

4

Geometric Constructions

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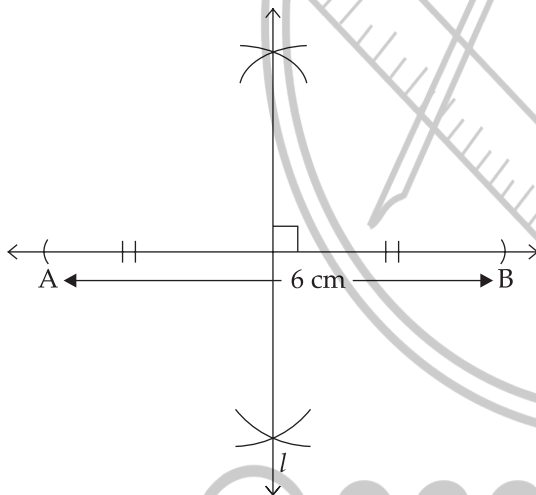
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**Points to Remember:**

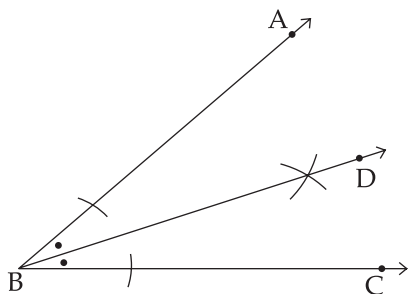
- Construction of various geometrical figures is a very important part of the study of geometry for understanding the concepts learnt in theoretical geometry.

Basic Constructions

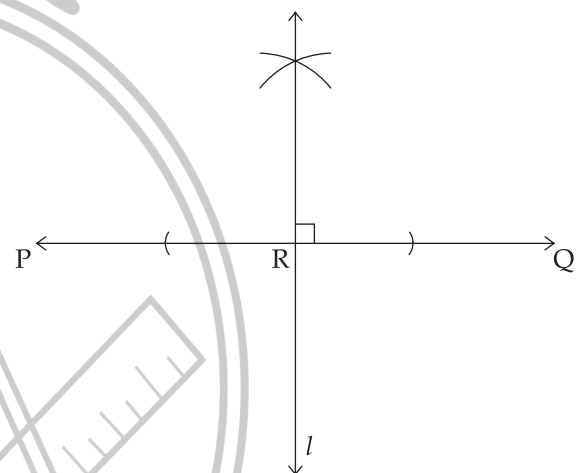
- (i) To draw a perpendicular bisector of a given line segment.



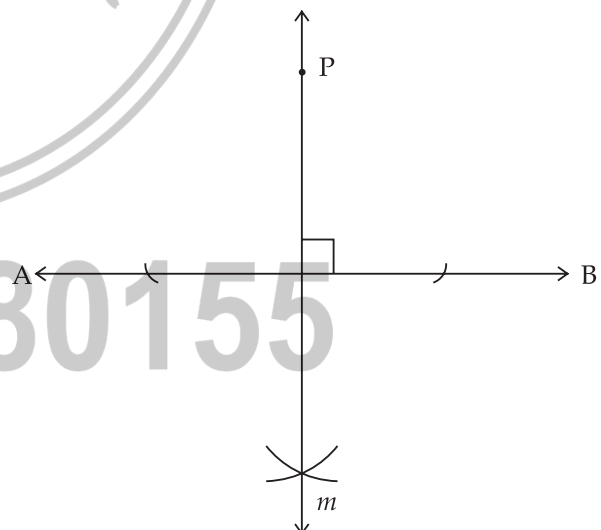
- (ii) To draw an angle bisector of a given angle.



- (iii) To draw a perpendicular to a line at a given point on it.



- (iv) To draw a perpendicular to a given line from a point outside it.



Construction of similar triangle to a given triangle: For a given one-to-one correspondence between the vertices of two triangles, if their corresponding angles are congruent and corresponding sides are in proportion, then these two triangles are called '**Similar triangles**'.

Here, we shall see two types of constructions as discussed below.

Example: $\triangle ABC \sim \triangle XYZ$. $AB = 8$ cm, $BC = 6$ cm and $AC = 10$ cm. $AB : XY = 2 : 1$ Construct $\triangle XYZ$.

$$\frac{AB}{XY} = \frac{BC}{YZ} = \frac{AC}{XZ} \quad \dots(i) \text{ (c.s.s.t.)}$$

$$\frac{AB}{XY} = \frac{2}{1} \quad \dots(ii) \text{ (Given)}$$

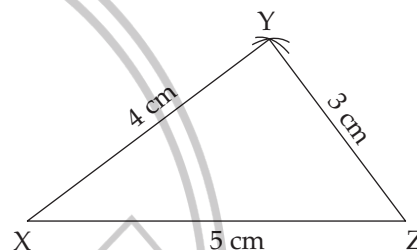
$$\therefore \frac{AB}{XY} = \frac{BC}{YZ} = \frac{AC}{XZ} = \frac{2}{1} \quad \dots[\text{From (i) and (ii)}]$$

$$\therefore \frac{8}{XY} = \frac{6}{YZ} = \frac{10}{XZ} = \frac{2}{1}$$

$$\frac{8}{XY} = \frac{2}{1}; \frac{6}{YZ} = \frac{2}{1}; \frac{10}{XZ} = \frac{2}{1}$$

$$\therefore XY = \frac{8}{2} ; YZ = \frac{6}{2} ; XZ = \frac{10}{2}$$

$$\therefore XY = 4 \text{ cm}, YZ = 3 \text{ cm}, XZ = 5 \text{ cm}$$



$\triangle XYZ$ is the required triangle.

Practice Set - 4.1 (Textbook Page No. 96)

(1) $\triangle ABC \sim \triangle LMN$, In $\triangle ABC$ $AB = 5.5$ cm, $BC = 6$ cm and $CA = 4.5$ cm. Construct $\triangle ABC$ and $\triangle LMN$, such that $\frac{BC}{MN} = \frac{5}{4}$. (4 marks)

$$\triangle ABC \sim \triangle LMN \quad (\text{Given})$$

$$\therefore \frac{AB}{LM} = \frac{BC}{MN} = \frac{AC}{LN} \quad \dots (i) \quad (c.s.s.t.)$$

$$\frac{BC}{MN} = \frac{5}{4} \quad \dots (ii) \quad (\text{Given})$$

$$\therefore \frac{AB}{LM} = \frac{BC}{MN} = \frac{AC}{LN} = \frac{5}{4} \quad [\text{from (i) and (ii)}]$$

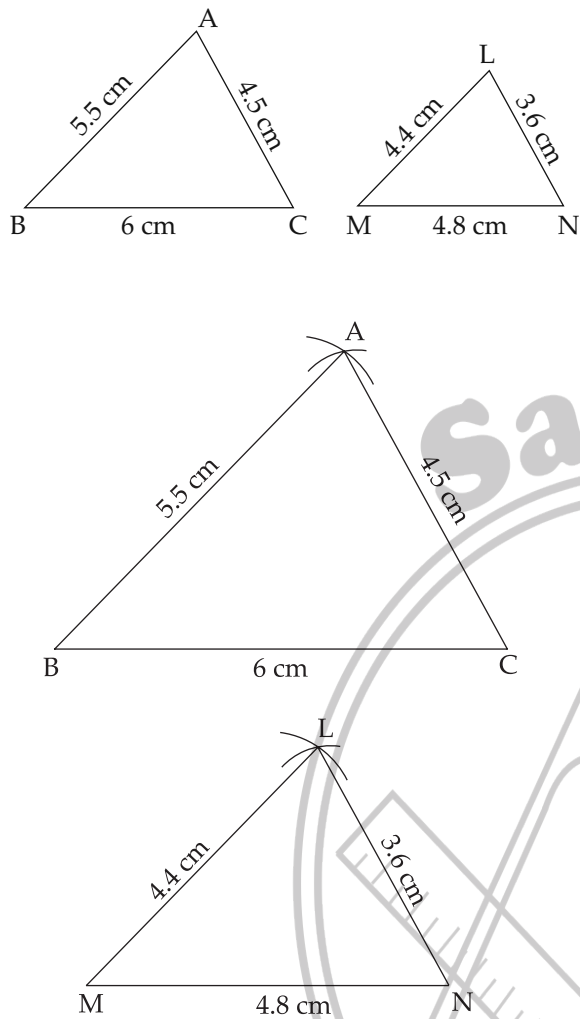
$$\therefore \frac{AB}{LM} = \frac{5}{4}, \frac{BC}{MN} = \frac{5}{4}, \frac{AC}{LN} = \frac{5}{4}$$

$$\therefore \frac{5.5}{LM} = \frac{5}{4}, \quad \frac{6}{MN} = \frac{5}{4}, \quad \frac{4.5}{LN} = \frac{5}{4}$$

$$\therefore \text{LM} = \frac{5.5 \times 4}{5} ; \text{MN} = \frac{6 \times 4}{5} ; \text{LN} = \frac{4.5 \times 4}{5}$$

$$\therefore \text{LM} = 4.4 \text{ cm, MN} = 4.8 \text{ cm, LN} = 3.6 \text{ cm}$$

