 3300, t_{28} = ?$$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$\therefore S_{55} = \frac{55}{2} [2a + (55-1)d]$$

$$\therefore 3300 = \frac{55}{2} [2a + 54d]$$

$$\therefore 3300 = \frac{55}{2} \times 2 [a + 27d]$$

$$\therefore 3300 = 55 (a + 27d)$$

$$\therefore a + 27d = \frac{3300}{55}$$

$$\therefore a + 27d = 60 \quad \dots(i)$$

$$\text{Now, } t_n = a + (n-1)d$$

$$\therefore t_{28} = a + (28-1)d$$

$$\therefore t_{28} = a + 27d$$

$$\therefore t_{28} = 60 \quad \dots[\text{from (i)}]$$

Thus, the 28th term is 60.

(4) In an A.P. 19th term is 52 and 38th term is 128, find sum of first 56 terms. (3 marks)

$$\text{Given : } t_{19} = 52, \text{ and } t_{38} = 128$$

$$\text{To find } S_{56} = ?$$

Solution:

$$t_n = a + (n-1)d$$

$$\therefore t_{19} = a + (19-1)d$$

$$\therefore 52 = a + 18d$$

$$\therefore a + 18d = 52 \quad \dots(i)$$

$$\text{Also, } t_{38} = a + (38-1)d$$

$$\therefore 128 = a + 37d$$

$$a + 37d = 128$$

$$\dots(ii)$$

Adding equations (i) and (ii) we get,

$$a + 18d = 52$$

$$a + 37d = 128$$

$$2a + 55d = 180$$

$$\dots(iii)$$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$\therefore S_{56} = \frac{56}{2} [2a + (56-1)d]$$

$$= 28 [2a + 55d]$$

$$= 28 \times 180$$

$$\dots[\text{From (iii)}]$$

$$\therefore S_{56} = 5040$$

Hence, the required sum is 5040.

Problem Set - 3 (Textbook Page No. 79)

***(7)** In an A.P., first term is - 5 and last term is 45. If sum of all the numbers in the A.P. is 120, then how many terms are there? What is the common difference. (3 marks)

Solution:

$$\text{Here, } t_1 = a = -5, t_n = 45, S_n = 120, n = ?, d = ?$$

$$\therefore \text{Now } S_n = \frac{n}{2} [t_1 + t_n]$$

$$\therefore 120 = \frac{n}{2} [-5 + 45]$$

$$\therefore 240 = n [40]$$

$$\therefore n = \frac{240}{40}$$

$$\therefore n = 6$$

$$t_n = a + (n-1)d$$

$$\therefore 45 = -5 + (6-1)d$$

$$\therefore 45 = -5 + 5d$$

$$\therefore 45 + 5 = 5d$$

$$\therefore 5d = 50$$

$$\therefore d = \frac{50}{5}$$

$$\therefore d = 10$$

Thus, there are 6 terms in the A.P. and the common difference is 10.

Practice Set - 3.3 (Textbook Page No. 73)

- * (7)** In an A.P., sum of three consecutive terms is 27 and their product is 504, find the terms.
(Assume that three consecutive terms in A.P. are $a-d, a, a+d$) (4 marks)

Solution:

Let the three consecutive terms be $a-d, a, a+d$.

Their sum is 27.

$$\therefore a-d + a + a+d = 27$$

$$\therefore 3a = 27$$

$$\therefore a = 9$$

The product of all three terms is 504.

$$\therefore (a-d) \times a \times (a+d) = 504$$

$$\therefore (a^2 - d^2) \times a = 504$$

...[Using $(a+b)(a-b) = a^2 - b^2$]

$$\therefore (9^2 - d^2) \times 9 = 504$$

$$\therefore 81 - d^2 = \frac{504}{9}$$

$$\therefore 81 - d^2 = 56$$

$$\therefore d^2 = 81 - 56$$

$$\therefore d^2 = 25$$

$$\therefore d = \pm 5 \quad \dots \text{(Taking square root)}$$

When $d = 5, a = 9$, the required terms are $a-d, a, a+d$, i.e. 4, 9, 14

When $d = -5, a = 9$ the required terms are $a-d, a, a+d$, i.e. 14, 9, 4

Hence, the required terms are 4, 9, 14 or 14, 9, 4

- * (8)** Find four consecutive terms in an A.P. whose sum is 12 and the sum of 3rd and 4th term is 14.
(Assume the four consecutive terms in A.P. are $a-d, a, a+d, a+2d$) (4 marks)

Solution:

