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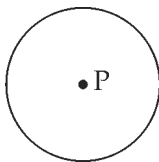
Circle

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**Points to Remember:**

- Circle** : The set of all points equidistant from a fixed point in a plane is called **circle**.

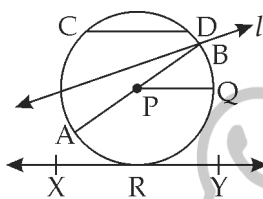


The fixed point is called the **centre** of the circle.

In the above figure, point P is the centre of the circle.

Basic Terms used in circle

- Radius** : Distance between centre of a circle and any point on the circle is called the **radius**.



Seg PQ, Seg PA and Seg PB are radii.

- Chord** : A segment whose end points lie on a circle is called the **chord**.

Seg CD and Seg AB are chords.

- Diameter** : A chord which passes through the centre of the circle is called the **diameter**.

Seg AB is a diameter.

Length of the diameter is double the radius.

- Tangent** : A line in the plane of a circle which touches the circle exactly in only one point is called **tangent** of the circle.

The point at which the tangent touches the circle is called the **point of contact**.

Line XY is tangent to the circle and point R is the point of the contact.

- Secant** : A line which intersects the circle in two distinct points is called the **secant**.

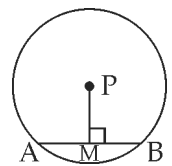
Line l is a secant.

Basic concepts related to circles.

- 'A Perpendicular segment drawn from the centre of a circle to the chord bisects the chord.'

Given : (1) A circle with centre P.

- Seg $PM \perp$ chord AB, A-M-B.

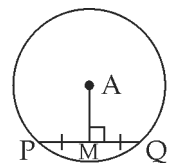


Conclusion : $AM = BM$.

- 'A segment joining centre of a circle and the midpoint of the chord is perpendicular to the chord.'

Given : (1) A circle with centre A.

- Point M is midpoint of chord PQ.



Conclusion : Seg $AM \perp$ chord PQ.

- 'In a circle (or in congruent circles), congruent chords subtend congruent angles at the centre.'

- Given :** (1) A circle with centre P
(2) Chord $AB \cong$ chord CD

Conclusion: $\angle APB \cong \angle CPD$

- (4) 'In a circle (or in congruent circles) if two or more chords subtend congruent angles at the centre, then they are congruent.'

- Given :** (1) A circle with centre O.
(2) $\angle POQ \cong \angle ROS$

Conclusion: Chord $PQ \cong$ chord RS

- (5) 'In a circle (or in congruent circles), congruent chords are equidistant from the centre.'

- Given :** (1) A circle with centre P.
(2) Seg $PM \perp$ chord AB ,
 $A-M-B$
(3) Seg $PN \perp$ chord CD ,
 $C-N-D$
(4) Chord $AB \cong$ chord CD

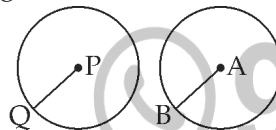
Conclusion: $PM = PN$

- (6) 'In a circle (or in congruent circles), chords which are equidistant from the centre are congruent.'

- Given :** (1) A circle with centre O.
(2) Seg $OM \perp$ chord PQ ,
 $P-M-Q$
(3) Seg $ON \perp$ chord RS ,
 $R-N-S$
(4) $OM = ON$

Conclusion: Chord $PQ \cong$ chord RS

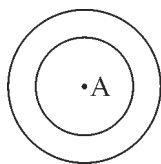
- (7) **Congruent circles :** Two or more circles are said to be congruent circles if their radii are equal.



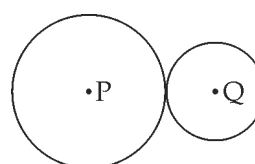
In the above figure, Radius $PQ \cong$ Radius AB .

$\therefore$ Both circles are congruent.

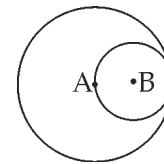
- (8) **Concentric circles :** Two or more circles with same centre but different radii are called concentric circles.



- (9) **Touching circles :** Two circles in the same plane having only one point in common are called touching circles.

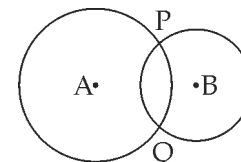


Externally touching circles



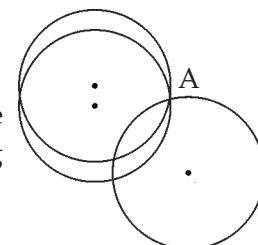
Internally touching circles

- (10) **Intersecting circles :** Two circles which have exactly two points in common are called intersecting circles.



- (11) **Circles passing through one point. Two points and Three points**

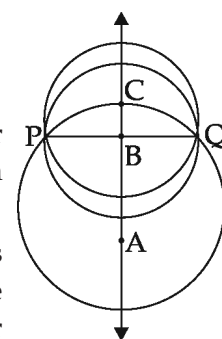
- (i) We shall first consider one point. There are infinite number of circles passing through a point.



- (ii) **Circles passing through two distinct points.**

There can be infinite number of circles passing through two distinct points.

Note : Centres of all these circles will pass through a line containing perpendicular bisector of the segment joining these two points.

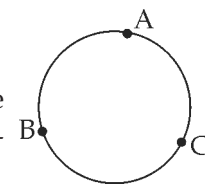


- (iii) **Circles passing through three points.**

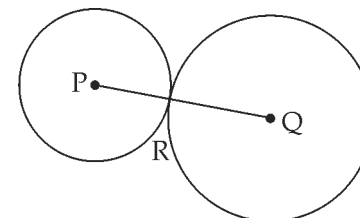
Here, arises to cases.

Case (I): When three points are non collinear.

We can draw exactly one circle passing through three non-collinear points.

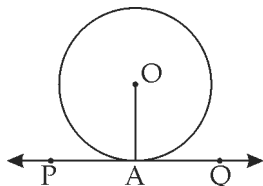


Case (II): When three points are collinear.



We can not draw any circle passing through three collinear points.

- (12) **Tangent theorem :** A tangent at any point of a circle is perpendicular to the radius, through the point of contact.



In above figure,

line PQ is a tangent at A and
Seg AO is the radius through
the point of contact A.

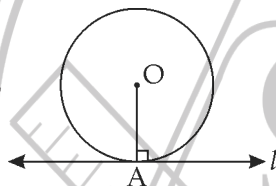
$\therefore \text{seg } OA \perp \text{line } PQ.$

- (13) **Converse of tangent theorem :**

A line perpendicular to a radius of a circle at its outer end is a tangent to the circle.

In adjoining figure,
line l is perpendicular to
radius OA at its outer
end A.

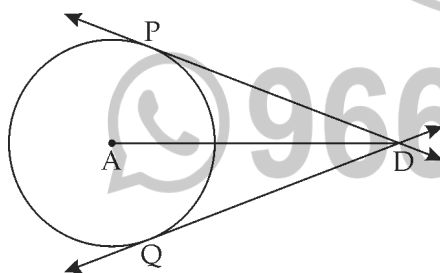
$\therefore$ line l is a tangent.



- (14) **Tangent segment Theorem :**

Statement : The lengths of the two tangent segments to a circle drawn from an external point are equal.

- Given :** (i) A circle with centre A.
(ii) D is a point in the exterior of the circle.
(iii) Points P and Q are the points of contact of the two tangents from D to the circle.



In $\triangle PAD$ and $\triangle QAD$,

Seg PA $\cong$ Seg QA ... (radii of the same circle)

Seg AD $\cong$ Seg AD ... (Common side)

$\angle APD \cong \angle AQD = 90^\circ$

... (by Tangent theorem)

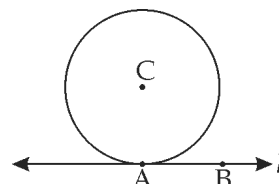
$\therefore \triangle PAD \cong \triangle QAD$... (by Hypotenuse side test)

$\therefore \text{Seg } DP \cong \text{Seg } DQ$... (c.s.c.t.)

MASTER KEY QUESTION SET - 3

Practice Set - 3.1 (Textbook Page No. 55)

- (1) In the adjoining figure, the radius of a circle with centre C is 6 cm, Line AB is a tangent at A? Answer the following questions.



- (i) What is the measure of $\angle CAB$? Why?
(ii) What is the distance of point C from line AB? Why?
(iii) $d(A, B) = 6$ cm, find $d(B, C)$.
(iv) What the measure of $\angle ABC$? Why? . (3 marks)

Solution :

- (i) Radius CA $\perp$ Line AB ... (Tangent Theorem)
 $\therefore m\angle CAB = 90^\circ$... (i)
(ii) $d(C, A) = 6$ cm ... (Radius of circle)
 $\therefore$ Distance of point C from line AB is 6 cm.
(iii) In $\triangle CAB$, $\angle CAB = 90^\circ$... [From (i)]
 $\therefore BC^2 = AC^2 + AB^2$... (By Pythagoras theorem)
 $= 6^2 + 6^2$
 $BC^2 = 36 + 36 = 72$
 $\therefore BC = 6\sqrt{2}$ cm ... (Taking square roots)
(iv) In $\triangle ABC$, $\angle A = 90^\circ$... [From (i)]
 $AC = AB$... (Given)
 $\therefore \angle ACB = \angle ABC$... (ii) (converse of isosceles triangle theorem)

In $\triangle CAB$,

$\angle ABC + \angle ACB + \angle CAB = 180^\circ$

... (Sum of all angles of a triangle is 180°)

$\therefore \angle ABC + \angle ABC + 90 = 180$... [From (i) and (ii)]

$\therefore 2\angle ABC = 180 - 90$

$\therefore 2\angle ABC = 90$

$\therefore \angle ABC = 45^\circ$

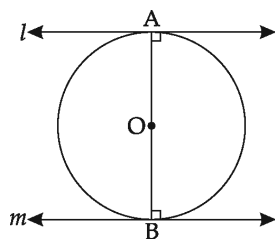
- (4) What is the distance between two parallel tangents of a circle having radius 4.5 cm. Justify your answer. (2 marks)

Given :

- (i) A circle with centre O and radius 4.5 cm.
(ii) Line l is tangent to the circle at point A
(iii) Line m is tangent to the circle at point B

(iv) Line $l \parallel$ line m

To find : Distance between line l and line m



Solution :

seg $OA \perp$ line l ... (i)
seg $OB \perp$ line m ... (ii) } (Tangent Theorem)

line $l \parallel$ line m ... (iii) ... (Given)

$\therefore$ A-O-B ... [From (i), (ii) and (iii)]

$\therefore$ $AB = AO + BO$

$\therefore$ $AB = 4.5 + 4.5$

$\therefore$ $AB = 9$ cm

$\therefore$ Distance between line l and line m is 9 cm.

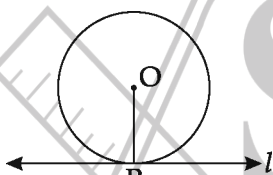
Problem Set - 3 (Textbook Pg No. 83)

(2) Line l touches the circle with centre O at P; radius of the circle is 9 cm. Answer the following.

(i) Find $d(O, P)$ = write reason

(ii) $d(O, Q) = 8$ cm. Where does the point Q lie?

(iii) $d(O, R) = 15$ How many such 'R' contained in line l . What is the distance of those points from 'R'? (3 marks)



Solution :

(i) Radius of the circle is 9 cm.

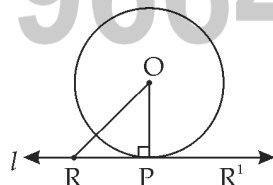
$\therefore d(O, P) = 9$ cm

(ii) $d(O, Q) = 8$ cm

$d(O, Q) <$ Radius

$\therefore$ Point Q lies in the interior of the circle.

(iii) Point can have two different positions on line l as shown in the adjoining figure.



In $\triangle OPR$, $\angle OPR = 90^\circ$... (Tangent Theorem)

$\therefore OR^2 = OP^2 + PR^2$... (Pythagoras theorem)

$\therefore 15^2 = 9^2 + PR^2$

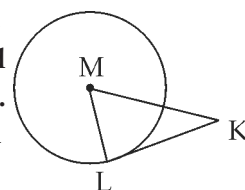
$\therefore PR^2 = 225 - 81 = 144$

$\therefore PR^2 = 144$

$\therefore$ PR = 12 units ... (Taking square roots)

$\therefore$ Two such 'R' contained in line l

(3) In the adjoining figure, M is the centre of the circle and seg KL is a tangent segment. If $MK = 12$, $KL = 6\sqrt{3}$, then



(i) Find radius of the circle.

(ii) Find measure of $\angle K$ and $\angle M$. (3 marks)

Solution :

(i) In $\triangle MLK$, $\angle MLK = 90^\circ$... (Tangent and radius $\perp$ at the point of contact) (i)

$\therefore MK^2 = ML^2 + LK^2$... (Pythagoras theorem)

$\therefore 12^2 = ML^2 + (6\sqrt{3})^2$

$\therefore 144 = ML^2 + 108$

$\therefore ML^2 = 144 - 108$

$\therefore ML^2 = 36$

$\therefore ML = 6$ units ... (ii) (Taking square roots)

$\therefore$ Radius of the circle is 6 units.

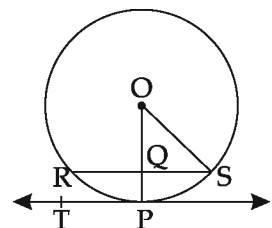
(ii) In $\triangle MLK$, $\angle MLK = 90^\circ$... [From (i)]

$ML = \frac{1}{2} MK$... [From (ii) and given]

$\therefore \angle K = 30^\circ$... (Converse of 30° - 60° - 90° theorem)

$\therefore \angle M = 60^\circ$... (Sum of all angles of a triangle is 180°)

(9) In the adjoining figure, line l touches the circle at P. O is the centre. Q is the mid point of radius OP. Chord $RS \parallel$ line l . $RS = 12$, find radius of the circle. (2 marks)



Solution :

Take a point T on line l as shown in the figure.

$\angle OPT = 90^\circ$... (i) (Tangent Theorem)

chord $RS \parallel$ line l ... (Given)

$\therefore \angle OPT \cong \angle OQR$... (ii) (Corresponding angles theorem)

$\therefore \angle OQR = 90^\circ$... (iii) [From (i) and (ii)]

$\therefore$ seg $OQ \perp$ chord RS ... [From (iii)]

$\therefore QR = \frac{1}{2} RS$

... (Perpendicular drawn from the centre of the circle to the chord bisects the chord.)