Analytical figure:



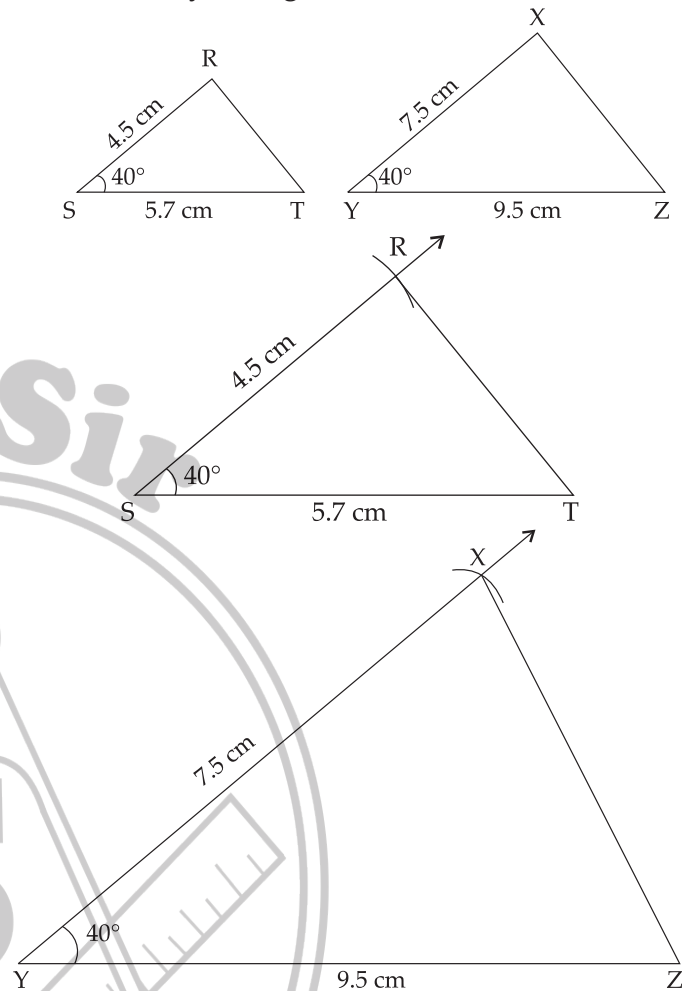
$\triangle LMN$ is the required triangle similar to the $\triangle ABC$.

- (3) $\triangle RST \sim \triangle XYZ$, In $\triangle RST$, $RS = 4.5$ cm, $\angle RST = 40^\circ$, $ST = 5.7$ cm. Construct $\triangle RST$ and $\triangle XYZ$, such that $\frac{RS}{XY} = \frac{3}{5}$ (4 marks)

Solution :

$$\begin{aligned} \triangle RST &\sim \triangle XYZ && \text{(Given)} \\ \therefore \angle RST &\cong \angle XYZ && \text{(c.a.s.t.)} \\ \therefore \angle XYZ &= 40^\circ && (\because \angle RST = 40^\circ, \text{ given}) \\ \frac{RS}{XY} &= \frac{ST}{YZ} && \dots (i) \quad \text{(c.s.s.t.)} \\ \text{But, } \frac{RS}{XY} &= \frac{3}{5} && \dots (ii) \quad \text{(given)} \\ \therefore \frac{RS}{XY} &= \frac{ST}{YZ} = \frac{3}{5} && \text{[from (i) and (ii)]} \\ \therefore \frac{RS}{XY} &= \frac{3}{5}; \frac{ST}{YZ} = \frac{3}{5} \\ \therefore \frac{4.5}{XY} &= \frac{3}{5} && \frac{5.7}{YZ} = \frac{3}{5} \\ \therefore XY &= \frac{4.5 \times 5}{3} && \therefore YZ = \frac{5.7 \times 5}{3} \\ \therefore XY &= 7.5 \text{ cm} && \therefore YZ = 9.5 \text{ cm} \end{aligned}$$

Analytical figure:



$\triangle XYZ$ is the required triangle similar to the $\triangle RST$.

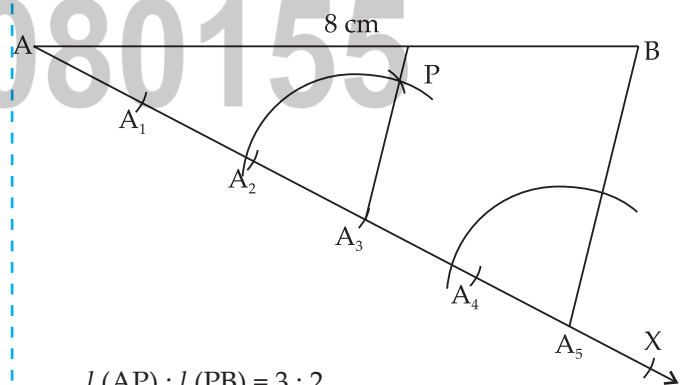


Points to Remember:

Basic Construction

To divide a line segment in the given ratio.

Example: Divide seg AB of length 8 cm in the ratio 3 : 2



$$l(AP) : l(PB) = 3 : 2$$

Steps of construction:

- (1) Draw seg AB of length 8 cm. Draw ray AX on either side of seg AB.

(2) Make five equal parts $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5$ on ray AX.

(3) Draw seg BA_5 .

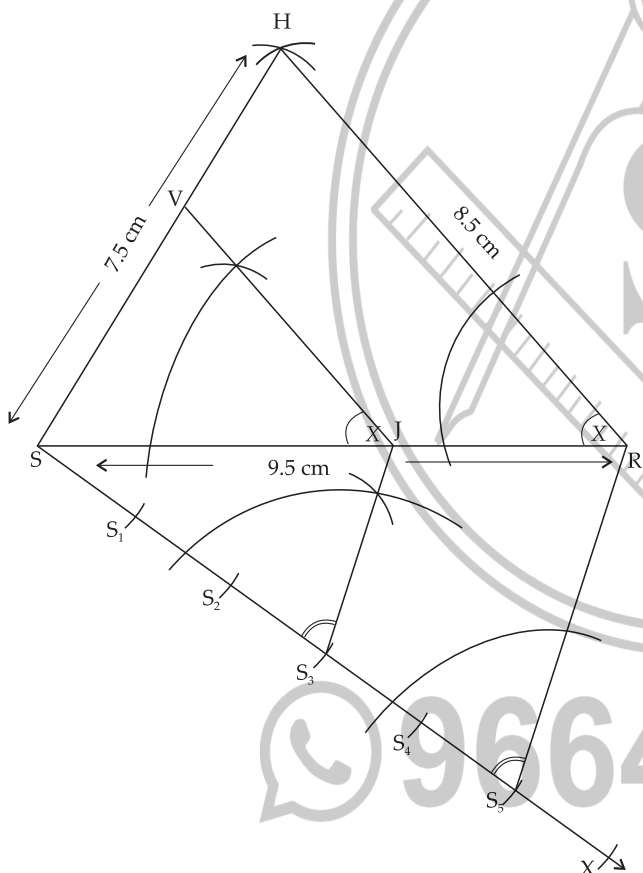
(4) Draw angle $\cong$ to $\angle AA_5B$ at vertex A_3 .