Let four consecutive terms in A.P. are $a-d, a, a+d, a+2d$

Their sum is 12

$$\therefore a-d + a + a+d + a+2d = 12$$

$$\therefore 4a + 2d = 12$$

$$\therefore 2(2a + d) = 12$$

$$\therefore 2a + d = 6 \quad \dots \text{(i)}$$

The sum of 3rd and 4th terms is 14.

$$\therefore a + d + a + 2d = 14$$

$$\therefore 2a + 3d = 14 \quad \dots \text{(ii)}$$

Subtracting (ii) from (i),

$$2a + d = 6$$

$$2a + 3d = 14$$

$$\begin{array}{r} (-) \quad (-) \quad (-) \\ \hline -2d = -8 \end{array}$$

$$d = \frac{-8}{-2}$$

$$\therefore d = 4$$

Substituting $d = 4$ in (i) we get,

$$2a + d = 6$$

$$\therefore 2a + 4 = 6$$

$$\therefore 2a = 6 - 4$$

$$\therefore 2a = 2$$

$$\therefore a = 1$$

Thus, the required terms are $a-d, a, a+d, a+2d$ i.e. -3, 1, 5, 9.

Problem Set - 3 (Textbook Page No. 80)

- (9)** Divide 207 in three parts, such that all parts are in A.P. and product of two smaller parts will be 4623. (4 marks)

Solution:

Let the three parts of 207, which are in A.P. be $a-d, a, a+d$

Sum of three parts is 207. ... (Given)

$$\therefore a-d + a + a+d = 207$$

$$\therefore 3a = 207$$

$$\therefore a = \frac{207}{3}$$

$$\therefore a = 69$$

Also, the product of two smaller parts is 4623

... (Given)

$$\therefore (a-d) \times a = 4623$$

$$\therefore (69-d) \times 69 = 4623$$

$$\therefore 69 - d = \frac{4623}{69}$$

$$\therefore 69 - d = 67$$

$$\therefore 69 - 67 = d$$

$$\therefore d = 2$$

Thus, the three parts of 207 are

$$a - d = 69 - 2 = 67, a = 69$$

$$a + d = 69 + 2 = 71$$

$$\therefore \text{Hence, three parts of 207 are 67, 69, 71}$$

- (10)** There are 37 terms in an A.P., the sum of three terms placed exactly at the middle is 225 and the sum of last three is 429. Write the A.P. (4 marks)

Solution:

Let the A.P. be $a, a + d, a + 2d, a + 3d, \dots, a + 36d$

Now, $n = 37$ (odd number)

$$\therefore \text{Exactly middle term} = \frac{n+1}{2} = \frac{37+1}{2} = 19^{\text{th}} \text{ term.}$$

$$\therefore 18^{\text{th}}, 19^{\text{th}}, 20^{\text{th}} \text{ terms are middle most terms.}$$

The sum of the three middle most terms is 225.

...(Given)

$$\therefore t_{18} + t_{19} + t_{20} = 225$$

$$\therefore (a + 17d) + (a + 18d) + (a + 19d) = 225$$

$$\dots [\because t_n = a + (n-1)d]$$

$$\therefore 3a + 54d = 225$$

$$\therefore a + 18d = 75 \quad \dots \text{(i) (Dividing both sides by 3)}$$

Now, $35^{\text{th}}, 36^{\text{th}}, 37^{\text{th}}$ terms are last three terms and their sum is 429.

$$\therefore t_{35} + t_{36} + t_{37} = 429$$

$$\therefore (a + 34d) + (a + 35d) + (a + 36d) = 429$$

$$\therefore 3a + 105d = 429$$

$$\therefore a + 35d = 143 \quad \dots \text{(ii) (dividing both sides by 3)}$$

Subtracting (i) from (ii) we get,

$$a + 35d = 143$$

$$a + 18d = 75$$

$$\begin{array}{r} (-) \quad (-) \quad (-) \\ \hline \end{array}$$

$$17d = 68$$

$$\therefore d = \frac{68}{17}$$

$$\therefore d = 4$$

Substituting $d = 4$ in (i) we get,

$$a + 18d = 75$$

$$\therefore a + 18 \times 4 = 75$$

$$\therefore a + 72 = 75$$

$$\therefore a = 75 - 72$$

$$\therefore a = 3$$

$$\therefore \text{Thus, the required A.P. is 3, 7, 11, 15, \dots, 147}$$

- * (12)** If the sum of first p terms of an A.P. is equal to the sum of first q terms, then show that the sum of its first $(p + q)$ terms is zero. ($p \neq q$) (4 marks)