$\therefore QR = \frac{1}{2} \times 12$

$\therefore QR = 6$ units

Let the radius of the circle be x units.

$\therefore OR = OP = x$ units ... (Radii of the same circle)

$$\therefore OQ = \frac{1}{2} OP \quad (\because Q \text{ is midpoint of seg } OP)$$

$$\therefore OQ = \frac{1}{2} \times x$$

$$\therefore OQ = \frac{x}{2}$$

In ΔOQR ,

$$\angle OQR = 90^\circ \quad \dots [\text{From (iii)}]$$

$$\therefore OR^2 = OQ^2 + QR^2 \dots (\text{By Pythagoras theorem})$$

$$\therefore x^2 = \left(\frac{x}{2}\right)^2 + (6)^2$$

$$\therefore x^2 = \frac{x^2}{4} + 36$$

$$\therefore 4x^2 = x^2 + 144 \quad \dots (\text{Multiplying throughout by 4})$$

$$\therefore 4x^2 - x^2 = 144$$

$$\therefore 3x^2 = 144$$

$$\therefore x^2 = \frac{144}{3}$$

$$\therefore x^2 = 48$$

$$\therefore x = \sqrt{48} \quad \dots (\text{taking square roots})$$

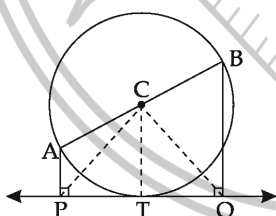
$$\therefore x = \sqrt{16 \times 3}$$

$$\therefore x = 4\sqrt{3}$$

$$\therefore OR = OP = 4\sqrt{3} \text{ units}$$

$$\therefore \text{Radius of the circle is } 4\sqrt{3} \text{ units.}$$

- (10) In the adjoining figure, seg AB is a diameter of a circle with centre C. Line PQ is a tangent, it touches the circle at T. segs AP and BQ are perpendiculars to line PQ. Prove seg CP $\cong$ seg CQ. (2 marks)



Construction :

Draw seg CT, seg CP and seg CQ.

Proof :

$$\text{seg } AP \perp \text{line } PQ \quad \dots (\text{i}) \text{ (Given)}$$

$$\text{seg } CT \perp \text{line } PQ \quad \dots (\text{ii}) \text{ (Tangent Theorem)}$$

$$\text{seg } BQ \perp \text{line } PQ \quad \dots (\text{iii}) \text{ (Given)}$$

$\therefore \text{seg } AP \parallel \text{seg } CT \parallel \text{seg } BQ \quad \dots [\text{Perpendiculars drawn to the same line are parallel to each other from (i), (ii) and (iii)}]$

On transversals AB and PQ,

$$\frac{PT}{QT} = \frac{AC}{BC} \quad \dots (\text{iv}) [\text{Property of intercepts made by three parallel lines}]$$

$$\text{But, } AC = BC \quad \dots (\text{Radii of the same circle})$$

$$\therefore \frac{AC}{BC} = 1 \quad \dots (\text{v})$$

$$\therefore \frac{PT}{QT} = 1 \quad \dots (\text{vi}) \dots [\text{From (iv) and (v)}]$$

$$\therefore PT = QT \quad \dots (\text{vi})$$

In ΔPTC and ΔQTC ,

$$\text{seg } CT \cong \text{seg } CT \quad \dots (\text{Common side})$$

$$\angle CTP \cong \angle CTQ \quad \dots [\text{Each is } 90^\circ \text{ from (ii)}]$$

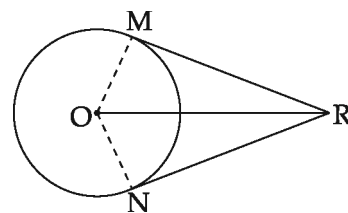
$$\text{seg } PT \cong \text{seg } QT \quad \dots [\text{From (vi)}]$$

$$\therefore \Delta PTC \cong \Delta QTC \quad \dots (\text{By SAS test of congruency})$$

$$\therefore \text{seg } CP \cong \text{seg } CQ \quad \dots (\text{c.s.c.t.})$$

Practice Set - 3.1 (Textbook Page No. 55)

- (2) In the adjoining figure, O is the centre of the circle. From point R, Seg RM and RN are tangent segments, touch the circle at M, N. $(O, R) = 10 \text{ cm}$, radius of the circle = 5 cm, then find (2 marks)



- (i) the length of each tangent segment?
(ii) Measure of $\angle MRO$?
(iii) Measure of $\angle MRN$

Construction : Draw seg OM and seg ON

Solution :

$$\text{Radius } OM \perp \text{tangent } RM \quad \dots (\text{i}) \text{ (Tangent Theorem)}$$

$$\text{In } \Delta OMR, \angle OMR = 90^\circ \quad [\text{From (i)}]$$

$$\therefore OR^2 = OM^2 + RM^2$$

$$\therefore 10^2 = 5^2 + RM^2$$

$$\therefore RM^2 = 100 - 25$$

$$\therefore RM^2 = 75$$

$$\therefore RM = 5\sqrt{3} \text{ cm} \quad \dots (\text{ii}) \text{ (Taking square roots)}$$

$$\therefore MR = RN \quad (\text{Tangent segments Theorem})$$

$$\therefore RM = 5\sqrt{3} \text{ cm} \quad \dots [\text{From (ii)}]$$

$$\text{In } \Delta OMR, \angle OMR = 90^\circ \quad [\text{From (i)}]$$

$$\therefore OM = \frac{1}{2} OR \quad (\text{Given})$$

$$\therefore \angle MRO = 30^\circ \quad \dots (\text{iii}) \text{ (Converse of } 30^\circ - 60^\circ - 90^\circ)$$

Similarly we can prove,

$$\angle NRO = 30^\circ \quad \dots (\text{iv})$$

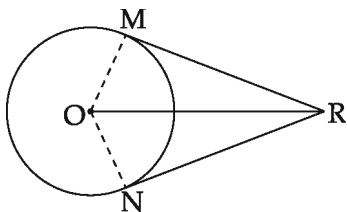
$$\angle MRN = \angle MRO + \angle NRO \quad \dots (\text{Angle addition property})$$

$$= 30^\circ + 30^\circ$$

$$\therefore \angle MRN = 60^\circ$$

- (3) In the figure, Seg RM and seg RN are tangent segments of a circle with centre O. Prove that seg OR divides $\angle MRN$ as well as $\angle MON$. (2 marks)

Construction : Draw seg OM and seg ON



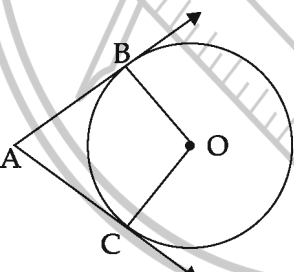
Proof :

In $\triangle OMR$ and $\triangle ONR$,

- (i) $\angle OMR = \angle ONR = 90^\circ$... (Tangent Theorem)
 - (ii) $seg OR \cong seg OR$ (Common side)
 - (iii) $seg OM \cong seg ON$ (Radii of same circle)
- $\therefore \triangle OMR \cong \triangle ONR$ [Hypotenuse – side test]
- $\therefore \angle MOR \cong \angle NOR$... (i)
- $\therefore \angle MRO \cong \angle NRO$... (ii)
- $\therefore seg OR$ bisects $\angle MRN$ and $\angle MON$... [From (i) and (ii)]

Problem Set - 3 (Textbook Pg No. 83)

- (4) In the adjoining figure, O is the centre of the circle. Seg AB, seg AC are tangent segments. A Radius of the circle is r and $l(AB) = r$, Prove $\square ABOC$ is a square.

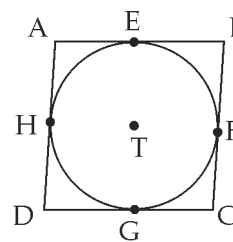


(2 marks)

Proof :

- $seg AB \cong seg AC$... (i) (Tangent segment Theorem)
- $l(AB) = r$... (ii) ... (Given)
- $l(AB) = l(AC) = r$... (iii) [From (i) and (ii)]
- $l(OB) = l(OC) = r$... (iv) (Radii of the same circle)
- In $\square ABOC$, $seg AB \cong seg AC \cong seg OB \cong seg OC$... [From (iii) and (iv)]
- $\therefore \square ABOC$ is a rhombus ... (v) (Definition)
- But, $\angle OBA = 90^\circ$... (vi) (Tangent Theorem)
- In rhombus $ABOC$, $\angle OBA = 90^\circ$... [From (v) and (vi)]
- $\therefore \square ABOC$ is a square (Definition)

- (5) In the adjoining figure, $\square ABCD$ is a parallelogram. It circumscribes the circle with centre T. Point E, F, G, H are touching points. $AE = 4.5$, $EB = 5.5$, find AD.



(3 marks)

Solution :

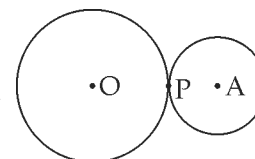
- $AE = AH = 4.5$... (i)
- $BE = BF = 5.5$... (ii)
- Let, $DH = DG = x$... (iii)
- $CG = CF = y$... (iv)
- (Tangent segment of Theorem and supposition)
- $\square ABCD$ is a parallelogram ... (Given)
- $\therefore AB = CD$... [Opposite sides of parallelogram are equal]
- $\therefore AE + BE = DG + CG$... [A-E-B and D-G-C]
- $\therefore 4.5 + 5.5 = x + y$... [From (i), (ii), (iii) and (iv)]
- $\therefore x + y = 10$... (v)
- $\therefore AD = BC$... [Opposite sides of parallelogram are equal]
- $\therefore AH + DH = BF + CF$... [A-H-D and B-F-C]
- $\therefore 4.5 + x = 5.5 + y$... [From (i), (ii), (iii) and (iv)]
- $\therefore x - y = 5.5 - 4.5$
- $\therefore x - y = 1$... (vi)
- Adding (v) and (vi),
- $x + y + x - y = 10 + 1$
- $\therefore 2x = 11$
- $\therefore x = \frac{11}{2}$
- $\therefore x = 5.5$

- $AD = AH + DH$... (A-H-D)
- $\therefore AD = 4.5 + x$... [From (iii)]
- $\therefore AD = 4.5 + 5.5$... [From (iii)]
- $\therefore AD = 10$ units



Points to Remember:

- Theorem**
If two circles are touching circles, then the common point lies on the line joining their centres.
- Externally touching circles :**
In the adjoining figure, two circles with centres O and A are touching externally at point P.

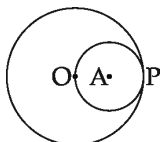


$\therefore O - P - A$

• **Internally touching circles :**

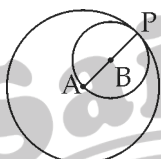
In the adjoining figure, two circles with centres O and A are touching internally at point P.

$\therefore O - A - P$



Practice Set - 3.2 (Textbook Page No. 58)

- (1) Two circles having radii 3.5 cm and 4.8 cm touch each other internally. Find the distance between their centres. (2 marks)



Given :

- Two circles with centers A and B touch each other internally at point P.
- Radius of circle with centre A is 4.8 cm.
- Radius of circle with centre B is 3.5 cm.

To Find : AB

Solution :

AP = 4.8 cm, BP = 3.5 cm ... (Given)

A-B-P ... (When two circles touch each other, the point of contact lies on the line joining the centres.)

$$\therefore AP = AB + BP$$

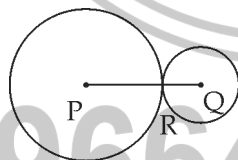
$$\therefore 4.8 = AB + 3.5$$

$$\therefore AB = 4.8 - 3.5$$

$$\therefore AB = 1.3 \text{ cm}$$

$$\therefore \boxed{AB = 1.3 \text{ cm}}$$

- (2) Two circles having radii 5.5 cm, 4.2 cm touch each other externally. Find distance between their centres? (2 marks)



Given :

- Two circles with centres P and Q touch each other externally at point R.
- Radius of circle with centre P is 5.5 cm
- Radius of circle with centre Q is 4.2 cm

To Find : PQ, QR

Solution :

PR = 5.5 cm, QR = 4.2 cm ... (Given)

P-R-Q ... (When two circles touch each other, the point of contact lies on the line joining the two centres.)

$$PQ = PR + QR$$

$$\therefore PQ = 5.5 + 4.2$$

$$\therefore PQ = 9.7 \text{ cm}$$

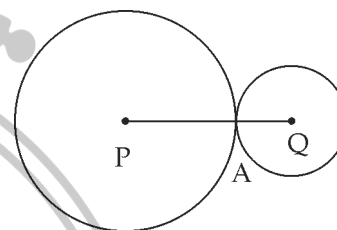
$$\therefore \boxed{PQ = 9.7 \text{ cm}}$$

- (3) If radii of two circles are 4 cm, 2.8 cm. Draw figures of circles touching each other, (i) externally (ii) internally. (2 marks)

Solution :

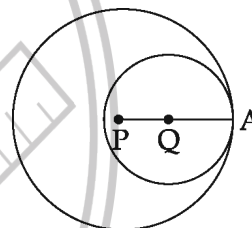
Case (i): Externally touching circles.

PA = 4 cm, QA = 2.8 cm



Case (ii) Internally touching circles.

PA = 4 cm, QA = 2.8 cm

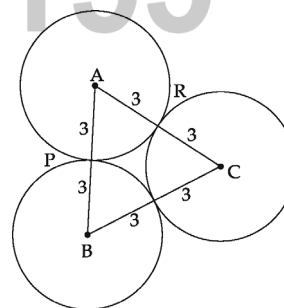


Problem Set - 3 (Textbook Pg No. 83)

- (11) Draw circles with centres A, B, C each of radius 3 cm. Such that each circle touches the remaining 2 circles. (2 marks)

Solution :

Draw an equilateral triangle ABC with each side measuring 6 cm. Taking A as centre draw a circle with radius 3 cm. Repeat same thing taking B and C as centres.



A-P-B

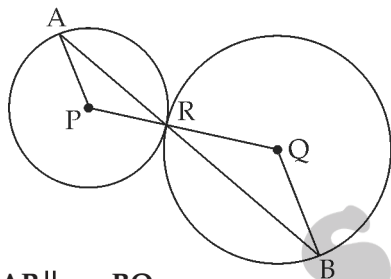
B-Q-C

C-R-A

} ... (For two touching circles point of contact lies on the line joining the centres.)