Let its arm intersect seg AB at P. Point P divides seg AB in the ratio $AP : PB$; i.e., 3 : 2.

(B) To construct similar triangle when one pair of angle is common.

Example: Construct ΔSVJ , if $\Delta SHR \sim \Delta SVJ$, $SH = 7.5$ cm, $HR = 8.5$ cm, $SR = 9.5$ cm, $SV : SH = 3 : 5$

[Note : In this case, we need not do any calculation, we have to use the basic construction that we did in the previous question.]



Steps of construction:

- (1) Draw seg SR as base [Note: Base should be taken that side in which vertex of common angle is contained.]
- (2) On either side of seg SR, draw ray SX and on ray SX make five equal parts $SS_1 = S_1S_2 = S_2S_3 = S_3S_4 = S_4S_5$. [As bigger triangle has to be divided in five equal parts.]

(3) Draw seg RS_5 . Draw an angle $\cong$ to $\angle SS_5R$ at vertex S_3 . Let its arm intersect side SR at point J.

(4) Draw $\angle SJV \cong \angle SRH$, intersecting side SH at point 'V'.

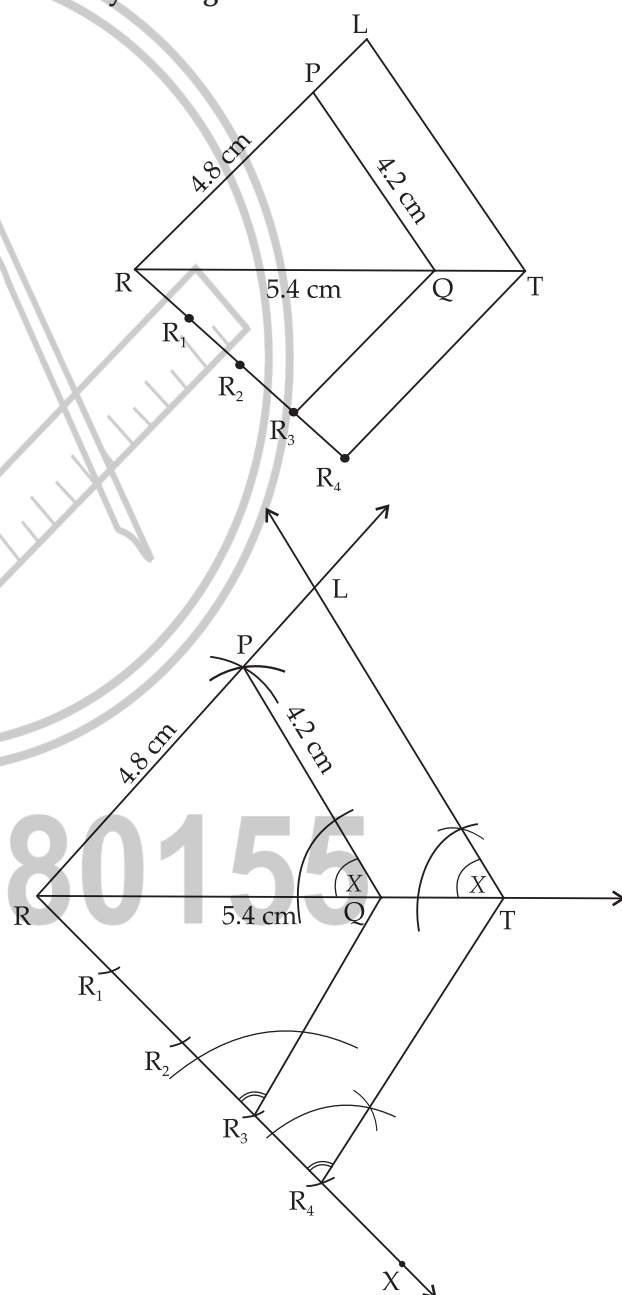
ΔSVJ is the required triangle.

Practice Set - 4.1 (Textbook Page No. 96)

- (2) $\Delta PQR \sim \Delta LTR$, In ΔPQR , $PQ = 4.2$ cm, $QR = 5.4$ cm, $PR = 4.8$ cm. Construct ΔPQR and ΔLTR , such that $\frac{PQ}{LT} = \frac{3}{4}$. (4 marks)

Solution :

Analytical figure:

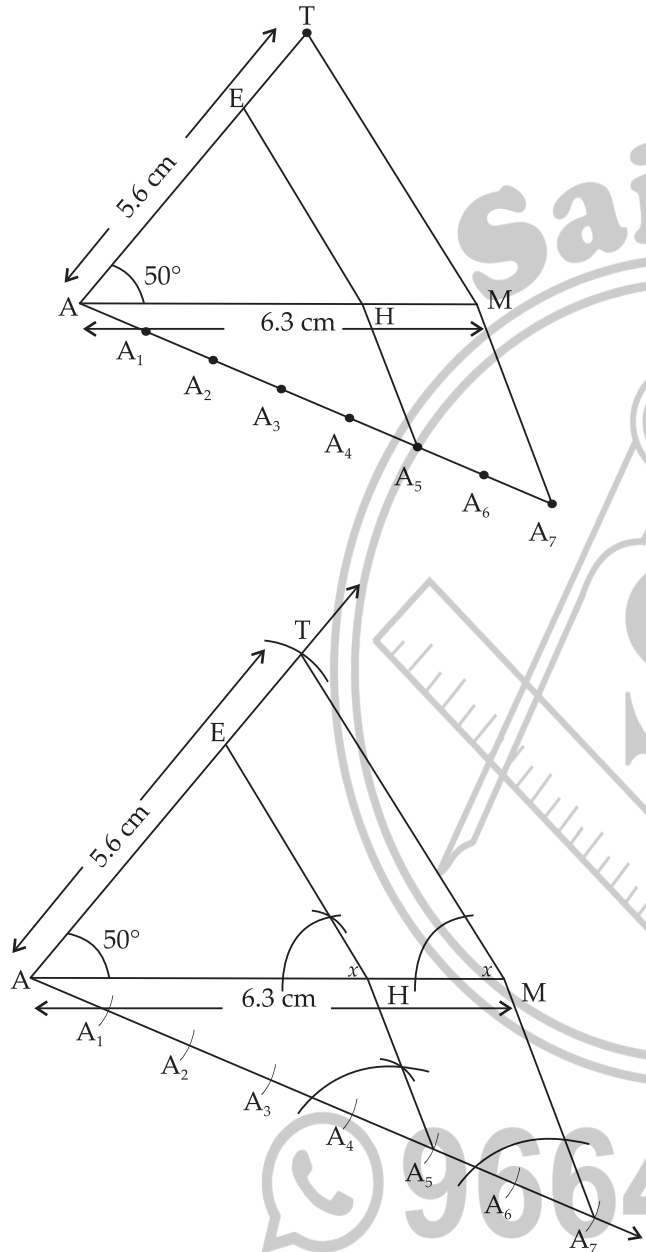


ΔLTR is the required triangle similar to the ΔPQR .