Solution:

$$\text{Given : } S_p = S_q$$

$$\text{To prove } S_{p+q} = 0$$

Proof : Let a be the first term and d be the common difference of the A.P.

$$\therefore S_n = \frac{n}{2} [2a + (n-1)d]$$

The sum of first p terms of the A.P. is

$$S_p = \frac{p}{2} [2a + (p-1)d] \quad \dots \text{(i)}$$

The sum of first q terms of the A.P. is

$$S_q = \frac{q}{2} [2a + (q-1)d] \quad \dots \text{(ii)}$$

$$\text{But } S_p = S_q \quad \dots \text{(Given)}$$

$$\therefore \frac{p}{2} [2a + (p-1)d] = \frac{q}{2} [2a + (q-1)d]$$

...[From (i) and (ii)]

$$\therefore p[2a + pd - d] = q[2a + qd - d]$$

... (multiplying both sides by 2)

$$\therefore 2ap + p^2d - pd = 2aq + q^2d - qd$$

$$\therefore 2ap - 2aq + p^2d - q^2d - pd + qd = 0$$

$$\therefore 2a(p - q) + (p^2 - q^2)d - (p - q)d = 0$$

$$\therefore 2a(p - q) + d(p + q)(p - q) - d(p - q) = 0$$

...[Using $a^2 - b^2 = (a + b)(a - b)$]

$$\therefore (p - q)(2a + pd + qd - d) = 0$$

$$\therefore 2a + (p + q - 1)d = \frac{0}{p - q}$$

$$\therefore 2a + (p + q - 1)d = 0 \dots \text{(iii) } [\because p \neq q \therefore p - q \neq 0]$$

$$\text{Now } S_{p+q} = \left(\frac{p+q}{2} \right) [2a + (p+q-1)d]$$

$$= \left(\frac{p+q}{2} \right) \times 0 \quad \dots \text{[From (iii)]}$$

$$\therefore S_{p+q} = 0$$

$$\therefore \text{The sum of first } (p + q) \text{ terms is zero.}$$

- * (11)** If first term of an A.P. is a , second term is b and last term is c , then show that sum of all the terms is $\frac{(a+c)(b+c-2a)}{2(b-a)}$. (4 marks)

Solution:

The A.P. is $a, b, \dots, c$

Here, $t_1 = a, d = b - a, t_n = c$

We know,

$$t_n = a + (n-1)d$$

$$c = a + (n - 1)(b - a)$$

$$\therefore c - a = (n - 1)(b - a)$$

$$\therefore \frac{c - a}{b - a} = n - 1$$

$$\therefore \frac{c - a}{b - a} + 1 = n$$

$$\therefore \frac{c - a + b - a}{b - a} = n$$

$$\therefore \frac{c + b - 2a}{b - a} = n \quad \dots(i)$$

Method - 1

We know,

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

$$S_n = \frac{c + b - 2a}{2(b - a)} \left[2a + \left(\frac{c + b - 2a}{b - a} - 1 \right) (b - a) \right]$$

...[from (i) and given]

$$= \frac{c + b - 2a}{2(b - a)} \left[2a + \left(\frac{c + b - 2a - (b - a)}{b - a} \right) (b - a) \right]$$

$$= \frac{c + b - 2a}{2(b - a)} [2a + c + b - 2a - b + a]$$

$$= \frac{c + b - 2a}{2(b - a)} [c + a]$$

$$\therefore S_n = \frac{(a + c)(b + c - 2a)}{2(b - a)}$$

Method - II

We know,

$$S_n = \frac{n}{2} [t_1 + t_n]$$

$$= \frac{c + b - 2a}{2(b - a)} [a + c]$$

$$\therefore S_n = \frac{(a + c)(b + c - 2a)}{2(b - a)}$$

Practice Set - 3.4 (Textbook Page No. 78)

- * (4)** There is an auditorium with 27 rows of seats. There are 20 seats in the first row, 22 seats in the second row, 24 seats in the third row and so on. Find the number of seats in 15th row and also find how many total seats are there in the auditorium? (4 marks)

Solution:

The number of seats in each row are as follows: 20, 22, 24,....

The number of seats in each row form an A.P. with number of seats in first row (a) = 20.

Difference between number of seats in two successive rows (d) = 2

Total number of rows (n) = 27

Number of seats in 15th row (t_{15}) = ?