Practice Set - 3.2 (Textbook Page No. 58)

- (4) In the adjoining figure, the circles with centres P and Q touch each other at R. A line passing through R meets the circles at A and B respectively then



Prove that :

- (i) $\text{Seg AP} \parallel \text{seg BQ}$.
(ii) $\triangle APR \sim \triangle RQB$.
(iii) Find $\angle RQB$, if $\angle PAR = 35^\circ$ (3 marks)

Proof and Solution :

P-R-Q ... (When two circles touch each other, then point of contact lies on the line joining the two centres.)

$\therefore \angle PRA \cong \angle QRB$... (i) [Vertically opposite angles]

In $\triangle PRA$, $\text{seg PA} \cong \text{seg PR}$... (Radii of same circle)

$\therefore \angle PRA \cong \angle PAR$... (ii) [Isosceles triangle theorem]

In $\triangle QRB$, $\text{seg QR} \cong \text{seg QB}$... (Radii of same circle)

$\therefore \angle QRB \cong \angle QBR$... (iii) [Isosceles triangle theorem]

$\therefore \angle PRA \cong \angle PAR \cong \angle QRB \cong \angle QBR$... (iv)
[From (i), (ii) and (iii)]

$\therefore \angle PAR \cong \angle QBR$... [From (iv)]

$\text{Seg AP} \parallel \text{seg BQ}$... (Alternate angles test)

In $\triangle APR$ and $\triangle RQB$,

(i) $\angle PAR \cong \angle QRB$... [From (iv)]

(ii) $\angle PRA \cong \angle QBR$... [From (iv)]

$\therefore \triangle APR \sim \triangle RQB$... (By AA test of similarity)

$\therefore \angle PAR = 35^\circ$... (v) (Given)

$\therefore \angle QRB = \angle QBR = 35^\circ$... (vi) [From (iv) and (v)]

In $\triangle QRB$

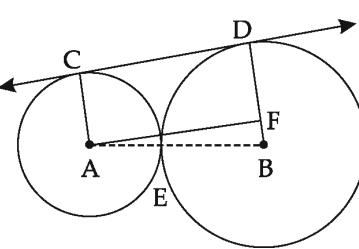
$\angle RQB + \angle QRB + \angle QBR = 180^\circ$
... (Sum of all angles of a $\triangle$ is 180°)

$\therefore \angle RQB + 35^\circ + 35^\circ = 180^\circ$... [From (vi)]

$\therefore \angle RQB = 180^\circ - 70^\circ$

$\therefore \angle RQB = 110^\circ$

- (5) In the adjoining figure, the circles with centres A and B touch each other at E. Line l is a common tangent that touches the circle at C and D respectively.



Find length of seg CD if the radii of the circles are 4 cm, 6 cm (3 marks)

Construction :

Draw seg AC and seg BD

Draw seg $AF \perp$ seg BD, B-F-D.

Solution :

A-E-B ... (i) [When two circles touch each other, the point of contact lies on the line joining the centres.]

$\therefore AC = AE = 4$ cm ... (ii) [Given, radii of same circle]

$\therefore BD = BE = 6$ cm ... (iii) [Given, radii of same circle]

$AB = AE + BE$ [A-E-B, from (i)]

$\therefore AB = 4 + 6$ [From (ii) and (iii)]

$\therefore AB = 10$ cm ... (iv)

In $\square ACDF$, $\angle ACD = 90^\circ$
 $\angle FDC = 90^\circ$ } [Tangent and radius are $\perp$ to each other at the point of contact]

$\angle AFD = 90^\circ$... (Construction)

$\angle FAC = 90^\circ$... (Remaining angle)

$\therefore \square ACDF$ is a rectangle. ... (Definition)

$\therefore CD = AF$... (v)
 $FD = AC$... (vi) } [Opposite sides of rectangle are equal]

$\therefore FD = 4$ cm ... (vii) ... [From (ii) and (vi)]

$BD = BF + FD$... [B-F-D]

$\therefore 6 = BF + 4$... [From (iii) and (vii)]

$\therefore BF = 6 - 4 = 2$ cm ... (viii)

In $\triangle AFB$, $\angle AFB = 90^\circ$... (Construction)

$\therefore AB^2 = AF^2 + BF^2$... (By Pythagoras theorem)

$\therefore 10^2 = AF^2 + 2^2$

$\therefore AF^2 = 100 - 4$

$\therefore AF^2 = 96$

$\therefore AF = \sqrt{96}$... (Taking square roots)

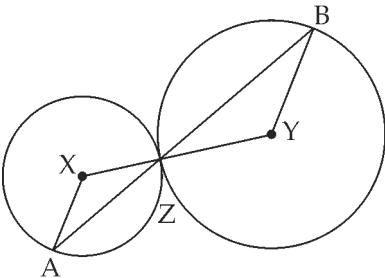
$\therefore AF = \sqrt{16 \times 6}$

$\therefore AF = 4\sqrt{6}$ cm

$\therefore CD = 4\sqrt{6}$ cm ... [From (v)]

Problem Set - 3 (Textbook Pg No. 83)

- (7) In the adjoining fig. circles with centres X, Y touch each other at Z. A secant passing through Z meets the circles at A and B respectively.



Prove that, Radius $XA \parallel$ radius YB .

Fill in the blanks and complete the proof. (3 marks)

Construction :

Draw segments XZ and $seg YZ$

Proof :

By theorem of touching circles,

points X, Z, Y are **collinear points**

$\angle XZA \cong \angle BZY$... (Vertically Opposite angles)

Let $\angle XZA = \angle BZY = a$... (i)

$seg XA \cong seg XZ$ **Radii of the same circle**

$\therefore \angle XAZ = \angle XZA = a$... (ii) (Isosceles triangle theorem)

$seg YB \cong seg YZ$ **$\therefore$ Radii of the same circle**

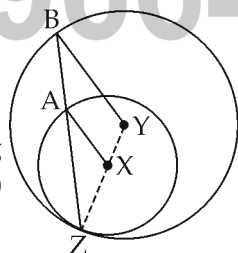
$\therefore \angle BZY = \angle YBZ = a$... (iii) (Isosceles triangle theorem)

$m\angle XAZ = m\angle YBZ = a$... [From (i), (ii) and (iii)]

$\therefore$ Radius $XA \parallel$ radius YB

...Converse of alternate angles test

- (8) In the adjoining fig., circles with centres X, Y touch at Z internally. Chord BZ of bigger circle, intersects inner circle at A. Prove : $seg AX \parallel seg BY$. (3 marks)


Construction :

Draw $seg ZY$.

Proof :

Y-X-Z ... (Theorem of touching circle)

In $\triangle XAZ$, $seg XA \cong seg XZ$

... (Radii of the same circle)

$\therefore \angle XAZ \cong \angle XZA$ (Isosceles triangle theorem)

$\therefore \angle XAZ \cong \angle YZB$... (i) [Y-X-Z, Z-A-B]

In $\triangle YBZ$, $seg YB \cong seg YZ$... (Radii of same circle)

$\therefore \angle YBZ \cong \angle YZB$... (ii) (Isosceles triangle theorem)

$\therefore \angle XAZ \cong \angle YBZ$... [From (i) and (ii)]

$\therefore seg AX \parallel seg BY$... (Corresponding angles test)


Points to Remember:

- Arc of a circle :** A part of a circle is called an arc of a circle.

In the adjoining figure, points A and B divide circle into two arcs, viz arc AXB and arc AYB.

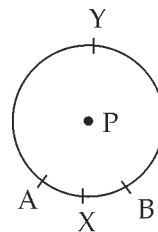


Fig . I

- Measure of an arc:** Measure of an arc equals to its corresponding central angle.

$$m(\text{arc } PXQ) = m \angle POQ$$

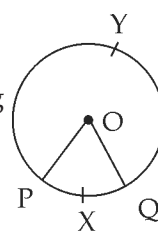


Fig . II

- Types of arc:**

On the basis of measure of arc, arcs can be classified into three types:

- Minor arc:** An arc whose measure is less than 180° is called the minor arc.

Note : Minor arcs are often named using two letters via in fig I and fig. II, arc AXB and arc PXQ can be named arc AB and arc PQ respectively.

- Major arc:** An arc measuring more than 180° is called the major arc.

Measure of major arc = 360° - Measure of minor arc.

- Semicircle:** An arc whose measure is 180° is called the semicircle.

Note: A diameter divides a circle into two semicircles.

Theorem : In a circle (or in congruent circles), congruent arcs have corresponding chords congruent.

Given : (1) A circle with centre P.

(2) $\text{arc } AXB \cong \text{arc } CYD$

To prove : chord $AB \cong$ chord CD

Proof :

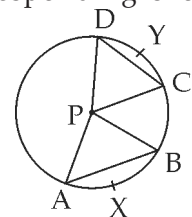
$\text{arc } AXB \cong \text{arc } CYD$

... (i) [Given]

$m(\text{arc } AXB) = m\angle APB$... (ii)

$m(\text{arc } CYD) = m\angle CPD$... (iii)

} [Definition of measure of a minor arc]



$$\therefore \angle APB = \angle CPD \quad \dots(iv)$$

[From (i), (ii) and (iii)]

In $\triangle APB$ and $\triangle CPD$,

(i) $seg PA \cong seg PC$ (Radii of a circle)

(ii) $\angle APB \cong \angle CPD$ [From (iv)]

(iii) $Seg PB \cong Seg PD$ Radii of same circle

$\therefore \triangle APB \cong \triangle CPD$... (by SAS Test of congruence)

$\therefore chord AB \cong chord CD$... (c.s.c.t.)

- Theorem:** In a circle (or in congruent circles), congruent chords have their corresponding minor arcs congruent.

Given: (i) A circle with centre P.

(ii) $chord DE \cong chord FG$

To prove: $arc DXE \cong arc FYG$

Proof:

- In $\triangle DPE$ and $\triangle FPG$,
 - (i) $side DP \cong side FP$
 - (ii) $side PE \cong side PG$
 - (iii) $side DE \cong side FG$

... (Radii of the same circle)

(2) $\therefore \triangle DPE \cong \triangle FPG$... (SSS Test)

(3) $\therefore \angle DPE \cong \angle FPG$... (i) [c.a.c.t.]

(4) $m(arc DXE) = m\angle DPE$... (ii)

(5) $m(arc FYG) = m\angle FPG$... (iii)

[Definition of measure of a minor arc]

(6) $\therefore m\angle DPE = m\angle FPG$... [From (i), (ii) and (iii)]

i.e. $\angle DPE \cong \angle FPG$

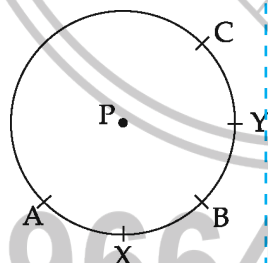
$\therefore arc DXE \cong arc FYG$

Arc addition Property:

In the adjoining figure, points

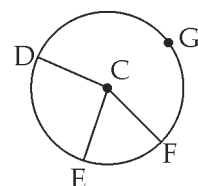
A, X, B, Y and C are on the same circle and $arc AXB$ and $arc BYC$ has exactly one end point common.

$\therefore m(arc AXB) + m(arc BYC) = m(arc ABC)$



Practice Set - 3.3 (Textbook Page No. 63)

- In the adjoining figure, G, D, E, F are concyclic points of a circle with centre C. $\angle ECF = 70^\circ$, $m(arc DGF) = 200^\circ$ find (i) $m(arc DE)$ (iii) $m(arc DEF)$. (2 marks)



Solution:

$$\therefore m(arc EF) = m\angle ECF \quad \text{(Definition of measure of a minor arc)}$$

$$\therefore m(arc EF) = 70^\circ \quad \dots(i)$$

$$m(arc DE) + m(arc EF) + m(arc DGF) = 360^\circ$$

...(Measure of a circle is 360°)

$$\therefore m(arc DE) + 70^\circ + 200^\circ = 360^\circ$$

...[From (i) and given]

$$\therefore m(arc DE) = 360^\circ - 270^\circ$$

$$\therefore m(arc DE) = 90^\circ$$

$$m(arc DEF) = m(arc DE) + m(arc EF) \quad \dots(\text{Arc addition property})$$

$$\therefore m(arc DEF) = 90^\circ + 70^\circ \quad \dots[\text{From (i) and given}]$$

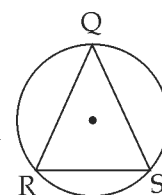
$$\therefore m(arc DEF) = 160^\circ$$

- In the adjoining figure, $\triangle QRS$ is an equilateral triangle.

Prove (1) $arc RS \cong arc QS \cong arc QR$

(2) $m(arc QRS) = 240^\circ$.

(3 marks)



Proof:

$\triangle QRS$ is an equilateral triangle. ... (Given)

$\therefore chord QR \cong chord RS \cong chord QS$... (Sides of equilateral $\triangle$ are equal)

$arc RS \cong arc QS \cong arc QR$... (i)

(In circle, congruent chords have corresponding minor arcs congruent)

Let $m(arc RS) = m(arc QS) = m(arc QR) = x$

... (ii) [From (i) and supposition]

$$\therefore m(arc RS) + m(arc QS) + m(arc QR) = 360^\circ$$

...(Measure of a circle is 360°)

$$\therefore x + x + x = 360^\circ$$

$$\therefore 3x = 360^\circ$$

$$\therefore x = 120^\circ$$

$$\therefore m(arc RS) = m(arc QS) = m(arc QR) = 120^\circ$$

... (iii)

$$\therefore m(arc QRS) = m(arc QR) + m(arc RS)$$

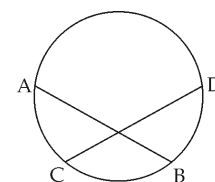
... (Arc addition property)

$$\therefore m(arc QRS) = 120^\circ + 120^\circ \quad \text{[From (iii)]}$$

$$\therefore m(arc QRS) = 240^\circ$$

- In the adjoining figure, $chord AB = chord CD$, prove that, $arc AC = arc BD$

(2 marks)



Proof:

$chord AB \cong chord CD$... (Given)

$arc ACB \cong arc DBC$

[In a circle, congruent chords have their corresponding minor arcs congruent]

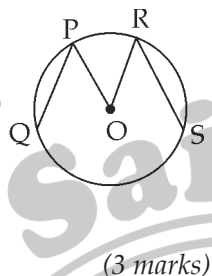
$$\begin{aligned} m(\text{arc ACB}) &= m(\text{arc DBC}) \\ m(\text{arc AC}) + m(\text{arc CB}) &= m(\text{arc CB}) + m(\text{arc BD}) \\ &\dots(\text{Arc addition property}) \\ m(\text{arc AC}) &= m(\text{arc BD}) \\ \text{arc AC} &\cong \text{arc BD} \end{aligned}$$

Problem Set - 3 (Textbook Pg No. 87)

- (14) In a circle with centre 'O', chord PQ $\cong$ chord RS.