- (4) $\triangle ATM \sim \triangle AHE$. In $\triangle AMT$, $AM = 6.3$ cm, $\angle TAM = 50^\circ$, and $AT = 5.6$ cm. $\frac{AM}{AH} = \frac{7}{5}$.
Construct $\triangle AHE$. (4 marks)

Solution :

Analytical figure:



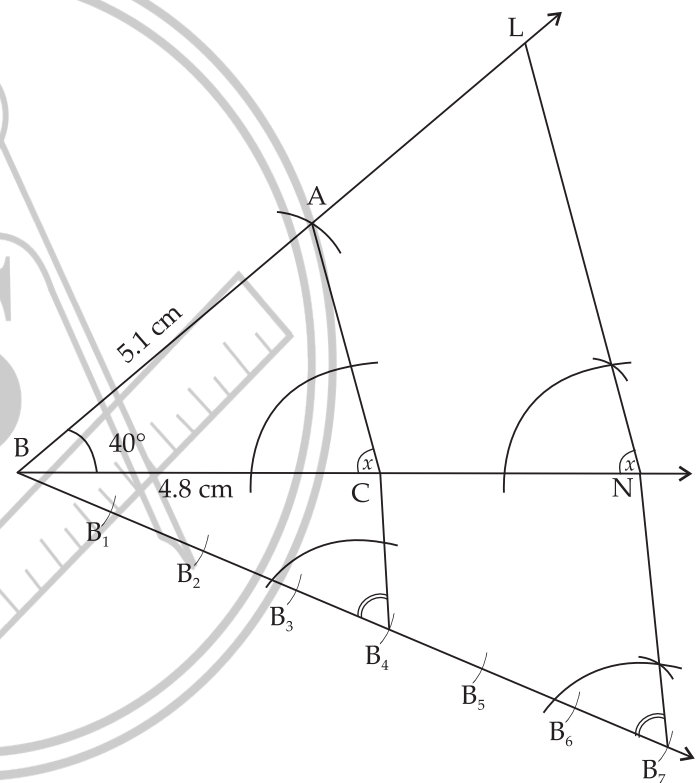
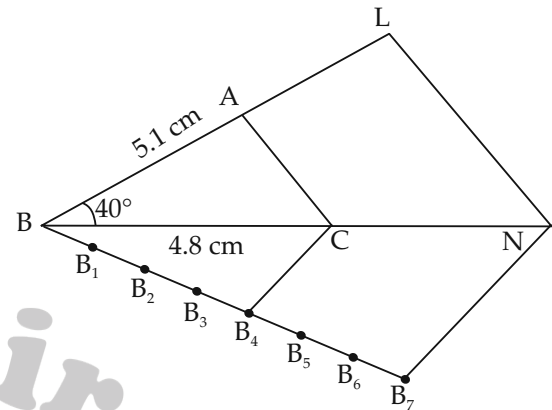
$\triangle AHE$ is the required triangle similar to the $\triangle AMT$.

Problem Set - 4 (Textbook Pg No. 99)

- (7) $\triangle ABC \sim \triangle LBN$. In $\triangle ABC$, $AB = 5.1$ cm, $\angle B = 40^\circ$, $BC = 4.8$ cm, $\frac{AC}{LN} = \frac{4}{7}$. Construct $\triangle ABC$ and $\triangle LBN$. (4 marks)

Solution :

Analytical figure:

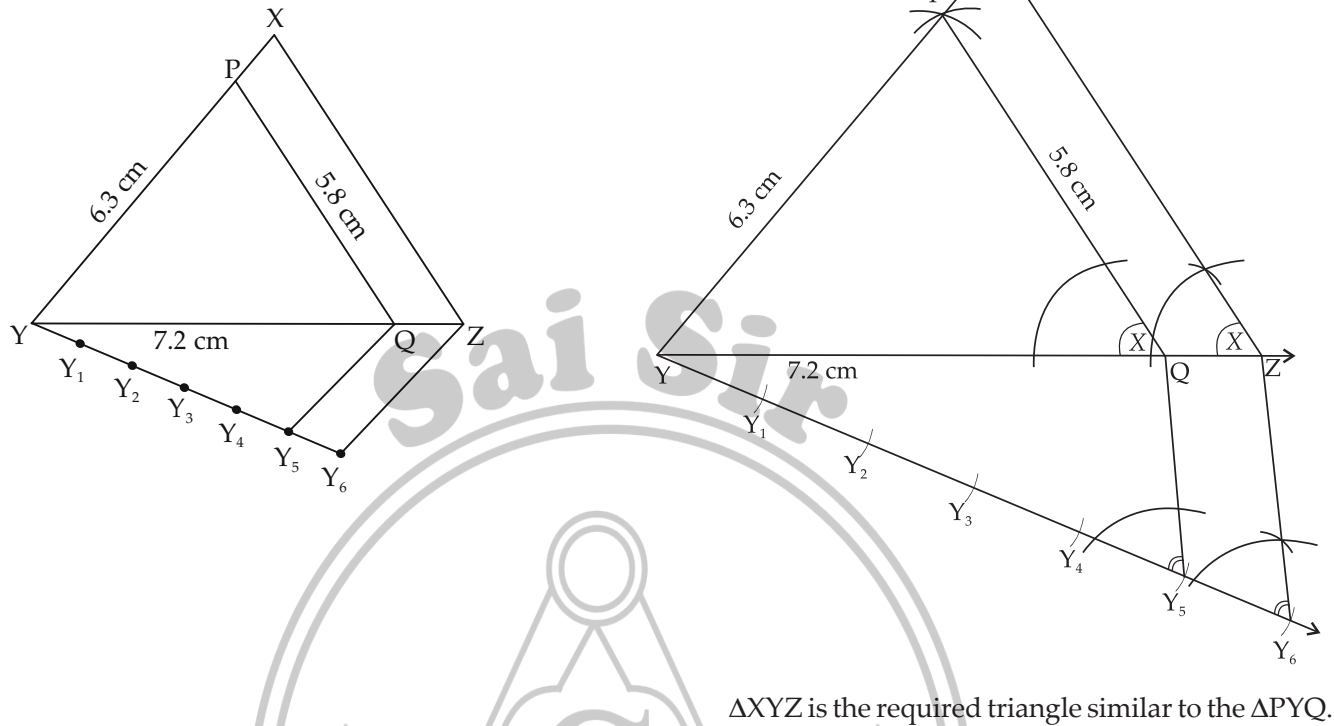


$\triangle LBN$ is the required triangle similar to the $\triangle ABC$.

- (8) Construct $\triangle PYQ$ such that $PY = 6.3$ cm, $YQ = 7.2$ cm, $PQ = 5.8$ cm. If $\frac{YZ}{YQ} = \frac{6}{5}$, then construct $\triangle XYZ$ similar to $\triangle PYQ$. (4 marks)

Solution :

Analytical figure:



Points to Remember:

(II) Construction of a tangent to the circle.

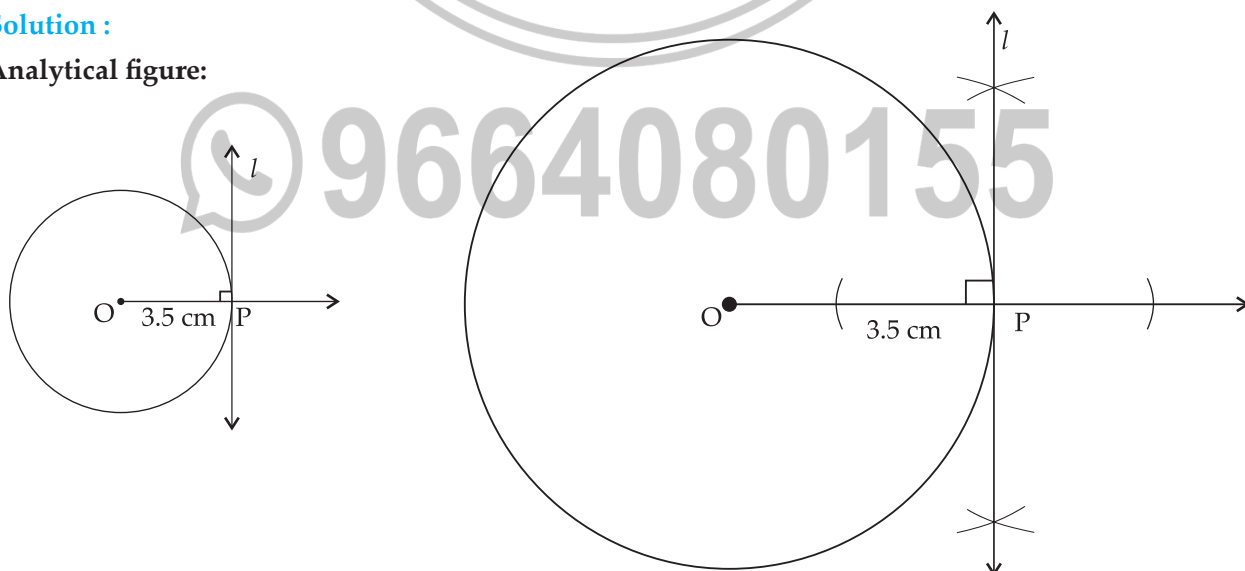
(A) Construction of tangent to a circle at a point on the circle using the centre of the circle.