Total number of seats in 27 rows (S_{27}) = ?

$$t_n = a + (n - 1)d$$

$$\therefore t_{15} = 20 + (15 - 1) \times 2$$

$$\therefore t_{15} = 20 + 14 \times 2$$

$$\therefore t_{15} = 20 + 28$$

$$\therefore t_{15} = 48$$

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

$$\therefore S_{27} = \frac{27}{2} [2 \times 20 + (27 - 1) 2]$$

$$\therefore S_{27} = \frac{27}{2} [40 + 52]$$

$$\therefore S_{27} = \frac{27}{2} \times 92$$

$$\therefore S_{27} = 1242$$

There are 48 seats in 15th row and 1242 seats in the auditorium.

(5)

Kargil's temperature was recorded for a week i.e. Monday to Saturday. All readings were in A.P. The sum of temperatures of Monday and Saturday was 5°C more than the sum of temperatures of Tuesday and Saturday. If temperature of Wednesday was -30°C then find the temperature on the other five days. (4 marks)

Solution:

Let the temperatures of Kargil from Monday to Saturday (in°C) which forms an A.P. be $a - d$, a , $a + d$, $a + 2d$, $a + 3d$, $a + 4d$ respectively.

Days	Mon.	Tue.	Wed.	Thu.	Fri.	Sat.
Temp (in °c)	$a - d$	a	$a + d$	$a + 2d$	$a + 3d$	$a + 4d$

As per the given condition,

$$(a - d) + (a + 4d) = a + (a + 4d) + 5$$

$$\therefore a - d + a + 4d = a + a + 4d + 5$$

$$\therefore 2a + 3d = 2a + 4d + 5$$

$$\therefore 3d = 4d + 5$$

$$\therefore 4d - 3d = -5$$

$$\therefore d = -5$$

New temperature on Wednesday is -30°C

...(Given)

$$\therefore a + d = -30$$

$$\therefore a - 5 = -30$$

$$\therefore a = -30 + 5$$

$$\therefore a = -25$$

$$\text{Hence, } a - d = -25 - (-5) = -25 + 5 = -20$$

$$a = -25$$

$$a + d = -25 + (-5) = -30$$

$$a + 2d = -25 + 2(-5) = -25 + (-10) = -35$$

$$a + 3d = -25 + 3(-5) = -25 + (-15) = -40$$

$$a + 4d = -25 + 4(-5) = -25 + (-20) = -45$$

$\therefore$ The temperatures from Monday to Saturday are -20°C , -25°C , -30°C , -35°C , -40°C , -45°C respectively.

Problem Set - 3 (Textbook Page No. 80)

- (14) ₹ 1000 is invested at 10 percent simple interest. Check at the end of every year if the total interest amount is in A.P. If this is an A.P. then find interest amount after 20 years. For this complete the following activity. (3 marks)

Solution:

$$\begin{aligned} \text{Simple interest} &= \frac{P \times R \times N}{100} \\ \text{Simple interest after 1 year} &= \frac{1000 \times 10 \times 1}{100} \\ &= 100 \\ \text{Simple interest after 2 year} &= \frac{1000 \times 10 \times 2}{100} \\ &= 200 \\ \text{Simple interest after 3 year} &= \frac{1000 \times 10 \times 3}{100} \\ &= 300 \end{aligned}$$

According to this the simple interest for 4, 5, 6 years will be 400, 500, 600 respectively.

From this $d = 100$ and $a = 100$

Amount of simple interest after 20 years.

$$\begin{aligned} t_n &= a + (n-1)d \\ \therefore t_{20} &= 100 + (20-1)100 \\ \therefore t_{20} &= 2000 \end{aligned}$$

$\therefore$ Amount of Simple interest after 20 years is ₹ 2000

Practice Set - 3.4 (Textbook Page No. 78)

- (1) On 1st Jan. 2016, Sanika decides to save ₹ 10, ₹ 11 on second day, ₹ 12 on third day. If she decides to save like this, then on 31st December 2016 what would be her total saving?

(3 marks)

Solution:

2016 is a leap year.

$\therefore$ Number of days from 1st January 2016 to 31st December 2016 = 366

The saving of Sanika per day, beginning with the first day are 10, 11, 12, ... for 366 days.