If $m\angle POR = 70^\circ$ and $m(\text{arc RS}) = 80^\circ$, then find.

- (1) $m(\text{arc PR})$
- (2) $m(\text{arc QSR})$
- (3) $m(\text{arc QS})$



(3 marks)

Solution :

$$\begin{aligned} m(\text{arc PR}) &= m\angle POR \\ &\dots(\text{Definition of measure of minor arc}) \end{aligned}$$

$$\therefore m(\text{arc PR}) = 70^\circ \dots(i)$$

$$\begin{aligned} \text{chord PQ} &\cong \text{chord RS} && \dots(\text{Given}) \\ (\text{arc PQ}) &\cong (\text{arc RS}) && \dots(\text{In a circle, congruent chords have corresponding minor arcs congruent}) \end{aligned}$$

$$\therefore m(\text{arc PQ}) = 80^\circ \dots(ii)$$

$$\begin{aligned} m(\text{arc PR}) + m(\text{arc RS}) + m(\text{arc PQ}) + m(\text{arc QS}) &= 360^\circ \\ &\dots(\text{Measure of circle}) \end{aligned}$$

$$\therefore 70^\circ + 80^\circ + 80^\circ + m(\text{arc QS}) = 360^\circ$$

$$\therefore m(\text{arc QS}) = 360^\circ - 230^\circ$$

$$\therefore m(\text{arc QS}) = 130^\circ \dots(iii)$$

$$\begin{aligned} m(\text{arc QSR}) &= m(\text{arc QS}) + m(\text{arc SR}) \\ &\dots(\text{Arc addition property}) \end{aligned}$$

$$\therefore m(\text{arc QSR}) = 130^\circ + 80^\circ$$

$$\therefore m(\text{arc QSR}) = 210^\circ$$



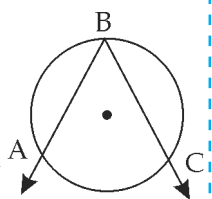
Points to Remember:

Inscribed angle :

An angle is said to be an inscribed angle, if

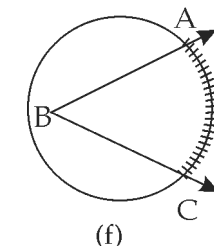
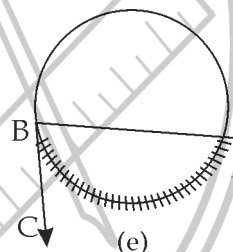
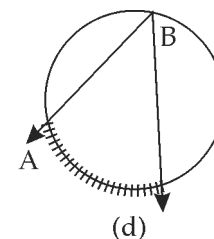
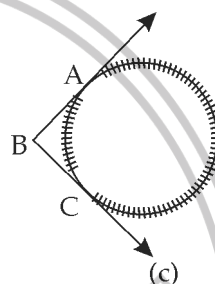
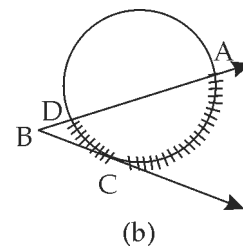
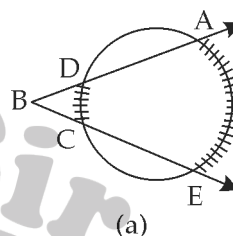
- (i) the vertex is on the circle
- (ii) both the arms are secants. In the adjoining figure, $\angle ABC$ is an inscribed angle, because vertex B lies on the circle and both the arms BA and BC are secants.

In other words, $\angle ABC$ is inscribed in arc AC.



Intercepted arc :

Given an arc of the circle and an angle, if each side of the angle contains an end point of the arc and all other points of the arc except the end points lie in the interior of the angle, then the arc is said to be intercepted by the angle.

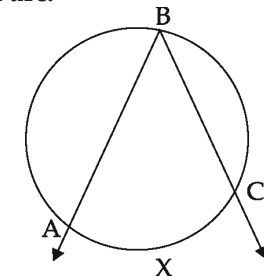


- (i) In figures (a), (b) and (c), $\angle ABC$ has its vertex B outside the circle and intercepts two arcs.
- (ii) In figures (d) and (e), $\angle ABC$ has its vertex on the circle and intercepts only one arc.
- (iii) In figure (f), $\angle ABC$ has its vertex B inside the circle and intercepts only one arc.

Inscribed Angle Theorem

The measure of an inscribed angle is half of the measure of its intercepted arc.

In the adjoining figure, $\angle ABC$ is an inscribed angle and arc AC is the intercepted arc.



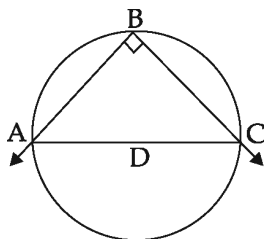
$$\therefore m\angle ABC = \frac{1}{2} m(\text{arc AC})$$

⇒ Corollary - 1 :

An angle inscribed in a semicircle is a right angle.

In the adjoining figure, $\angle ABC$ is inscribed in the semicircle ABC.

$$\therefore \boxed{m\angle ABC = 90^\circ}$$

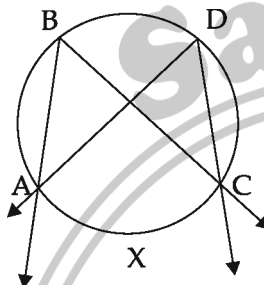


⇒ Corollary - 2 :

Angles inscribed in the same arc are congruent.

In the adjoining figure, $\angle ABC$ and $\angle ADC$, both are inscribed in the same arc ABC.

$$\therefore \boxed{\angle ABC \cong \angle ADC}$$



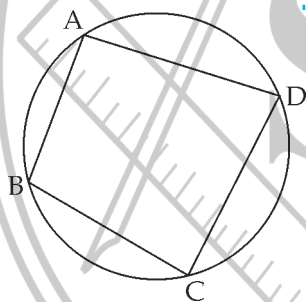
Note

Inscribed angles, $\angle ABC$ and $\angle ADC$ both intercept the same arc AC. $\therefore \angle ABC = \angle ADC$

• **Cyclic Quadrilateral :**

A quadrilateral whose all four vertices lie on a circle is called cyclic quadrilateral.

In the adjoining figure, $\square ABCD$ is cyclic, as all the four vertices A, B, C and D, lie on a circle.



• **Theorem**

Statement:

The opposite angles of a cyclic quadrilateral are supplementary.

Given : $\square ABCD$ is a cyclic quadrilateral.

To Prove :

$$m\angle ABC + m\angle ADC = 180^\circ$$

$$m\angle BAD + m\angle BCD = 180^\circ$$

Proof:

$$\left. \begin{aligned} m\angle ABC &= \frac{1}{2} m(\text{arc ADC}) \quad \dots(i) \\ m\angle ADC &= \frac{1}{2} m(\text{arc ABC}) \quad \dots(ii) \end{aligned} \right\} \begin{array}{l} \text{Inscribed} \\ \text{angle} \\ \text{theorem} \end{array}$$

Adding (i) and (ii), we get

$$m\angle ABC + m\angle ADC$$

$$= \frac{1}{2} m(\text{arc ADC}) + \frac{1}{2} m(\text{arc ABC})$$

$$\therefore m\angle ABC + m\angle ADC$$

$$= \frac{1}{2} [m(\text{arc ADC}) + m(\text{arc ABC})]$$

$$\therefore m\angle ABC + m\angle ADC = \frac{1}{2} \times 360$$

...(Measure of a circle is 360°)

$$\therefore m\angle ABC + m\angle ADC = 180^\circ$$

...(iii)

In $\square ABCD$,

$$m\angle BAD + m\angle BCD + m\angle ABC + m\angle ADC = 360^\circ$$

(Sum of measure of angles of a quadrilateral is 360°)

$$\therefore m\angle BAD + m\angle BCD + 180^\circ = 360^\circ \quad \dots[\text{From (iii)}]$$

$$\therefore m\angle BAD + m\angle BCD = 180^\circ$$

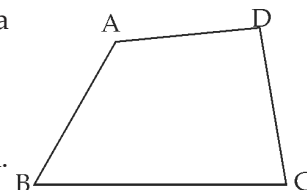
• **Converse of cyclic quadrilateral theorem :**

If the opposite angles of a quadrilateral are supplementary, then it is a cyclic quadrilateral.

In $\square ABCD$, if $m\angle A + m\angle C = 180^\circ$

or $m\angle B + m\angle D = 180^\circ$

then, $\square ABCD$ is a cyclic quadrilateral.



• **Corollary :**

An exterior angle of cyclic quadrilateral is congruent to the angle opposite to adjacent interior angle.

In the adjoining figure, $\square ABCD$ is cyclic $\angle DCE$ is an exterior angle.

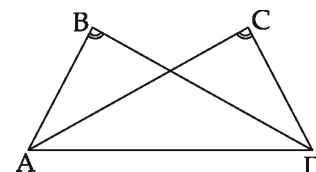
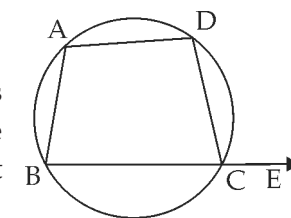
$$\therefore \angle DCE \cong \angle BAD$$

• If end points of a segment forms congruent angles on the same side of the segment, then the vertices of those angles and end points of the segment are concyclic.

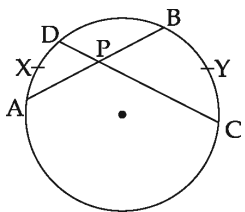
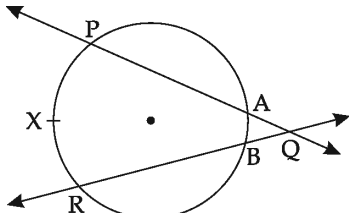
$$\angle ABD \cong \angle ACD$$

$\therefore$ Points A, B, C and D are concyclic.

Note: Concyclic means points lying on the same circle.

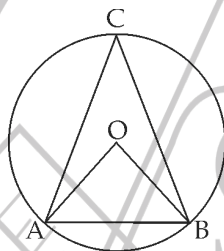


Two special properties

- (i)  $m \angle APD = \frac{1}{2} [m(\text{arc } AXD) + m(\text{arc } BYC)]$
- (ii)  $m \angle PQR = \frac{1}{2} [m(\text{arc } PXR) - m(\text{arc } AB)]$

Practice Set - 3.4 (Textbook Page No. 73)

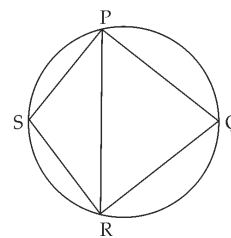
- (1) In the adjoining figure, point O is the centre of the circle. Length of chord AB is equal to the radius of the circle. Find (i) $\angle AOB$ (ii) $\angle ACB$ (iii) $m(\text{arc } AB)$ (iv) $m(\text{arc } ACB)$ (3 marks)



Solution :

- $OA = OB$... (i) (Radii of the same circle)
- $AB = \text{Radius of the given circle}$... (ii) (Given)
- $\therefore AB = OA = OB$... [From (i) and (ii)]
- $\therefore \triangle OAB$ is an equilateral triangle ... (By definition)
- $\therefore \angle AOB = 60^\circ$... (iii) (Measure of an angle of an equilateral triangle)
- $m(\text{arc } AB) = m \angle AOB$... (Definition of measure of minor arc)
- $\therefore m(\text{arc } AB) = 60^\circ$... (iv) [From (iii)]
- $\angle ACB = \frac{1}{2} (m \text{ arc } AB)$... (Inscribed angle theorem)
- $\angle ACB = \frac{1}{2} \times 60 = 30^\circ$... (v) [From (iv)]
- $m(\text{arc } ACB) + m(\text{arc } AB) = 360^\circ$... (Measure of a circle is 360°)
- $m(\text{arc } ACB) + 60^\circ = 360^\circ$... [From (v)]
- $m(\text{arc } ACB) = 360^\circ - 60^\circ$
- $\therefore m(\text{arc } ACB) = 300^\circ$

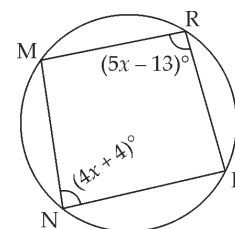
- (2) In the adjoining figure, $\square PQRS$ is a cyclic quadrilateral, side $PQ \cong$ side RQ . $\angle PSR = 110^\circ$. Find (i) $\angle PQR$ (ii) $m(\text{arc } PQR)$ (iii) $m(\text{arc } QR)$ (iv) $\angle PRQ$ (3 marks)



Solution :

- $\square PQRS$ is a cyclic quadrilateral (Given)
- $\therefore \angle PQR + \angle PSR = 180^\circ$... (Cyclic quadrilateral theorem)
- $\therefore \angle PQR + 110^\circ = 180^\circ$
- $\therefore \angle PQR = 180^\circ - 110^\circ$... (i)
- $m \angle PQR = 70^\circ$
- $\angle PSR = \frac{1}{2} m(\text{arc } PQR)$... (Inscribed angle theorem)
- $\therefore 110^\circ = \frac{1}{2} m(\text{arc } PQR)$... (v) [From (iv)]
- $\therefore m(\text{arc } PQR) = 220^\circ$... (ii)
- $\therefore$ In $\triangle PQR$, side $PQ \cong$ side RQ ... (Given)
- $\therefore \angle QPR \cong \angle QRP$... (iii) (Isosceles triangle theorem)
- In $\triangle PQR$,
- $\angle PQR + \angle PRQ + \angle QPR = 180^\circ$ (Sum of all angles of a triangle is 180°)
- $\therefore 70^\circ + \angle QPR + \angle QPR = 180^\circ$... (From (i) and (ii))
- $\therefore 2\angle QPR = 180^\circ - 70^\circ$
- $\therefore 2\angle QPR = 110^\circ$
- $\therefore \angle QPR = 55^\circ$... (iv)
- $\angle QPR = \frac{1}{2} m(\text{arc } QR)$... (Inscribed angle theorem)
- $\therefore 55 = \frac{1}{2} \times m(\text{arc } QR)$... [From (iv)]
- $\therefore m(\text{arc } QR) = 110^\circ$... [From (iii) and (iv)]
- $\therefore \angle PRQ = 55^\circ$

- (3) In cyclic $\square MRPN$, $\angle R = (5x - 13)^\circ$ and $\angle N = (4x + 4)^\circ$. Find the measure of $\angle R$ and $\angle N$. (2 marks)



Solution :

- $\square MRPN$ is a cyclic quadrilateral ... (Given)
- $\therefore \angle R + \angle N = 180^\circ$... (Cyclic quadrilateral theorem)
- $\therefore 5x - 13 + 4x + 4 = 180$
- $\therefore 9x - 9 = 180$

$$\begin{aligned}\therefore 9x &= 180 + 9 \\ \therefore 9x &= 189 \\ \therefore x &= \frac{189}{9} \\ \therefore x &= 21 \\ \therefore m\angle R &= 5x - 13 = 5 \times 21 - 13 = 105 - 13 = 92^\circ \\ \therefore m\angle N &= 4x + 4 = 4 \times 21 + 4 = 84 + 4 = 88^\circ\end{aligned}$$

(5) Prove that any rectangle is a cyclic quadrilateral.