The tangent of a circle is perpendicular to the radius at its outer end. We use the same property to do this construction.

Example: To construct a tangent to a circle of radius 3.5 cm at a point P on it.

Solution :

Analytical figure:



Steps of construction:

- (1) Draw circle with given radius with centre 'O'.
- (2) Take a point 'P' on the circle.
- (3) Draw ray OP.
- (4) Draw perpendicular to ray OP at point P. Name line as l .

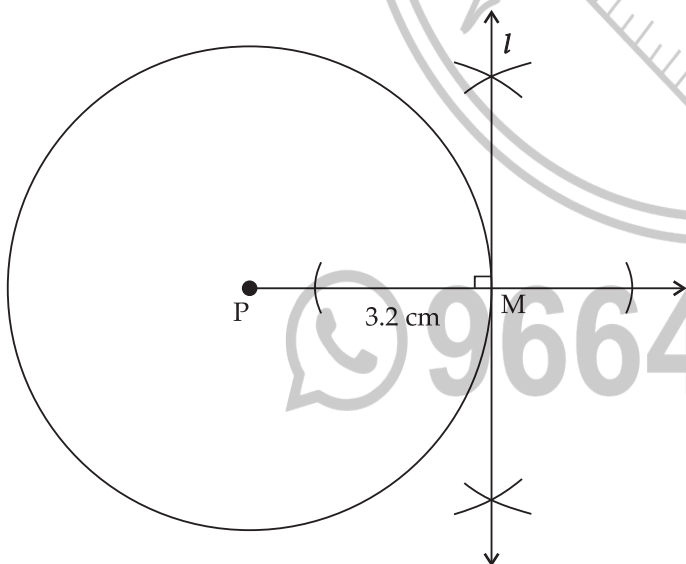
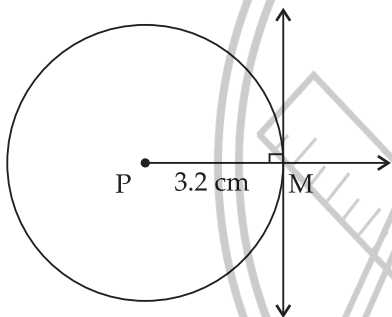
Line ' l ' is tangent to the circle (as perpendicular at outer end of radius is tangent.)

Practice Set - 4.2 (Textbook Page No. 98)

- (1) Construct a tangent to a circle with centre P and radius 3.2 cm at any point M on it. (2 marks)

Solution :

Analytical figure

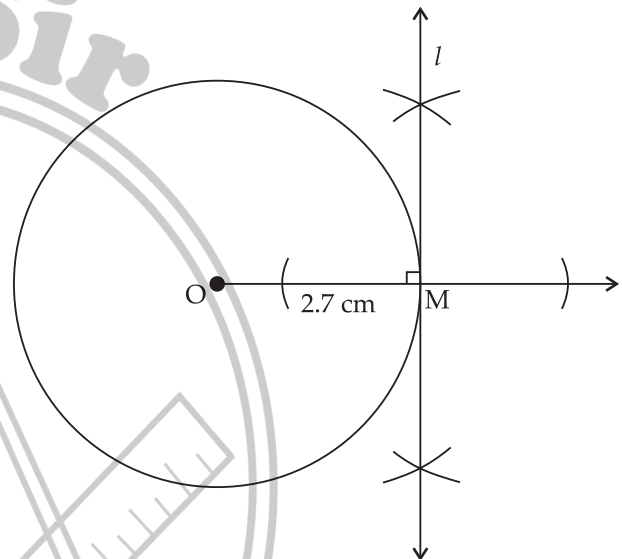
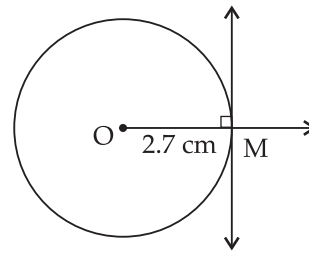


line l is the required tangent to the circle passing through point M on the circle.

- (2) Draw a circle of radius 2.7 cm. Draw a tangent to the circle at any point on it. (2 marks)

Solution :

Analytical figure:



line l is the required tangent to the circle passing through point M on the circle.

- (4) Draw a circle of radius 3.3 cm. Draw a chord PQ of length 6.6 cm. Draw tangents to the circle at points P and Q. Write your observation about the tangents. (3 marks)

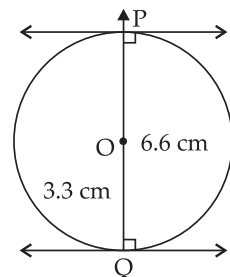
Solution : Analytical figure:

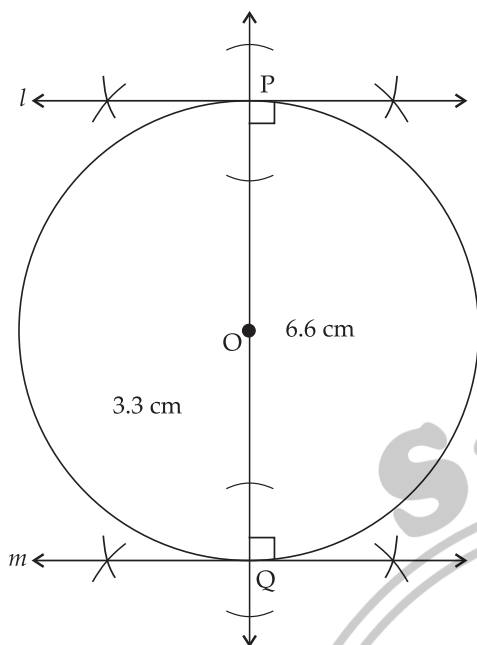
Radius = 3.3 cm (Given)

Chord = 6.6 cm (Given)

$\therefore$ Chord is twice of radius.

$\therefore$ Chord PQ is a diameter.





line l and m are the required tangents to the circle at point P and point Q.

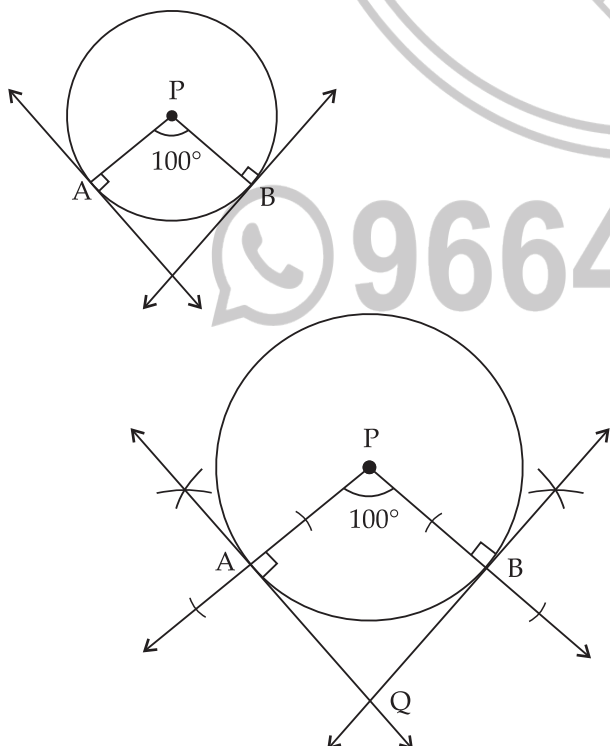
Tangents at the end points of a diameter are parallel to each other.