These every day savings form an A.P. with first days saving (a) = 10

Difference in saving made in two successive days (d) = 1

Total number of days in the year 2016 (n) = 366

$\therefore$ Total saving for the year 2016 (S_{366}) = ?

$$\begin{aligned} S_n &= \frac{n}{2} [2a + (n-1)d] \\ \therefore S_{366} &= \frac{366}{2} [2 \times 10 + (366-1)1] \\ &= 183 [20 + 365] \\ &= 183 \times 385 \\ \therefore S_{366} &= 70,455 \\ \therefore \text{Sanika saved ₹ 70,455 in the year 2016.} \end{aligned}$$

- (6) On the world environment day tree plantation programme was arranged on a land which is triangular in shape. Trees are planted such that in the first row there is one tree, in the second row there are two trees, in the third row three trees and so on. Then find the total number of trees in the 25 rows. (3 marks)

Solution:

The number of trees in each row upto the 25th row are as follows: 1, 2, 3, 4, ...

The no. of trees planted in each row form an A.P. with no. of trees in first row (a) = 1

Difference between no. of trees planted in two successive rows (d) = 1

No. of rows (n) = 25

Total no. of trees planted (S_{25}) = ?

$$\begin{aligned} S_n &= \frac{n}{2} [2a + (n-1)d] \\ \therefore S_{25} &= \frac{25}{2} [2 \times 1 + (25-1)1] \\ \therefore S_{25} &= \frac{25}{2} [2 + 24] \\ \therefore S_{25} &= \frac{25}{2} \times 26 \\ \therefore S_{25} &= 25 \times 13 \\ \therefore S_{25} &= 325 \\ \therefore \text{325 trees were planted in 25 rows.} \end{aligned}$$

- (3) Sachin invested in a National Saving Certificate scheme. In the 1st year, he invested ₹ 5000, in 2nd year ₹ 7000, in 3rd year ₹ 9000 and so on. Find the total amount that he invested in 12 years.

(4 marks)

Solution:

Yearly investments of Sachin are as follows:

5000, 7000, 9000, ...

The yearly investments form an A.P. with first year investment (a) = 5000Difference between investments done in two successive years (d) = 2000No. of years (n) = 12Total investment done in 12 years $S_{12} = ?$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$\therefore S_{12} = \frac{12}{2} [2 \times 5000 + (12-1)2000]$$

$$\therefore S_{12} = 6 [10000 + 11 \times 2000]$$

$$\therefore S_{12} = 6 \times 32,000$$

$$\therefore S_{12} = 1,92,000$$

Total investment done in 12 years is ₹ 1,92,000

- (2) A man borrows ₹ 8000 and agrees to repay with a total interest of ₹ 1360 in 12 monthly installments. Each installment is being less than the preceding one by ₹ 40. Find the amount of the first and last installment.

(4 marks)

Solution:

Each installment is less than preceding investment by ₹ 40.

Therefore the installments are in A.P.

Total money repaid in 12 installments (S_{12})

$$= 8000 + 1360 = ₹ 9360$$

No. of installments (n) = 12Difference between two consecutive installments (d) = -40First installment (a) = ?, Last installment (t_{12}) = ?

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$\therefore S_{12} = \frac{12}{2} [2a + (12-1)(-40)]$$

$$\therefore 9360 = 6 [2a + 11 \times (-40)]$$

$$\therefore \frac{9360}{6} = 2a - 440$$

$$\therefore 1560 = 2a - 440$$

$$\therefore 1560 + 440 = 2a$$

$$\therefore 2a = 2000$$

$$\therefore a = \frac{2000}{2}$$

$$\therefore a = 1000$$

$$t_n = a + (n-1)d$$

$$\therefore t_{12} = 1000 + (12-1)(-40)$$

$$\therefore t_{12} = 1000 + 11 \times (-40)$$

$$\therefore t_{12} = 1000 - 440$$

$$\therefore t_{12} = 560$$

First installment is ₹ 1000 and last installment is ₹ 560.

Problem Set - 3 (Textbook Page No. 78)**MCQ's**

Choose the correct alternative answer for each of the following sub questions. (1 mark each)