(2 marks)

Given : $\square PQRS$ is a rectangle

To Prove : $\square PQRS$ is a cyclic quadrilateral



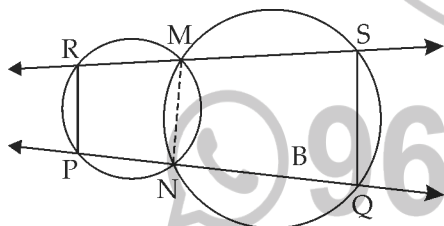
Proof : $\square PQRS$ is a rectangle ...(Given)
 $\therefore \angle P = \angle Q = \angle R = \angle S = 90^\circ$...(i) (Angles of rectangle)
 $\angle P + \angle R = 90^\circ + 90^\circ$ [From (i)]
 $\therefore \angle P + \angle R = 180^\circ$
 i.e. opposite angles of $\square PQRS$ are supplementary
 $\square PQRS$ is cyclic quadrilateral. (converse of cyclic quadrilateral theorem)

Problem Set - 3 (Textbook Pg No. 83)

(23) In the adjoining figure, two circles intersect each other at points M and N. Secants drawn from points M and N intersect circles at point R, S, P and Q as shown in the figure.

(3 marks)

To Prove : $\text{seg PR} \parallel \text{seg QS}$



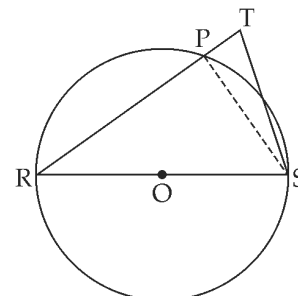
Construction : Draw seg MN.

Proof : $\square PNMR$ is a cyclic quadrilateral ...(Definition)
 $\angle MNQ$ is an exterior angle of $\square PNMR$...(Definition)
 $\therefore \angle MNQ \cong \angle PRM$...(i)
 $\square MNQS$ is cyclic quadrilateral ...(Definition)
 $\therefore \angle MNQ + \angle MSQ = 180^\circ$...(ii) (Opposite angles of a cyclic quadrilateral are supplementary)

$$\begin{aligned}\therefore \angle PRM + \angle MSQ &= 180^\circ \quad \dots[\text{From (i) and (ii)}] \\ \therefore \angle PRS + \angle RSQ &= 180^\circ \quad \dots(R - M - S) \\ \therefore \text{seg PR} &\parallel \text{seg QS} \quad \dots(\text{Interior angles test})\end{aligned}$$

Practice Set - 3.4 (Textbook Page No. 73)

(4) In the adjoining figure, seg RS is the diameter of the circle with centre 'O'. Point T is in the exterior of the circle. Prove that $\angle RTS$ is an acute angle. (3 marks)

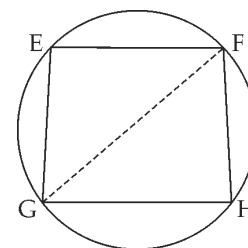


Construction : Draw seg PS

Proof :
 seg RS is the diameter of the circle. ...(Given)
 $\therefore \angle RPS = 90^\circ$...(i) (Diameter subtends a right angle at any point of the circle)
 $\angle RPS$ is an exterior angle of $\triangle PTS$...(Definition)
 $\therefore \angle RPS > \angle PTS$ (Exterior angle theorem)
 i.e. $\angle PTS < \angle RPS$
 $\therefore \angle PTS < 90^\circ$ [From (i)]
 i.e. $\angle RTS < 90^\circ$ (R - P - T)
 i.e. $\angle RTS$ is an acute angle.

Problem Set - 3 (Textbook Pg No. 83)

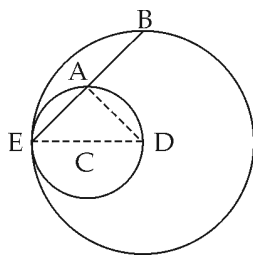
(17) In the adjoining diagram, chord $EF \parallel$ chord GH . Prove that chord $EG \cong$ chord FH .



[Complete the following for the proof] (2 marks)

Proof : Draw seg GF
 $\angle EFG = \angle FGH$...(i) (Alternate angles theorem)
 $\angle EFG = \frac{1}{2} m(\text{arc EG})$...(ii) (Inscribed angle theorem)
 $\angle FGH = \frac{1}{2} m(\text{arc FH})$...(iii) (Inscribed angle theorem)
 $\therefore m(\text{arc EG}) = m(\text{arc FH})$...[From (i), (ii) and (iii)]
 $\therefore \text{chord EG} \cong \text{chord FH}$...(In a circle, congruent arcs have their corresponding chords congruent)

- (19) A circle with centre C touches the circle with centre D internally in the point E. Point D lies on the smaller circle. Chord EB of the external circle intersects internal circle at point A. Prove that $\text{seg EA} \cong \text{seg AB}$.



(3 marks)

Construction : Draw seg DE and seg DA

Proof : E – C – D ... (Theorem on touching circles)

Seg DE is the diameter. (Definition)

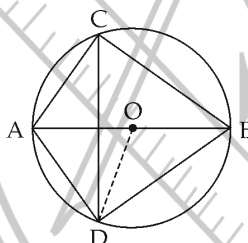
$\angle DAE = 90^\circ$... (i) (Diameter subtends a right angle at any point of circle other than its end points)

Consider circle with centre D.

$\therefore$ seg DA $\perp$ chord BE ... [From (i)]

$\therefore$ **seg EA $\cong$ seg AB** ... (Perpendicular drawn from the centre to the chord bisects the chord)

- (20) In the adjoining figure, seg AB is a diameter of a circle with centre O. Bisector of inscribed $\angle ACB$ intersects circle at point D. (3 marks)



Prove that: seg AD $\cong$ seg BD

Proof : Draw seg OD.

$\angle ACB = 90^\circ$ ($\because$ Angle inscribed in a semicircle)

$\angle DCB = 45^\circ$ ($\because$ CD bisects $\angle ACB$)

$m(\text{arc DB}) = 90^\circ$... (Inscribed angle theorem)

$\angle DOB = 90^\circ$... (i) (Definition of measure of an arc)

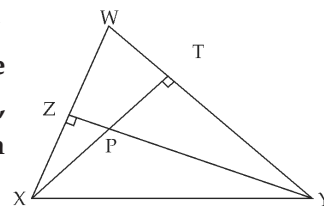
seg OA $\cong$ seg OB ... (ii) (Radii of same circle)

$\therefore$ seg OD is ... Perpendicular bisector of seg AB [From (i) and (ii)]

$\therefore$ **seg AD $\cong$ seg BD** ... ($\because$ Perpendicular bisector theorem)

Practice Set - 3.4 (Textbook Page No. 73)

- (6) In the adjoining figure, seg YZ and seg XT are altitudes of $\triangle WXY$, which intersect each other at point P.



(3 marks)

To Prove:

(i) $\square WZPT$ is a cyclic quadrilateral

(ii) Points X, Z, T and Y are concyclic points.

Proof :

$\angle XTW = 90^\circ$... (i) (Given)

$\angle YZW = 90^\circ$... (ii) (Given)

$\angle XTW + \angle YZW = 90^\circ + 90^\circ$... (Adding (i) and (ii))

$\therefore \angle PTW + \angle PZW = 180^\circ$ (X – P – T, Y – P – Z)

$\square WZPT$ is a cyclic quadrilateral ... (Convers of cyclic quadrilateral theorem)

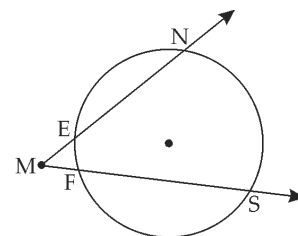
$\angle XZY = \angle XTY = 90^\circ$... (Given)

$\therefore \angle XZY = \angle XTY$

$\therefore$ seg XY subtends congruent angle at points Z and T which are on the same side of line XY.

$\therefore$ **Point X, Z, T and Y are concyclic points.**

- (7) In the adjoining figure, $m(\text{arc NS}) = 125^\circ$, $m(\text{arc EF}) = 37^\circ$. Find $m\angle NMS$. (1 mark)



Solution :

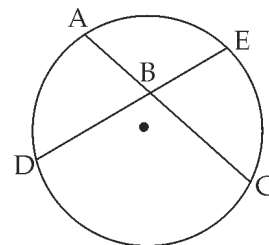
$$m\angle NMS = \frac{1}{2} [m(\text{arc NS}) - m(\text{arc EF})]$$

$$= \frac{1}{2} (125^\circ - 37^\circ)$$

$$= \frac{1}{2} \times 88$$

$$\therefore \boxed{m\angle NMS = 44^\circ}$$

- (8) In the adjoining figure, chord AC and chord DE intersect at point B. If $\angle ABE = 108^\circ$ and $m(\text{arc AE}) = 95^\circ$, then find $m(\text{arc DC})$.



(1 mark)

Solution :

$$m\angle ABE = \frac{1}{2} [m(\text{arc DC}) + m(\text{arc AE})]$$

$$\begin{aligned}\therefore 108 &= \frac{1}{2} [m(\text{arc DC}) + 95^\circ] \\ \therefore 216 &= m(\text{arc DC}) + 95^\circ \\ \therefore m(\text{arc DC}) &= 216 - 95^\circ \\ \therefore m(\text{arc DC}) &= 121^\circ\end{aligned}$$

Problem Set - 3 (Textbook Pg No. 83)

- (25) In the adjoining diagram, seg BD and seg CE are altitudes.

To Prove that:

- (i) $\square AEFD$ is cyclic quadrilateral
(ii) Points B, E, D, C are con-cyclic points.

[Complete the following for the proof] (3 marks)

Proof: $\angle AEF = 90^\circ$... (i) (Given)
 $\angle ADF = 90^\circ$... (ii) (Given)

Adding (i) and (ii),

$$\angle AEF + \angle ADF = 90^\circ + 90^\circ$$

$$\angle AEF + \angle ADF = 180^\circ$$

$\square AEFD$ is cyclic

(Converse of cyclic quadrilateral theorem)

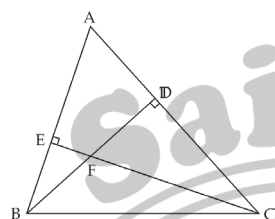
$$\angle BEC = \angle BDC = 90^\circ$$

... (Given)

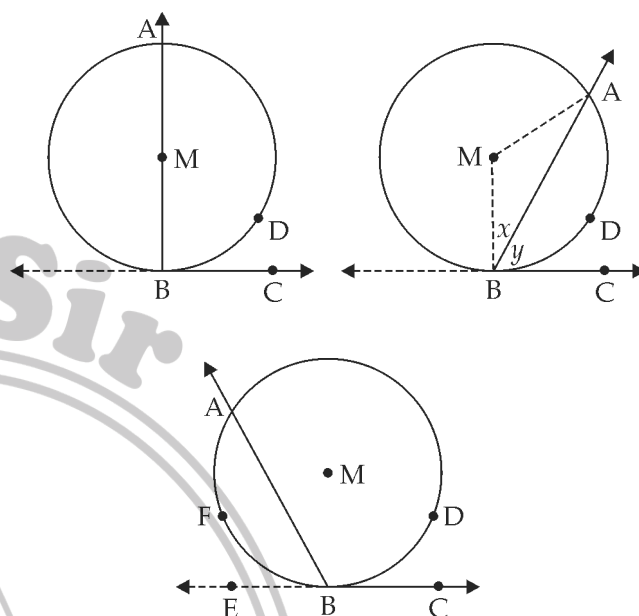
$$\therefore \angle BEC = \angle BDC$$

$\therefore$ Seg BC subtends congruent angles at points E and D which are on the same side of line BC.

$\therefore$ Points B, E, D and C are con-cyclic.



side touches the circle and the other intersects the circle in two points, then the measure of the angle is half the measure of its intercepted arc.



In the above three figures,

$\angle ABC$ has its vertex B on the circle, line BC is tangent to circle at B and ray BA is a secant.

$$\therefore m\angle ABC = \frac{1}{2} m(\text{arc ADB})$$

• **Segment of a circle:** A secant divides the circular region into two parts. Each part is called a segment of the circle.

• **Alternate segment:** Each of the two segments formed by the secant of a circle is called alternate segment in relation with the other.

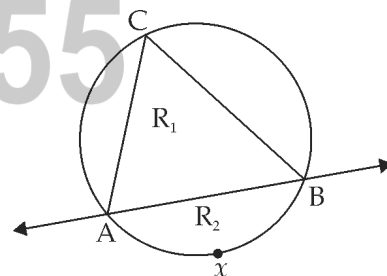
• **Angle formed in a segment:** An angle inscribed in the arc of a segment is called an angle formed in that segment.

In the adjoining figure, secant AB divides the circular region into two segments R_1 and R_2 .

R_1 and R_2 are alternate segments in relation with each other.

$\angle ACB$ is inscribed in arc ACB of segment R_1 .

$\therefore \angle ACB$ is an angle formed in segment R_1 .