Problem Set - 4 (Textbook Page No. 99)

- (5) Draw a circle with centre P. Draw an arc AB of 100° measure. Draw tangents to the circle at points A and point B. (3 marks)

Solution :

Analytical figure:



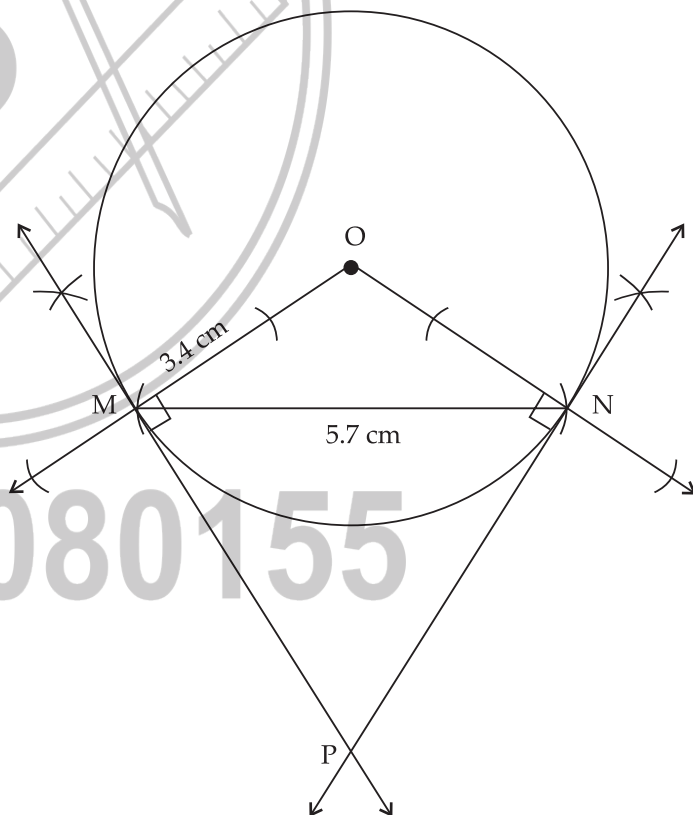
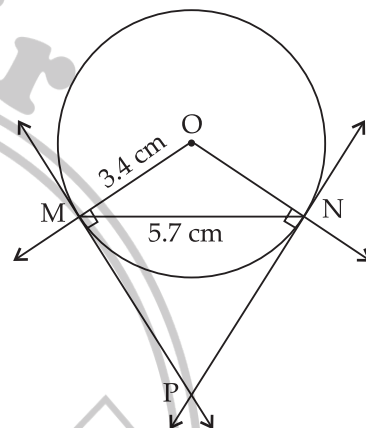
Line AQ and line BQ are tangents to the circle at points A and B respectively.

Practice Set - 4.2 (Textbook Page No. 99)

- (5) Draw a circle with radius 3.4 cm. Draw a chord MN of length 5.7 cm in it. Construct tangent at point M and N to the circle. (3 marks)

Solution :

Analytical figure:



Line MP and line NP are required tangents to the circle at point M and point N respectively.



Points to Remember:

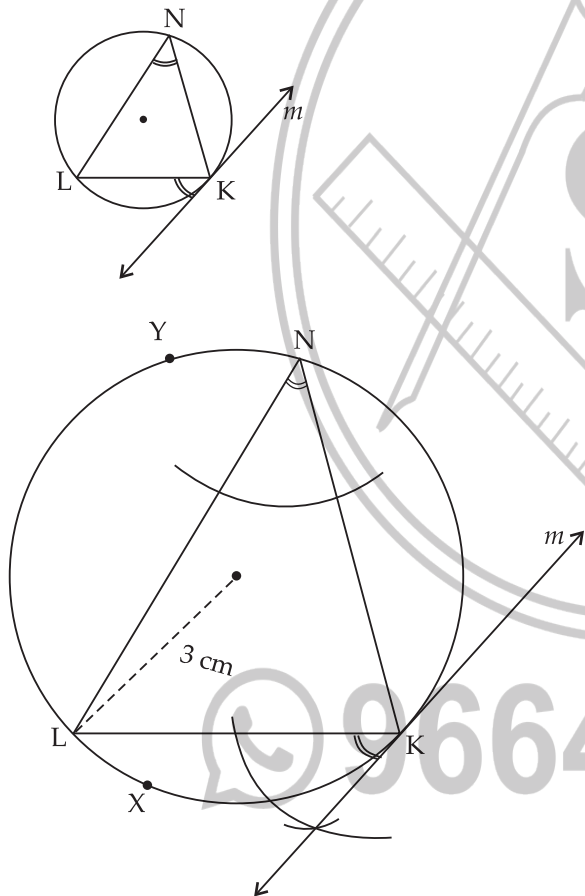
- (B) **Construction of a tangent to the circle from a point on the circle without using the centre.**

If a line drawn through an end point of a chord of a circle and an angle formed by it with the chord is equal to the angle subtended by the chord in the corresponding alternate segment, then the line is a tangent to the circle.

Example: Draw a circle of radius 3 cm. Take any point K on it. Draw a tangent to the circle at K without using centre of the circle.

Solution:

Analytical figure:



Line 'm' is the tangent to the circle

Steps of construction:

- (1) Draw a circle with radius 3 cm.
- (2) Take a point K on the circle. Draw chord KL.
- (3) Take a point N in the alternate arc of arc KXL.
- (4) Draw seg LN and seg KN to form $\angle LNK$.

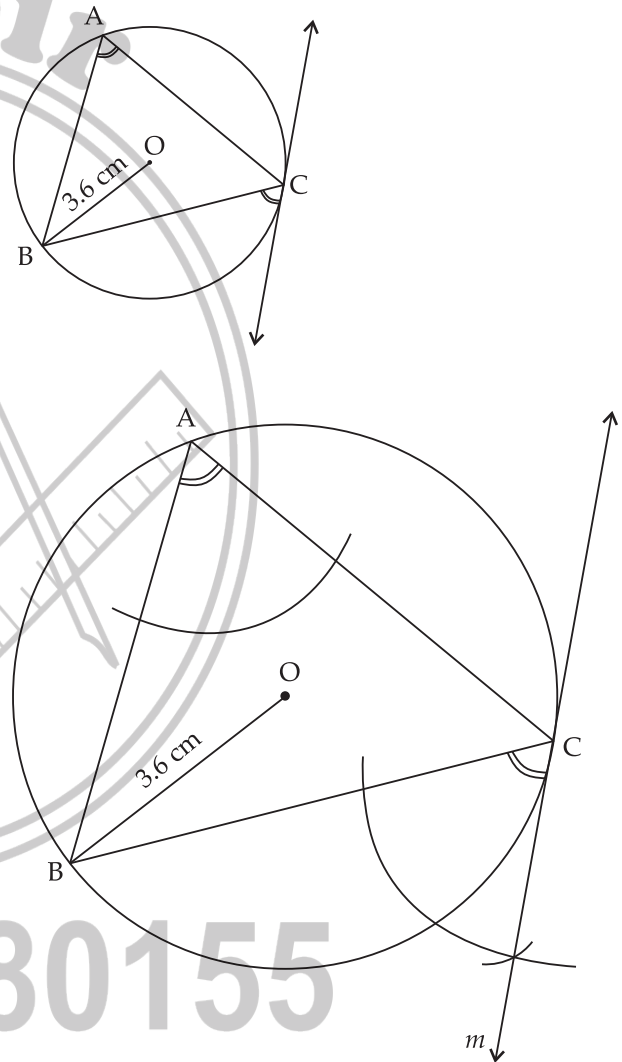
- (5) Draw an angle congruent to $\angle LNK$ at vertex K, taking LK as one side.
- (6) The line m is the required tangent.

Practice Set - 4.2 (Textbook Page No. 98)

- (3) Draw a circle of radius 3.6 cm. Draw a tangent to the circle at any point on it without using the centre. (2 marks)

Solution :

Analytical figure:



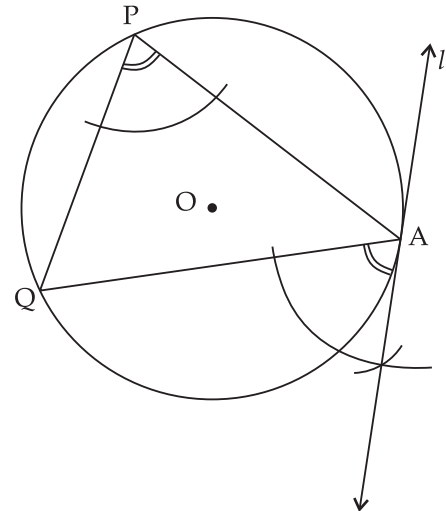
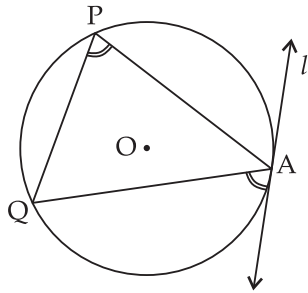
Line m is the required tangent to the circle at point C.