- (1) The sequence -10, -6, -2, 2, ...
(A) is an A.P., Reason $d = -16$
(B) is an A.P., Reason $d = 4$
(C) is an A.P., Reason $d = -4$
(D) is not an A.P.
- (2) First four terms of an A.P. are, whose first term is -2 and common difference is -2.
(A) -2, 0, 2, 4 (B) -2, 4, -8, 16
(C) -2, -4, -6, -8 (D) -2, -4, -8, -16
- (3) What is the sum of first 30 natural numbers?
(A) 464 (B) 465 (C) 462 (D) 461
- (4) For an given A.P., $t_7 = 4$, $d = -4$ then $a =$
(A) 6 (B) -7 (C) 20 (D) 28
- (5) For an given A.P., $a = 3.5$, $d = 0$, $n = 101$ then $t_n =$
(A) 0 (B) 3.5 (C) 103.5 (D) 104.5
- (6) In an A.P., first two terms are -3, 4 then 21st term is
(A) -143 (B) 143 (C) 137 (D) 17
- (7) If for an any A.P., $d = 5$ then $t_{18} - t_{13} =$
(A) 5 (B) 20 (C) 25 (D) 30
- (8) Sum of first five multiples of 3 is
(A) 45 (B) 55 (C) 15 (D) 75
- (9) 15, 10, 5, ... In this A.P. the sum of first 10 terms is
(A) -75 (B) -125 (C) 75 (D) 125
- (10) In an A.P., 1st term is 1 and the last term is 20. The sum of all terms is 399 then $n =$
(A) 42 (B) 38 (C) 21 (D) 19

Additional MCQ's

- (11) For an A.P. 4, 9, 14, ... $t_{11} =$
(A) 49 (B) 54 (C) 59 (D) 44

- (12) If $a = 6$, $d = 3$ then $S_{10} = \dots\dots$
 (A) 192 (B) 195 (C) 198 (D) 201
- (13) The sum of first 10 natural number is
 (A) 55 (B) 155 (C) 310 (D) 210
- (14) Which of the following sequence is not an A.P.?
 (A) 0.5, 2, 3.5, 5 ... (B) 22, 26, 28, 31
 (C) 3, 5, 7, 9, ... (D) 1, 4, 7, 10, ...
- (15) Find the missing term in the A.P.: -5, ... , 13, ...
 (A) 1 (B) 2 (C) 3 (D) 4
- (16) Find the t_2 of the following sequence for which $S_1 = 2$, $S_2 = 12$ and $S_3 = 36$.
 (A) 24 (B) 2
 (C) 10 (D) none of these
- (17) For an A.P. if $t_4 = 12$ and $d = -10$ then find a .
 (A) -18 (B) 42 (C) -5 (D) 21
- (18) The next two terms of the given sequence 1, 3, 7, 15, 31, ...
 (A) 42, 54 (B) 62, 124 (C) 64, 128 (D) 63, 127
- (19) In an A.P., $a = 8$, $t_n = 62$, $S_n = 210$ then find n .
 (A) 5 (B) 6 (C) 7 (D) 8
- (20) The sum of first n terms of an A.P. $S_n = \dots\dots\dots$
 (A) $\frac{n}{2} [t_1 + t_n]$ (B) $\frac{n}{2} [a + (n-1)d]$
 (C) $\frac{n}{2} [2 + (n-1)d]$ (D) none of these
- (21) A meeting hall has 30 rows in all. There are 20 seats in the first row, 24 seats in the second row and 28 seats in the third row and so on. How many seats are there in the hall?
 (A) 136 (B) 4640 (C) 2340 (D) 192
- (22) State whether the given sequence is an A.P. or not: $1^3, 2^3, 3^3, 4^3, 5^3, \dots\dots$
 (A) an A.P. with $d = 3$ (B) not an A.P.
 (C) an A.P. with $d = 7$ (D) can't say

ANSWERS

- (1) (B) is an A.P., Reason with $d = 4$
 (2) (C) -2, -4, -6, -8 (3) (B) 465 (4) (D) 28 (5) (B) 3.5
 (6) (C) 137 (7) (C) 25 (8) (A) 45 (9) (A) -75
 (10) (B) 38 (11) (B) 54 (12) (B) 195 (13) (A) 55
 (14) (B) 22, 26, 28, 31, ... (15) (D) 4 (16) (C) 10
 (17) (B) 42 (18) (D) 63, 127 (19) (B) 6
 (20) (A) $\frac{n}{2} [t_1 + t_n]$ (21) (C) 2340 (22) (B) not an A.P.

PROBLEMS FOR PRACTICE

Based on Practise Set 3.1

- (1) Which of the following sequences are Arithmetic progression? If it is an A.P. then

write common difference. (1 mark each)

- (i) 0, 1, 0, 1, 0, 1, ... (ii) -10, -13, -16, -19, ...
 (iii) $1^3, 2^3, 3^3, 4^3, \dots$ (iv) 31, 26, 21, 15, ...
 (v) $-1, \frac{-3}{2}, -2, \frac{-5}{2}, \dots$

- (2) Write an A.P. when the common difference d and the first term a are given. (1 mark each)

- (i) $a = 11$, $d = 1.5$ (ii) $a = 5$, $d = -5$
 (iii) $a = -8$, $d = 0$ (iv) $a = -3.5$, $d = -3.5$
 (v) $a = 10$, $d = -3$