Points to Remember:

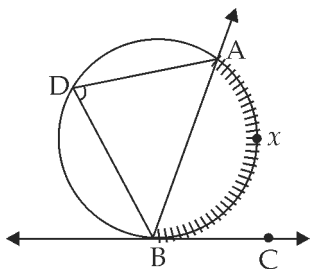
(1) $m\angle APD = \frac{1}{2} [m(\text{arc AXD}) + m(\text{arc BYC})]$
 $m\angle APC = \frac{1}{2} [m(\text{arc DB}) + m(\text{arc AC})]$
 $m\angle PQR = \frac{1}{2} [m(\text{arc PXR}) - m(\text{arc AB})]$

• Tangent Secant Theorem

If an angle with its vertex on the circle whose one

Angles in Alternate Segment

If a line touches a circle and from the point of contact a chord is drawn, then the angles which this chord makes with the given line is equal respectively to the angle formed in the corresponding alternate segment.



In the above figure,

$$m \angle ABC = \frac{1}{2} m (\text{arc AXB}) \quad \dots(i) \quad \dots(\text{Tangent secant theorem})$$

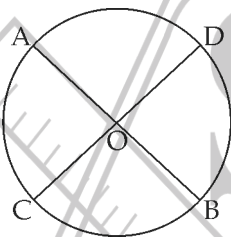
$$m \angle ADB = \frac{1}{2} m (\text{arc AXB}) \quad \dots(ii) \quad \dots(\text{Inscribed angle theorem})$$

$$\therefore m \angle ABC = m \angle ADB \quad \dots[\text{From (i) and (ii)}]$$

$$\therefore \angle ABC \cong \angle ADB$$

Theorem

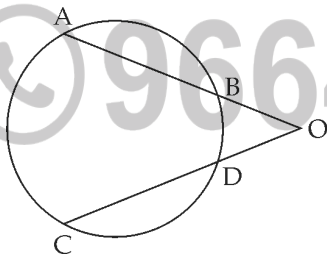
If two secants of a circle intersect inside or outside the circle then the area of the rectangle formed by the two line segments corresponding to one secant is equal in area to the rectangle formed by the two line segments corresponding to the other.



In the adjoining figure, chords AB and CD intersect each other at point O inside the circle.

$$\therefore OA \times OB = OC \times OD$$

In the adjoining figure, chords AB and CD intersect each other at point P outside the circle.



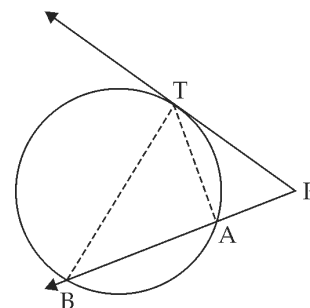
$$\therefore OA \times OB = OC \times OD$$

Tangent Secant Segment Theorem

Statement: If a secant and a tangent of a circle intersect in a point outside the circle, then the area of the rectangle formed by the two line segments corresponding to the secant is equal to the area of the square formed by the line segment corresponding to the other tangent.

Given :

- line PAB is a secant intersecting the circle at points A and B.
- line PT is a tangent to the circle at point T.



To Prove: $PA \times PB = PT^2$

Construction: Draw seg BT and seg AT.

Proof :

In $\triangle PTA$ and $\triangle PBT$,

$$\angle TPA \cong \angle BPT \quad (\text{Common angle})$$

$$\angle PTA \cong \angle PBT \quad (\text{Angles in alternate segment})$$

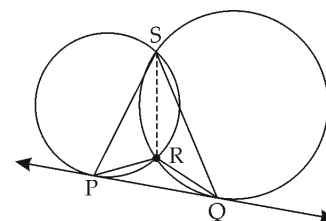
$$\therefore \triangle PTA \sim \triangle PBT \quad (\text{By AA test of similarity})$$

$$\therefore \frac{PT}{PB} = \frac{PA}{PT} \quad (\text{Corresponding sides of similar triangles})$$

$$\therefore PA \times PB = PT^2$$

Problem Set - 3 (Textbook Pg No. 83)

- (22) In the adjoining figure, two circles intersect each other in point R and point S. Line PQ is a common tangent touching circle at points P and Q.



is a common tangent touching circle at points P and Q. (2 marks)

Prove that: $\angle PRQ + \angle PSQ = 180^\circ$

Construction: Draw seg RS

$$\text{Proof: } \left. \begin{array}{l} \angle PSR \cong \angle RPQ \quad \dots(i) \\ \angle QSR \cong \angle RQP \quad \dots(ii) \end{array} \right\} \quad (\text{Angles in alternate segments})$$

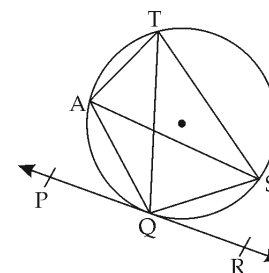
In $\triangle PRQ$,

$$\angle PRQ + \angle RPQ + \angle RQP = 180^\circ \quad \dots(\text{sum of angles of a triangle is } 180^\circ)$$

$$\therefore \angle PRQ + \angle PSR + \angle QSR = 180^\circ \quad \dots[\text{From (i) and (ii)}]$$

$$\therefore \angle PRQ + \angle PSQ = 180^\circ \quad \dots(\text{Angle addition property})$$

- (13) In the adjoining figure, line PR touches the circle at the point Q. Using the information given in the diagram, answer the following questions.

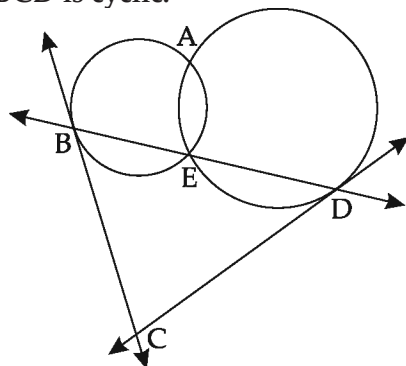


- (i) What is the sum of $\angle TAQ$ and $\angle TSQ$?
(ii) Write names of angles congruent to $\angle AQP$.
(iii) Write names of angles congruent to $\angle QTS$.
(iv) If $\angle TAQ = 65^\circ$, then find $\angle TQS$ and arc TS.
(v) If $\angle AQP = 42^\circ$ and $\angle SQR = 58^\circ$, then find $\angle ATS$.

Solution : (4 marks)

- (i) $\square TAQS$ is a cyclic quadrilateral ... (Definition)
 $\therefore \angle TAQ + \angle TSQ = 180^\circ$... (Cyclic quadrilateral theorem)
(ii) $\angle ATQ \cong \angle AQP$
 $\angle ASQ \cong \angle AQP$ } (Angles in alternate segments)
(iii) $\angle QAS \cong \angle QTS$... (Angles inscribed in the same arc)
 $\angle RQS \cong \angle QTS$... (Angles in alternate segment)
(iv) $\angle TQS \cong \angle TAS$... (Angles inscribed in the same arc)
 $\angle TAS = 65^\circ$... (Given)
 $\therefore \angle TQS = 65^\circ$
 $\angle TQS = \frac{1}{2} m(\text{arc TS})$... (Inscribed angle theorem)
 $\therefore 65^\circ = \frac{1}{2} m(\text{arc TS})$
 $\therefore m(\text{arc TS}) = 65^\circ \times 2$
 $\therefore m(\text{arc TS}) = 130^\circ$
(v) $\angle ATQ \cong \angle AQP$... (Angles in alternate segments)
 $\therefore \angle ATQ = 42^\circ$... (i)
 $\angle STQ \cong \angle SQR$... (Angles in alternate segments)
 $\therefore \angle STQ = 58^\circ$... (ii)
 $\angle ATS = \angle ATQ + \angle STQ$... (Angle addition property)
 $\angle ATS = 42^\circ + 58^\circ$... [From (i) and (ii)]
 $\therefore \angle ATS = 100^\circ$

- (24) In the adjoining figure, two circles intersect each other at points A and E. Their common secant through E intersects the circle at points B and D. The tangents of the circles at point B and D intersect each other at point C. Prove that $\square ABCD$ is cyclic.



Proof: $\angle BAE = \angle EBC$... (i)
 $\angle DAE = \angle EDC$... (ii) } (Angles in alternate segments)

In $\triangle BCD$,

$$\angle BCD + \angle DBC + \angle BDC = 180^\circ \text{ ... (Sum of angle of a triangle is } 180^\circ)$$

$$\therefore \angle BCD + \angle EBC + \angle EDC = 180^\circ \text{ ... (B-E-D)}$$

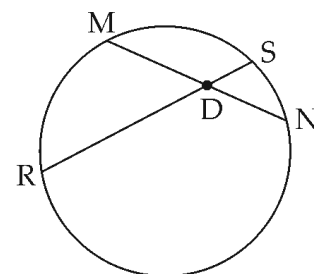
$$\angle BCD + \angle BAE + \angle DAE = 180^\circ \text{ ... [From (i) \& (ii)]}$$

$$\angle BCD + \angle BAD = 180^\circ \text{ ... (Angle addition property)}$$

$\therefore \square ABCD$ is a cyclic ... (Converse of cyclic quadrilateral Theorem)

Practice Set - 3.5 (Textbook Page No. 82)

- (2) In the adjoining figure, chord MN and RS intersect each other at point D.



- (i) If $RD = 15$, $DS = 4$, $MD = 8$ then $DN = ?$
(ii) If $RS = 18$, $MD = 9$, $DN = 8$, then find DS .

(3 marks)

Solution :

- (i) Chord MN and Chord RS intersect each other in the interior of the circle at point D.

$$\therefore DM \times DN = DR \times DS \text{ ... (Theorem of internal division of chords)}$$

$$\therefore 8 \times DN = 15 \times 4$$

$$\therefore DN = \frac{15 \times 4}{8}$$

$$\therefore \boxed{DN = 7.5 \text{ units}}$$

- (ii) Let $DS = x$... (i) (Supposition)
 $RS = DR + DS$... (R - D - S))

$$\therefore 18 = DR + x$$

$$\therefore DR = (18 - x) \text{ ... (ii)}$$

Chord MN and Chord RS intersect each other in the interior of the circle at point D.

$$\therefore DM \times DN = DR \times DS \text{ (Theorem of internal division of chords)}$$

$$\therefore 9 \times 8 = x(18 - x)$$

$$\therefore 72 = 18x - x^2$$

$$\therefore x^2 - 18x + 72 = 0$$

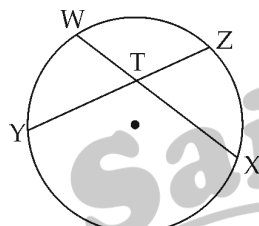
$$\therefore x^2 - 12x - 6x + 72 = 0$$

$$\therefore x(x - 12) - 6(x - 12) = 0$$

$$\begin{aligned}\therefore (x - 12)(x - 6) &= 0 \\ \therefore x - 12 = 0 \text{ or } x - 6 &= 0 \\ \therefore x = 12 \text{ or } x &= 6 \\ \therefore \boxed{DS = 12 \text{ units}} \text{ or } \boxed{DS = 6 \text{ units}}\end{aligned}$$

Problem Set - 3 (Textbook Pg No. 87)

- (15) In the adjoining figure,
 $m(\text{arc } WY) = 44$,
 $m(\text{arc } ZX) = 68$ then,
 (i) Find $m\angle ZTX$ and $m(\text{arc } WZ)$
 (ii) If $l(WT) = 4.8$, $l(TX) = 8$,
 $l(YT) = 6.4$, then find $l(TZ)$
 (iii) If $l(WX) = 12.8$, $l(YT) = 6$,
 $l(TX) = 6.4$, then find $l(WT)$



(3 marks)

Solution:

$$\begin{aligned}\text{(i) } m\angle ZTX &= \frac{1}{2} [m(\text{arc } WY) + m(\text{arc } ZX)] \\ &= \frac{1}{2} [44 + 68] \\ &= \frac{1}{2} \times 112 \\ &= 56^\circ\end{aligned}$$

$$\boxed{m\angle ZTX = 56^\circ}$$

- (ii) $WT \times TX = TZ \times YT$... (Theorem on internal division of chords)

$$\begin{aligned}\therefore 4.8 \times 8 &= TZ \times 6.4 \\ \therefore TZ &= \frac{4.8 \times 8}{6.4}\end{aligned}$$

$$\boxed{TZ = 6 \text{ units}}$$

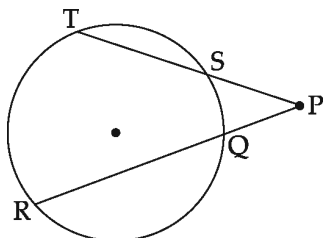
- (iii) $WX = WT + TX$... (W - T - X)

$$\begin{aligned}\therefore 12.8 &= WT + 6.4 \\ \therefore WT &= 12.8 - 6.4\end{aligned}$$

$$\boxed{WT = 6.4 \text{ units}}$$

Practice Set - 3.5 (Textbook Page No. 82)

- (4) In the adjoining figure,
 if $PQ = 6$, $QR = 10$,
 $PS = 8$, then find TS .
 (2 marks)



(P - Q - R)

Solution :

$$PR = PQ + QR$$

$$\therefore PR = 6 + 10$$

$$\therefore PR = 16 \text{ units}$$

... (i)

Secants PST and PQR intersect each other in the exterior of the circle at point P.

$$\therefore PT \times PS = PR \times PQ \quad \dots (\text{Theorem of external division of chords})$$

$$\therefore PT \times 8 = 16 \times 6 \quad \dots [\text{From (i) and given}]$$

$$\therefore PT = \frac{16 \times 6}{8}$$

$$\therefore PT = 12 \text{ units} \quad \dots (\text{ii})$$

$$\therefore PT = PS + TS \quad \dots (P - S - T)$$

$$\therefore 12 = 8 + TS$$

$$\therefore TS = 12 - 8$$

$$\therefore \boxed{TS = 4 \text{ units}}$$

Problem Set - 3 (Textbook Pg No. 88)

- (16) In the adjoining figure,

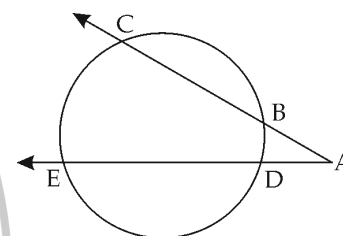
- (i) If $m(\text{arc } CE) = 54^\circ$,
 $m(\text{arc } BD) = 23^\circ$,
 then find $\angle CAE$

- (ii) If $AB = 4.2$,
 $BC = 5.4$

$AE = 12$, then find AD

- (iii) If $AB = 3.6$, $AC = 9.0$, $AD = 5.4$, then find AE .