Based on Practice Set 3.2

- (3) How many terms are there in the A.P. 187, 194, 201, ..., 439? (2 marks)
- (4) Find n , if the n^{th} term of the following sequence is 68.
 5, 8, 11, 14, ... (2 marks)
- (5) If 10th term and the 18th term of an A.P. are 25 and 41 respectively, then find the 38th term. (3 marks)
- (6) How many three digit natural numbers are divisible by 4? (3 marks)
- (7) Find the eighteenth term of the A.P. 1, 7, 13, 19. (3 marks)
- (8) Find t_{11} from the following A.P. 4, 9, 14, ... (2 marks)
- (9) Find the 10th term from the end of the A.P.
 4, 9, 14, ..., 254. (3 marks)
- (10) For what value of n , the n^{th} term of the following two A.P.s are equal?
 23, 25, 27, 29, ... and -17, -10, -3, 4, ... (3 marks)
- (11) The sixth term of an A.P. is 5 times the 1st term and the eleventh term exceeds twice the fifth term by 3. Find the 8th term. (3 marks)

Based on Practice Set 3.3

- (12) How many two digit natural numbers are divisible by 5? (3 marks)
- (13) Obtain the sum of 56 terms of an A.P. whose 19th and 38th terms are 52 and 148 respectively. (3 marks)
- (14) In an A.P., if the 5th and 12th terms are 30 and 65 respectively, what is the sum of the first 20 terms. (3 marks)
- (15) Split 69 in three parts such that they are in A.P. and product of two smaller parts is 483. (3 marks)
- (16) The first and the last terms of an A.P. are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum? (3 marks)

- (17) Find four consecutive terms in an A.P. such that the sum of the middle two terms is 18 and product of the two end terms is 45. (3 marks)
- (18) Find three consecutive terms in an A.P. whose sum is -3 and the product of their cubes is 512. (3 marks)
- (19) How many terms of the A.P.: 9, 17, 25, ... must be taken to give a sum of 636? (3 marks)
- (20) Find the sum of all natural numbers between 100 and 1000 which are multiples of 7. (3 marks)
- (21) A meeting hall has 20 seats in the first row, 20 seats in the second, 28 seats in the third row and so on and has in all 30 rows. How many seats are there in the meeting hall? (4 marks)

Based on Practice Set 5.4

- (22) In winter, the temperature at a hill station from Monday to Friday is in A.P., The sum of the temperatures of Monday, Tuesday and Wednesday is zero and the sum of the temperature of Thursday and Friday is 15. Find the temperature of each of the five days. (4 marks)
- (23) Neeta saves in a 'Mahila Bachat Gat' ₹ 2 on the first day, ₹ 4 on the second day, ₹ 6 on the third day and so on. What will be her saving in the month of February 2010? (4 marks)
- (24) Mr. Shah borrows ₹ 4000 and agrees to repay with a total interest of ₹ 500 in 10 installments, each installment being less than the preceding installment by ₹ 10. What should be the first and the last installment? (4 marks)
- (25) A farmer borrows ₹ 1000 and agrees to repay with a total interest of ₹ 140 in 12 installments, each installment being less than the preceding installment by ₹ 10. What should be his first installment? (4 marks)

ANSWERS

- (1) (i) It is not an A.P. (ii) It is an A.P., $d = -3$
 (iii) It is not an A.P. (iv) It is not an A.P.
 (v) It is an A.P., $d = \frac{-1}{2}$
- (2) (i) 11, 12.5, 14, 15.5, ... (ii) 5, 0, -5 , -10 , ...
 (iii) -8 , -8 , -8 , -8 , ... (iv) -3.5 , -7 , -10.5 , -14 , ...
 (v) 10, 7, 4, 1, ...
- (3) 37 terms (4) $n = 22$ (5) $t_{38} = 81$ (6) 225
 (7) $t_{18} = 103$ (8) $t_{11} = 54$ (9) 209 (10) $n = 9$ (11) $t_8 = 33$
 (12) 18 (13) 5600 (14) 1150 (15) 21, 23, 25
 (16) 38, 6973 (17) 3, 7, 11, 15 or 15, 11, 7, 3
 (18) -4 , -1 , 2 or 2, -1 , -4 (19) 12 (20) 70, 336
 (21) 2340 (22) -3 , 0, 3, 6, 9 (23) ₹ 812 (24) ₹ 495, ₹ 405
 (25) ₹ 150

ASSIGNMENT - 3

Time : 1 Hr.

Marks : 20

Q.1. (A) Choose the proper alternative answer for the questions given below:

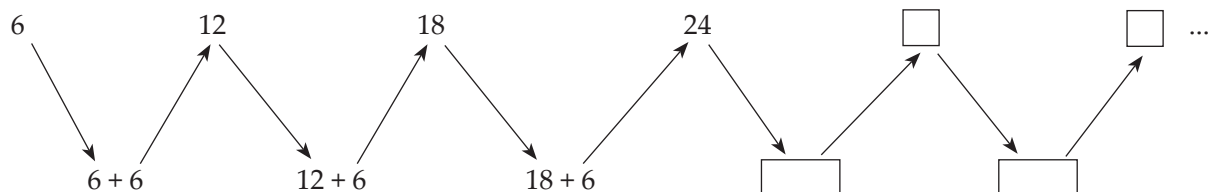
(2)

- (1) The sequence $-10, -6, -2, 2, \dots$
 (A) is an A.P., Reason $d = -16$ (B) is an A.P., Reason $d = 4$
 (C) is an A.P., Reason $d = -4$ (D) is not an A.P.
- (2) Sum of first five multiples of 3 is
 (A) 45 (B) 55 (C) 15 (D) 75

(B) Perform the following activities:

(3)

- (1) Write the next two terms of the following A.P.:



(2) For an A.P. 2, -2, -6, -10, ... find d

Solution:

Here, $t_1 = 2, t_2 = -2, t_3 = -6, t_4 = -10$

$$\begin{aligned}\therefore d &= t_2 - t_1 \\ &= \square - \square \\ \therefore d &= \square\end{aligned}$$

(3) For an A.P., if $a = -3$ and $d = 4$ then find t_n

Solution:

$$\begin{aligned}t_n &= a + (n - 1)d \\ \therefore t_n &= \square + (n - 1) \square \\ \therefore t_n &= \square\end{aligned}$$

Q.2. Perform the following activities: (Any 2)

(4)

(1) Determine whether following sequence is an A.P.

2, -2, -6, -10, ...

Solution: Here $t_1 = 2, t_2 = -2, t_3 = -6, t_4 = -10$

$$t_2 - t_1 = \square - \square = \square$$

$$t_3 - t_2 = \square - \square = \square$$

$$\therefore t_3 - t_2 \square t_2 - t_1$$

$\therefore$ The given sequence is

(2) In an A.P., $a = 6$ and $d = 3$ then find S_{27} .

$$S_n = \frac{n}{2} [\square + (n - 1)d]$$

$$\therefore S_{27} = \frac{27}{2} [12 + (n - 1) \square]$$

$$= \frac{27}{2} \times \square$$

$$= 27 \times 45$$

$$\therefore S_{27} = \square$$

(3) Find 19th term of the following A.P.: 7, 13, 19, 25, ...

Q.3. Perform the following activities. (Any 1)

(3)

(1) Find the sum of all numbers from 1 to 140 which are divisible by 4.

Solution:

The numbers between 1 to 140 that are divisible by 4

4, 8, 12, ..., 136

$$a = \square, d = \square, t_n = \square$$

$$t_n = a + (n - 1)d$$

$$136 = \square + (n - 1) \times \square$$

$$n = \square$$

$$S_n = \frac{n}{2} [t_1 + t_n]$$

$$\begin{aligned}S_{\square} &= \frac{\square}{2} [\square] \\ &= \square\end{aligned}$$

- (2) A village has 4000 literate people in the year 2010 and this number increases by 400 per year. How many literate people will be there till year 2020?

Solution:

Year	2010	2011	2012		2020
Literate people	4000	4400	4800		

Here $a = \square$, $d = \square$, $n = \square$

$$t_n = a + (n - 1) d$$

$$\therefore = \square + (\square - 1) \times \square$$

$$\therefore = \square + \square$$

$$\therefore = \square$$

$\therefore$ There will be literate people till year 2020.

Q.4. Attempt the following: (Any 2)

(8)

- Find four consecutive terms in an A.P. whose sum is 12 and the sum of 3rd and 4th term is 14. (Let four consecutive terms be $a - d, a, a + d, a + 2d$)
- If the sum of p terms of an A.P. is equal to the sum of q terms then show that the sum of its $p + q$ terms is zero.
- Mr. Ajay Sharma borrows ₹ 3,25,000. He paid ₹ 30,500 in the first month and then each installment being less than the preceding installment by ₹1500. How long will it take to clear his loan?

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