(3 marks)



Solution:

$$\begin{aligned}\text{(i) } \angle CAE &= \frac{1}{2} [m(\text{arc } CE) - m(\text{arc } BD)] \\ &= \frac{1}{2} (54 - 23) \\ &= \frac{1}{2} \times 31 \\ &= 15.5^\circ\end{aligned}$$

$$\boxed{\angle CAE = 15.5^\circ}$$

- (ii) $AD \times AE = AB \times AC$... (Theorem of external division of chords)

$$\begin{aligned}\therefore AD \times 12 &= 4.2 \times 9.6 & AC &= AB + BC \\ \therefore AD &= \frac{4.2 \times 9.6}{12} & &= 4.2 + 5.4 \\ & & &= 9.6\end{aligned}$$

$$\therefore \boxed{AD = 3.36 \text{ units}}$$

- (iii) $AE \times AD = AC \times AB$... (Theorem of external division of chords)

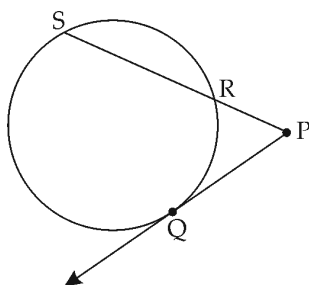
$$\therefore AE \times 5.4 = 9 \times 3.6$$

$$\therefore AE = \frac{9 \times 3.6}{5.4}$$

$$\therefore \boxed{AE = 6 \text{ units}}$$

Practice Set - 3.5 (Textbook Page No. 82)

- (1) In the adjoining figure, point Q is the point of contact. If $PQ = 12$, $PR = 8$, then find PS and RS. (2 marks)



Solution :

Tangent PQ and secant PRS intersect each other at point P.

$$\therefore PQ^2 = PR \times PS \quad \dots(\text{Tangent secant segments theorem})$$

$$\therefore 12^2 = 8 \times PS$$

$$\therefore \frac{12 \times 12}{8} = PS$$

$$\therefore \boxed{PS = 18 \text{ units}}$$

$$PS = PR + RS$$

$$\therefore 18 = 8 + RS$$

$$\therefore RS = 18 - 8$$

$$\therefore \boxed{RS = 10 \text{ units}}$$

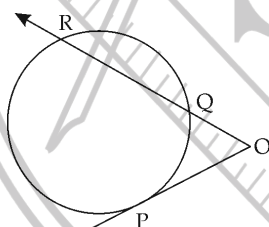
Problem Set - 3 (Textbook Pg No. 88)

- (18) In the adjoining diagram,

(i) $m(\text{arc PR}) = 140^\circ$, $m\angle POR = 36^\circ$. Find $m(\text{arc PQ})$

(ii) If $OP = 7.2$, $OQ = 3.2$, then find QR

(iii) If $OP = 7.2$, $OR = 16.2$ then find QR (3 marks)



Solution:

(i) $\angle POR = \frac{1}{2} [m(\text{arc PR}) - m(\text{arc PQ})]$

$$\therefore 36^\circ = \frac{1}{2} [140^\circ - m(\text{arc PQ})]$$

$$\therefore 36 \times 2 = 140^\circ - m(\text{arc PQ})$$

$$\therefore m(\text{arc PQ}) = 140^\circ - 72^\circ$$

$$\therefore \boxed{m \text{ arc PQ} = 68^\circ}$$

(ii) $OP^2 = OQ \times OR \quad \dots(\text{Tangent secant segment theorem})$

$$\therefore 7.2 \times 7.2 = 3.2 \times OR$$

$$\therefore OR = \frac{7.2 \times 7.2}{3.2}$$

$$\therefore \boxed{OR = 16.2 \text{ units}}$$

$$OR = OQ + QR$$

$$\dots(O - Q - R)$$

$$\therefore 16.2 = 3.2 + QR$$

$$\therefore QR = 16.2 - 3.2$$

$$\therefore QR = 13 \text{ units}$$

(iii) $OP^2 = OQ \times OR \quad \dots(\text{Tangent secant segment theorem})$

$$\therefore 7.2 \times 7.2 = OQ \times 16.2$$

$$\therefore OQ = \frac{7.2 \times 7.2}{16.2}$$

$$\therefore OQ = 3.2$$

$$OQ + QR = OR$$

$$\dots(O - Q - R)$$

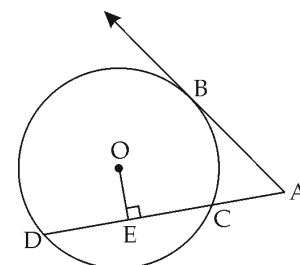
$$\therefore 3.2 + QR = 16.2$$

$$\therefore QR = 16.2 - 3.2$$

$$\therefore QR = 13 \text{ units}$$

Practice Set - 3.5 (Textbook Page No. 82)

- (3) In the adjoining figure, point B is the point of contact and point O is the centre of the circle. Seg OE $\perp$ Seg AD, if $AB = 12$, $AC = 8$, then find (i) AD (ii) DC and (iii) DE (3 marks)



Solution :

Tangent AB and Secant ACD intersect at point A.

$$\therefore AB^2 = AC \times AD \quad \dots(\text{Tangent secant segments theorem})$$

$$\therefore 12^2 = 8 \times AD$$

$$\therefore \frac{12 \times 12}{8} = AD$$

$$\therefore \boxed{AD = 18 \text{ units}}$$

$$\therefore AD = AC + DC$$

$$\dots[A - C - D]$$

$$\therefore 18 = 8 + DC$$

$$\therefore DC = 18 - 8$$

$$\therefore \boxed{DC = 10 \text{ units}}$$

$$\therefore \text{Seg DE} \perp \text{chord CD}$$

$$\dots(\text{Given})$$

$$\therefore OE = \frac{1}{2} \times CD \quad (\text{Perpendicular drawn from the centre of the circle to the chord bisects the chord})$$

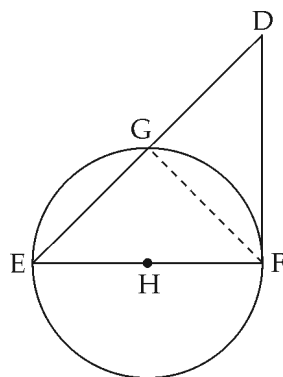
$$\therefore DE = \frac{1}{2} \times 10$$

$$[\text{From (i)}]$$

$$\therefore \boxed{DE = 5 \text{ units}}$$

- (5) In the adjoining figure, seg EF is the diameter of the circle with centre H. Line DF is tangent at point F. If r is the radius of the circle, then prove that $DE \times GE = 4r^2$

(3 marks)



To prove: $DE \times GE = 4r^2$

Construction: Draw seg GF

Proof: $\angle EGF = 90^\circ$... (i) (Diameter subtends a right angle at any point on the circle)

In $\triangle EFD$,

$\angle EFD = 90^\circ$... (Tangent and radius are perpendicular at the point of contact)

seg $FG \perp$ hypotenuse ED ... [From (i)]

$\therefore \triangle EFD \sim \triangle EGF \sim \triangle FGD$... (ii) (similarity in right angled triangles)

$\therefore \triangle EFD \sim \triangle EGF$... [From (ii)]

$$\therefore \frac{EF}{GE} = \frac{DE}{EF} \quad \dots (\text{c.s.c.t.})$$

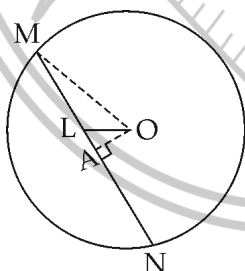
$$\therefore DE \times GE = EF^2$$

$$\therefore DE \times GE = (2r)^2 \quad \dots (\text{diameter is twice the radius})$$

$$\therefore DE \times GE = 4r^2$$

Problem Set - 3 (Textbook Pg No. 89)

- (21) In the adjoining figure, seg MN is a chord of a circle with centre O. $l(MN) = 25$. Point L on chord MN such that $l(ML) = 9$ and $l(OL) = 5$, then find radius of the circle.



(3 marks)

Construction:

Draw seg OM and seg $OA \perp$ chord MN

(M-A-N)

Solution:

Seg $OA \perp$ chord MN ... (Construction)

$$\therefore MA = \frac{1}{2} \times MN \quad \dots (\text{Perpendicular drawn from the centre to the chord bisects the chord})$$

$$\therefore MA = \frac{1}{2} \times 25 = 12.5 \text{ units} \quad \dots (i)$$

$$MA = ML + LA \quad \dots (M-L-A)$$

$$\therefore 12.5 = 9 + LA \quad \dots [\text{From (i) and given}]$$

$$\therefore 12.5 - 9 = LA$$

$$\therefore LA = 3.5 \text{ units} \quad \dots (ii)$$

In $\triangle OAL$, $\angle OAL = 90^\circ$... (Construction)

$$\therefore OL^2 = OA^2 + LA^2 \quad \dots (\text{Pythagoras theorem})$$

$$\therefore 5^2 = OA^2 + (3.5)^2 \quad \dots [\text{From (ii) and given}]$$

$$\therefore OA^2 = 25 - 12.25$$

$$\therefore OA^2 = 12.75 \quad \dots (iii)$$

In $\triangle OAM$, $\angle OAM = 90^\circ$... (Construction)

$$\therefore OM^2 = OA^2 + MA^2 \quad \dots (\text{Pythagoras theorem})$$

$$\therefore OM^2 = 12.75 + (12.5)^2 \quad \dots [\text{From (iii) and given}]$$

$$\therefore OM^2 = 12.75 + 156.25$$

$$\therefore OM^2 = 169$$

$$\therefore OM = 13 \text{ units} \quad \dots (\text{Taking square roots})$$

Radius of the circle is 13 units

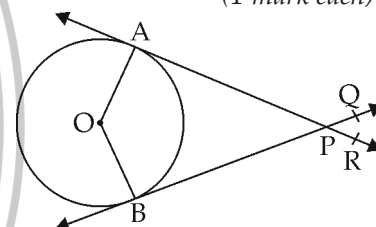
Problem Set - 3 (Textbook Pg No. 89)

MCQ's

- (1) Choose the correct alternative for each of the following. (1 mark each)

(1) $\angle QPR = 60^\circ$

$$\therefore \angle AOB = \dots$$



(A) 60° (B) 90° (C) 120° (D) Can not be found

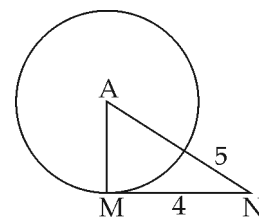
- (2) Angle between external end point of radius and tangent is

(A) 90° (B) Acute angle
(C) Obtuse angle (D) Can not say

- (3) Point P is on the circle. AB is diameter of the circle, $\angle APB$ is

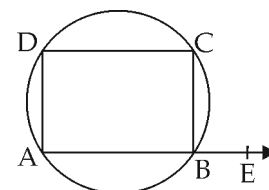
(a) Reflex angle (b) Acute angle
(c) Right angle (d) Obtuse angle

- (4) MN is tangent at M and AM is radius. Find AM.



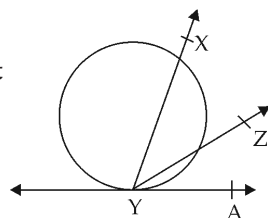
(A) 6 (b) 3 (c) $3\sqrt{3}$ (d) 1

- (5) $\angle ADC = 80^\circ$, then $\angle CBE = ?$



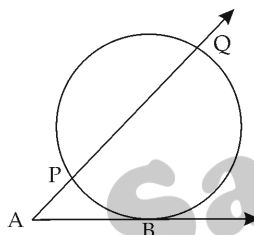
(A) 100° (B) 10° (C) 80° (D) 280°

- (6) $\angle XYZ = 40^\circ$, $\angle AYZ = 20^\circ$,
line AY is tangent at
point Y.
 $\therefore m(\text{arc XY}) = \dots\dots\dots$



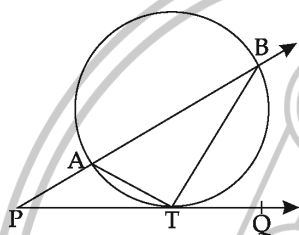
- (A) 80° (B) 40° (C) 60° (D) 120°

- (7) AB is tangent at B.
 $AB = 12$, $AP = 6$
 $\therefore PQ = \dots\dots\dots$



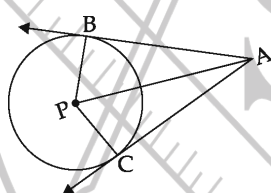
- (A) 18 (B) 6 (C) 12 (D) 20

- (8) Line PT is tangent at
point T. Which of the
following is true?



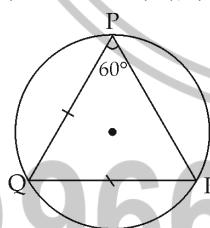
- (A) $\angle ABT \cong \angle APT$ (B) $\angle ABT \cong \angle BAT$
(C) $\angle BAT \cong \angle BTQ$ (D) None of the (A), (B), (C)

- (9) A circle with centre P.
Line AB and line AC
are tangents from point
A at points B and C
respectively. Which of
the following is/are true?



- (A) $\angle BPA \cong \angle CPA$ (B) $\angle BAP \cong \angle CAP$
(C) $\angle PBA \cong \angle PCA$ (D) All three (A), (B), (C)

- (10) In adjoining figure,
 $PQ = QR$. $\angle P = 60^\circ$
 $\therefore m(\text{arc PR}) = \dots\dots\dots$



- (A) 120° (B) 60° (C) 90° (D) 240°

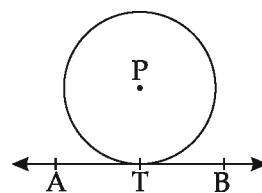
ANSWERS

- (1) (C) 120° (2) (A) 90°
(3) (C) Right angle (4) (B) 3
(5) (C) 80° (6) (D) 120°
(7) (A) 18 (8) (C) $\angle BAT \cong \angle BTQ$
(9) (D) All three (A), (B), (C) (10) (A) 120°

PROBLEMS FOR PRACTICE

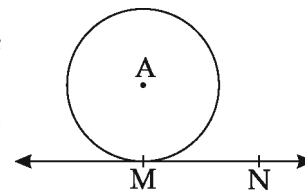
Based on Practice Set 3.1

- (1) In the adjoining figure,
point P is the centre of
the circle and line AB is
the tangent to the circle
at T. The radius of the
circle is 6 cm. Find PB if
 $\angle TPB = 60^\circ$.

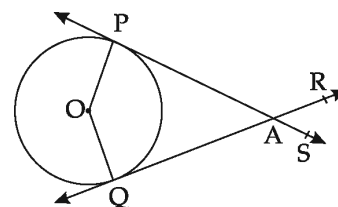


(2 marks)

- (2) In the adjoining figure,
point A is the centre of
the circle $AN = 10$ cm.
Line NM is tangent at
M. $MN = 5$ cm. Find the
radius. (2 marks)

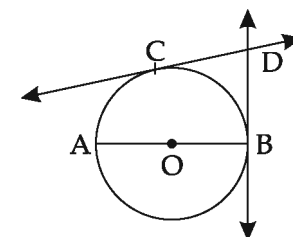


- (3) A circle with centre.
O Point A is in the
exterior of the circle.
Line AP and line
AQ are tangents at
point P and point Q
respectively P-A-S, Q-A-R $\angle PAR = 130^\circ$. Find
 $\angle AOP$. (2 marks)



- (4) Two tangents TP and TQ are drawn to a circle
with centre O from an external point T. Prove that
 $\angle PTQ = 2\angle OPQ$. (2 marks)

- (5) In the adjoining figure,
O is the centre and seg
AB is a diameter. At
point C on the circle, the
tangent CD is drawn.
Line BD is tangent at B.
Prove that seg OD $\parallel$ seg
AC. (2 marks)



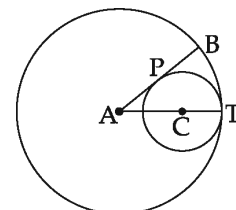
(2 marks)

Based on Practice Set 3.2

- (6) If two circles of radii 5 cm and 3 cm touch
externally. Find the distance between them. (1 mark)
(7) Find the distance between two internally
touching circles whose radii are 10 cm and 2 cm.
(1 marks)

- (8) The circles which are not congruent touch
externally. The sum of their areas is 130π cm^2 and
distance between their centres is 14 cm. Find the
radii of the two circles. (2 marks)

- (9) In the adjoining figure
circles with centres A
and C touch internally
at point T. Line AB is
tangent to the smaller

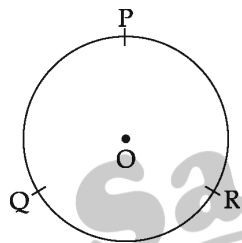


circle at point P. Point B lies on the bigger circle. Radii are 16 cm and 6 cm. Find AP. (2 marks)

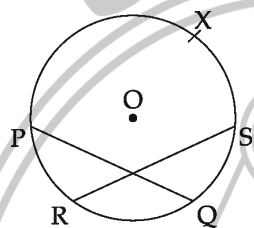
- (10) The radii of two circles are 25 cm and 9 cm. The distance between their centres is 34 cm. Find the length of the common tangent segment to these circles. (2 marks)

Based on Practice Set 3.3

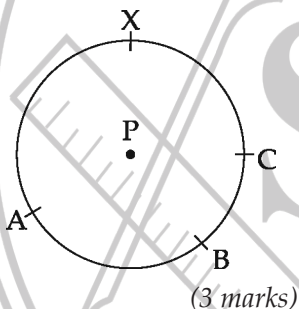
- (11) In the adjoining figure, A circle with centre 'O' arc PQ = arc QR = arc PR. Find measure of each of above arcs. (2 marks)



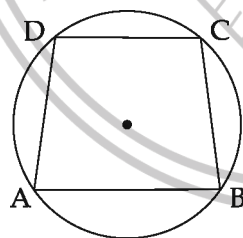
- (12) A circle with centre 'O' chord PQ $\cong$ chord RS. $m(\text{arc PXQ}) = 260^\circ$. Then find $m(\text{arc RXS})$. (1 marks)



- (13) A circle with centre P. arc AB = arc BC and arc AXC = 2 arc AB. Find measure of arc AB, arc BC and arc AXC. Prove chord AB $\cong$ chord BC. (3 marks)

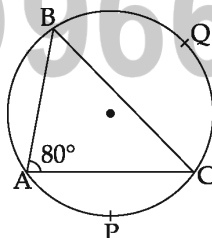


- (14) In the adjoining figure, chord AD $\cong$ chord BC. $m(\text{arc ADC}) = 100^\circ$, $m(\text{arc CD}) = 60^\circ$. Find $m(\text{arc AB})$ and $m(\text{arc BC})$. (3 marks)

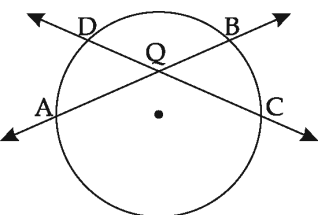


Based on Practice Set 3.4

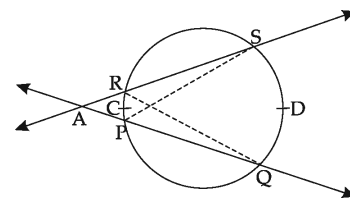
- (15) If $m(\text{arc APC}) = 60^\circ$ and $\angle BAC = 80^\circ$. Find (a) $\angle ABC$ (b) $m(\text{arc BQC})$. (2 marks)



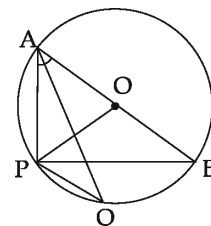
- (16) Chords AB and CD of a circle intersect in point Q in the interior of a circle of as shown in the figure. If $m(\text{arc AD}) = 20^\circ$, and $m(\text{arc BC}) = 36^\circ$, then find $\angle BQC$. (3 marks)



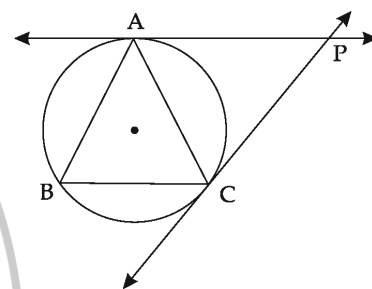
- (17) Secants containing chords RS and PQ of a circle intersect each other in point A in the exterior of a circle. If $m(\text{arc PCR}) = 26^\circ$ and $m(\text{arc QDS}) = 48^\circ$, then find (1) $\angle AQR$ (2) $\angle SPQ$ (3) $\angle RAQ$. (3 marks)



- (18) In the adjoining figure, O is the centre of the circle. Find the value of $\angle ABP$ if $\angle POB = 90^\circ$ (2 marks)

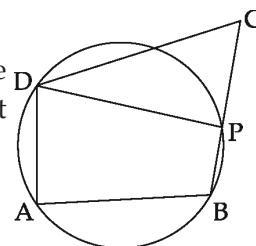


- (19) Tangents drawn at points A and C of a circle intersect each other in point P. If $\angle APC = 50^\circ$, then find $\angle ABC$. (2 marks)

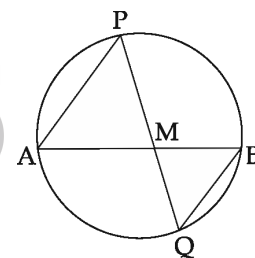


- (20) If two consecutive angles of cyclic quadrilateral are congruent, then prove that one pair of opposite sides is congruent and other is parallel. (4 marks)

- (21) $\square ABCD$ is a parallelogram. Side BC intersects circle at point P. Prove that $DC = DP$. (3 marks)

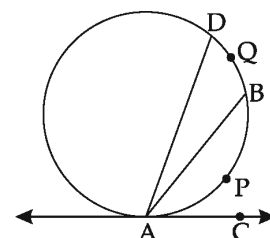


- (22) In the adjoining figure, chord PQ and chord AB intersect at point M. If $PM = AM$, then prove that $BM = QM$. (2 marks)

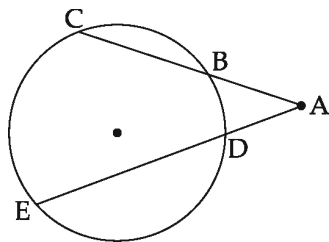


Based on Practice Set 3.5

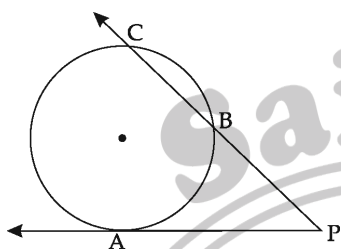
- (23) Seg AB and seg AD are the chords of the circle. C is a point on tangent of the circle at point A. If $m(\text{arc APB}) = 80^\circ$ and $\angle BAD = 30^\circ$. Then find (i) $\angle BAC$ (ii) $m(\text{arc BQD})$. (3 marks)



- (24) Secant AC and secant AE intersects in point A. Points of intersections of the circle and secants are B and D respectively. If $CB = 5$, $AB = 7$, $EA = 20$. Determine $ED - AD$. (3 marks)

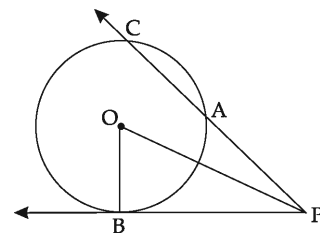


- (25) In the adjoining figure line PA is tangent at point A. Line PBC is a secant. If $AP = 15$ and $BP = 10$, find BC. (2 marks)



- (26) $\square ABCD$ is a rectangle. Taking AD as a diameter, a semicircle AXD is drawn which intersects the diagonal BD at X. If $AB = 12$ cm, $AD = 9$ cm, then find values of BD and BX. (3 marks)

- (27) In the adjoining figure, point O is the centre of the circle. Line PB is a tangent and line PAC is a secant. Find $PA \times PC$ if $OP = 25$ and radius is 7. (3 marks)



ANSWERS

- (1) 12 (2) $5\sqrt{3}$ (3) 65° (6) 8 cm (7) 8 cm
 (8) 11 cm, 3 cm (9) 8 units (10) 30 units
 (11) 120° (12) 260°
 (13) $m(\text{arc } AB) = 90^\circ$, $m(\text{arc } BC) = 90^\circ$, $m(\text{arc } AXC) = 180^\circ$
 (14) $m(\text{arc } AD) = m(\text{arc } BC) = 100^\circ$ (15) 30° , 160°
 (16) 30.5° (17) 13° , 24° , 11° (18) 45° (19) 65°
 (23) (i) 40° (ii) 60° (24) 11.6 units (25) 12.3 units
 (26) $BD = 15$, $BX = 9.6$ (27) 576 units

ASSIGNMENT - 3

Time : 1 Hr.

Marks : 20

Q.1. (A) Solve the following sub questions:

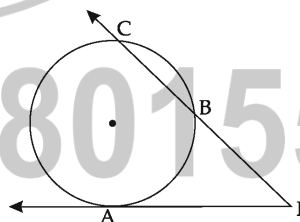
(2)

- (1) Radius of a circle with centre 'O' is 5 cm, $OA = 4$ cm, $OB = 5.5$ cm. Find the position of point A and B with respect to circle.
 (2) Two circles with diameters 6 cm and 9 cm touch each other externally. Find the distance between their centres.

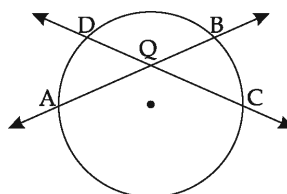
Q.1. (B) Solve the following any one questions:

(2)

- (1) Line PA is a tangent at point A. Line PBC is a secant $AP = 15$, $BP = 10$, find BC.



- (2) Secants AB and CD are intersecting in point Q. $m(\text{arc } AD) = 25^\circ$ and $m(\text{arc } BC) = 36^\circ$, then find: $\angle BQC$



Q.2. Perform any on of the activities

(2)

- (1) Measure of a major arc of a circle is four times the measure of corresponding minor arc. Complete the following activity to find the measure of each arc.

Sol. Let the measure of minor arc be x .

$\therefore$ Measure of major arc =

$\therefore x + \text{} = 360^\circ$ (Measure of a circle is 360°)

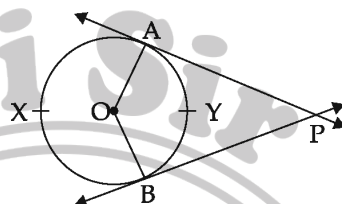
$\therefore \text{} x = 360^\circ$

$\therefore x = \text{}$

$\therefore$ Measure of minor arc = $x = \dots\dots\dots$

$\therefore$ Measure of major arc = = $\times x = \dots\dots\dots$

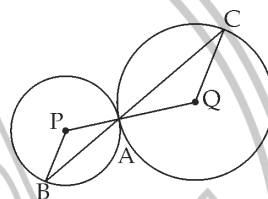
- (2) Line PA and line PB are tangents to the circle at points A and B. If $\angle APB = 60^\circ$, then find $m(\text{arc AXB})$.



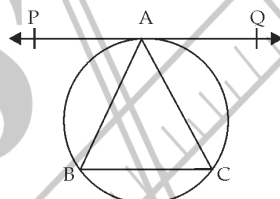
Q.3. Solve the following sub questions: (any two)

(6)

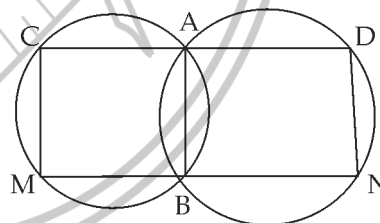
- (1) Two circles with centres P and Q touch each other at point A. $\angle BPA = 60^\circ$. Find $m\angle QCA$ and $m\angle CQP$.



- (2) Line PQ is a tangent to the circle at point A. $\text{arc AB} \cong \text{arc AC}$. Complete the following activity to prove $\triangle ABC$ as isosceles triangle.



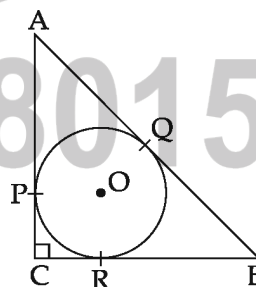
- (3) Two circles intersect each other in points A and B. Secants through A and B intersect circles in C, D and M, N as shown in the figure. Prove that: $CM \parallel DN$



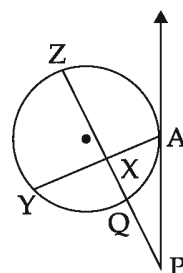
Q.4. Solve any two of the following questions:

(8)

- (1) A circle with centre 'O' is incircle of $\triangle ABC$. $\angle ACB = 90^\circ$. Radius of the circle is r . Prove that : $2r = a + b - c$.



- (2) In the adjoining figure line PA is a tangent to the circle at point A. Secant PQZ intersects chord AY in point X, such that $AP = PX = XY$. If $PQ = 1$ and $QZ = 8$. Find AX.



- (3) In the adjoining figure line PA is a tangent to the circle at