Problem Set - 4 (Textbook Page No. 99)

- (3) Draw any circle. Take any point A on it and construct tangent at A without using the centre of the circle. (2 marks)

Solution :

Analytical figure:



Line l is the required tangent to the circle at point A.



Points to Remember:

- (C) **Construction of tangents to a circle from a point outside the circle.**

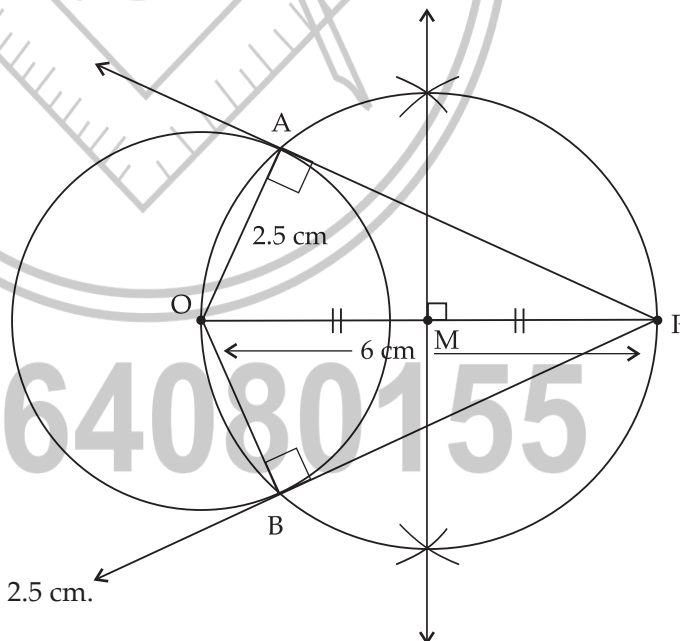
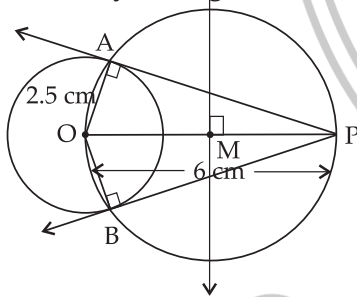
The angle inscribed in a semicircle is a right angle, using this property we shall draw a tangent to a circle from a point outside it.

Note: We can draw two tangents from a point outside the circle.

Example: Draw tangent to the circle of radius 2.5 cm from a point 'P' at a distance 6 cm from the centre.

Solution:

Analytical figure:



Steps of construction:

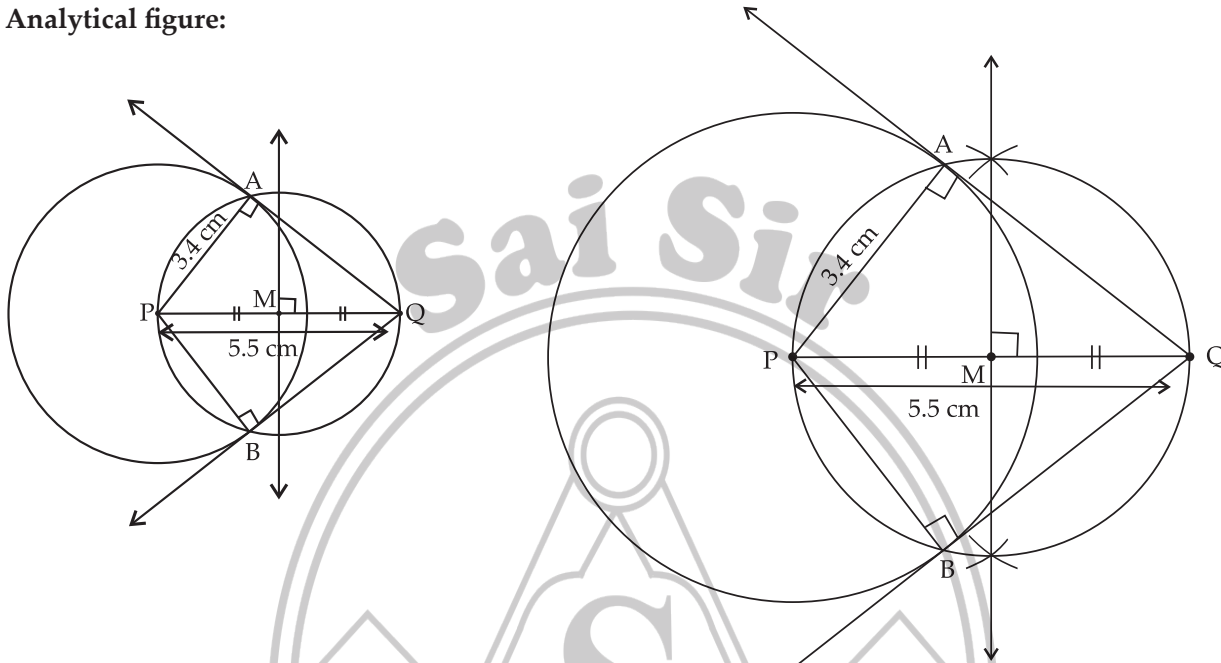
- (1) Draw a circle with centre 'O' and radius 2.5 cm.
- (2) Take a point 'P' such that $OP = 6$ cm.
- (3) Draw perpendicular bisector of seg OP and obtain midpoint M of seg OP.
- (4) Taking 'M' as centre and MO as radius draw a circle, intersecting the circle at points A and B.
- (5) Draw ray PA and ray PB.
- (6) Line PA and line PB are tangents to the circle at points A and B respectively from point 'P'.

Practice Set - 4.2 (Textbook Page No. 99)

- (6) Draw a circle with centre P and radius 3.4 cm. Take point Q at a distance 5.5 cm from the centre. Construct tangents to the circle from point Q. (3 marks)

Solution :

Analytical figure:

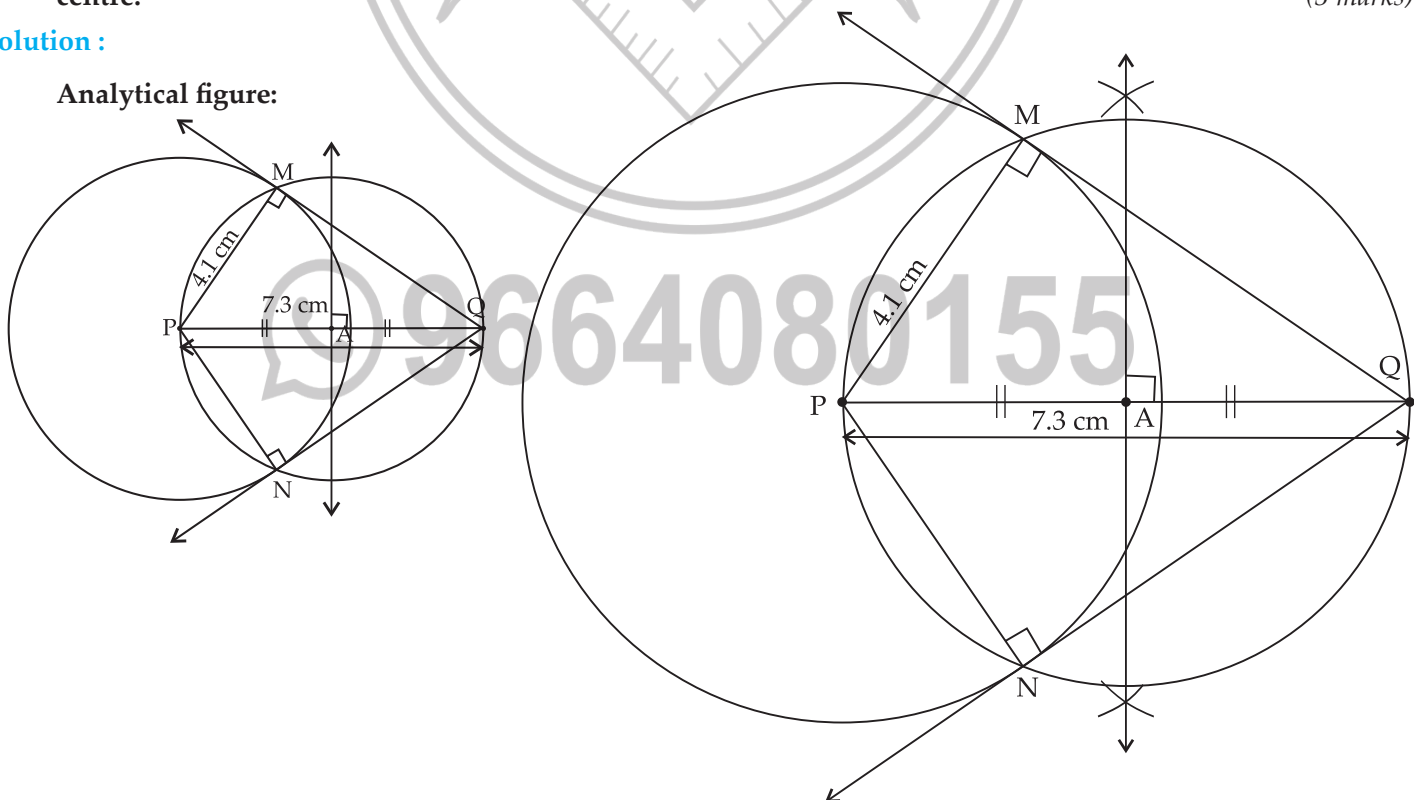


line MQ and line NQ are the required tangents to the circle from point Q.

- (7) Draw a circle with radius 4.1 cm. Construct tangents to the circle from a point at a distance 7.3 cm from the centre. (3 marks)

Solution :

Analytical figure:



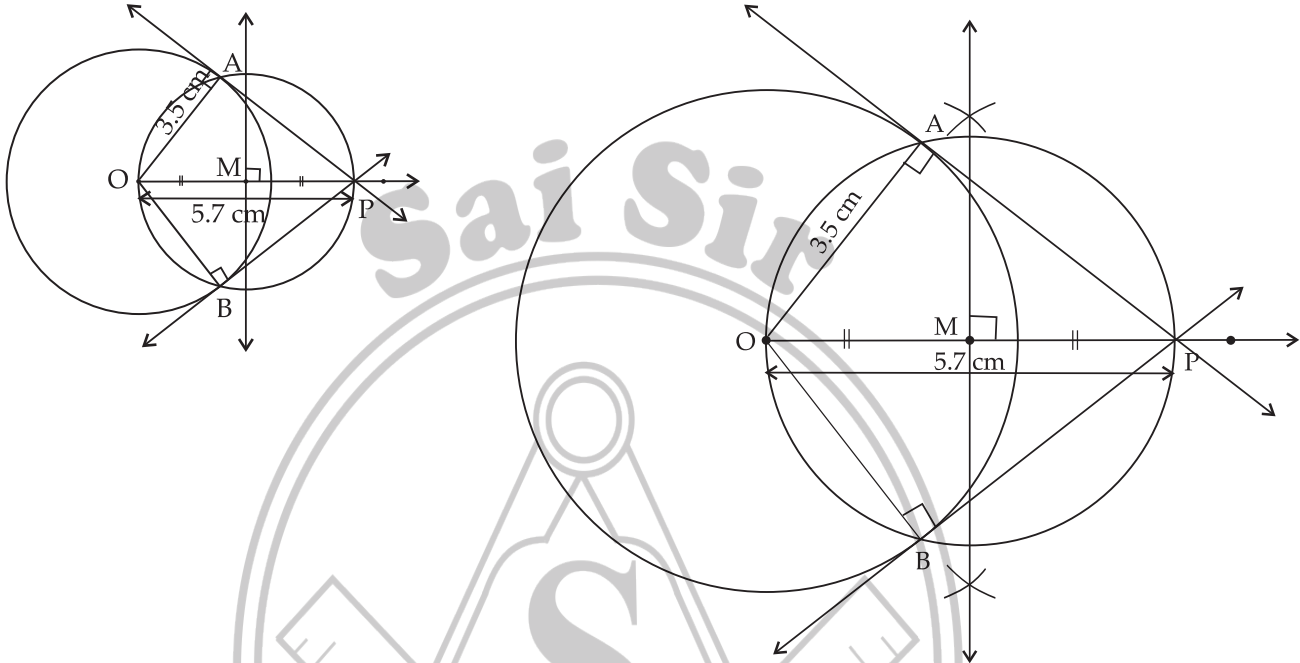
Line MQ and line NQ are the required tangents to the circle from point Q.

Problem Set - 4 (Textbook Pg No. 99)

- (2) Draw a circle with centre O and radius 3.5 cm. Take a point P at a distance 5.7 cm from the centre. Draw tangents to the circle from point P. (3 marks)

Solution :

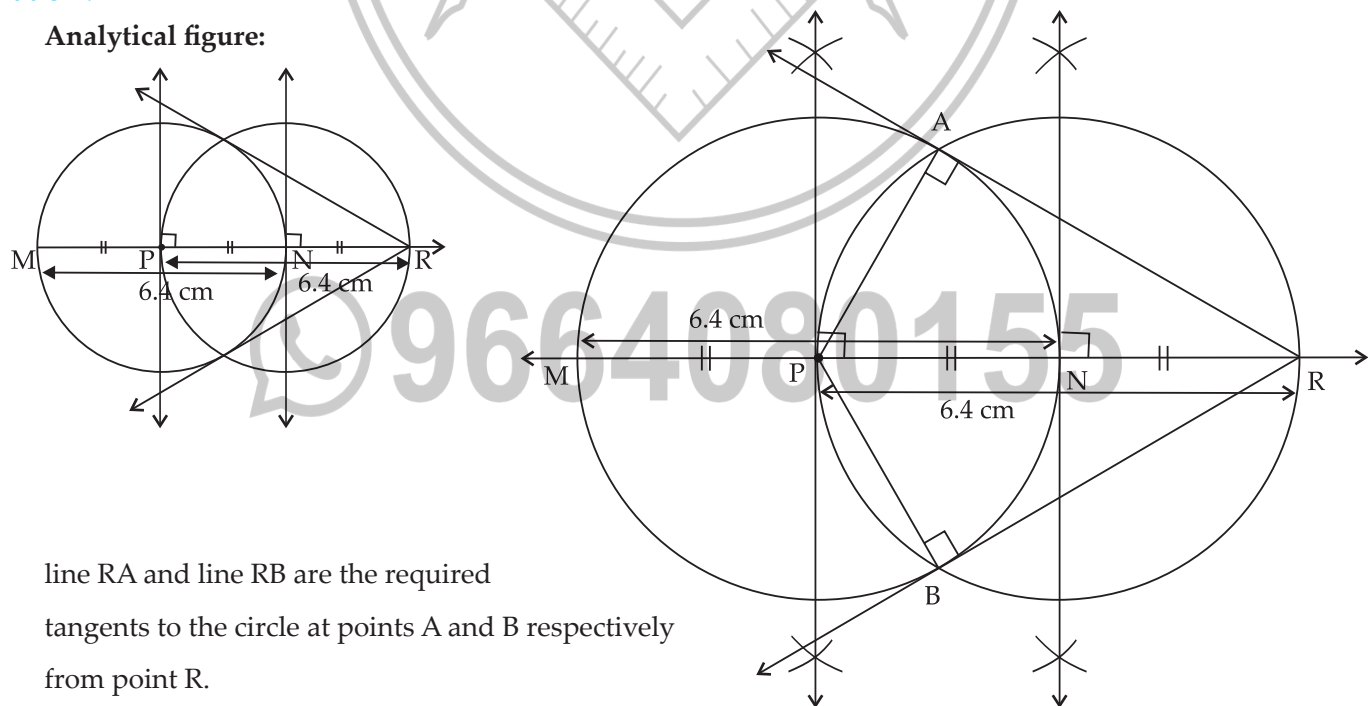
Analytical figure:



- (4) Draw a circle of diameter 6.4 cm. Take a point R at a distance equal to its diameter from the centre. Draw tangents from point R. (3 marks)

Solution :

Analytical figure:

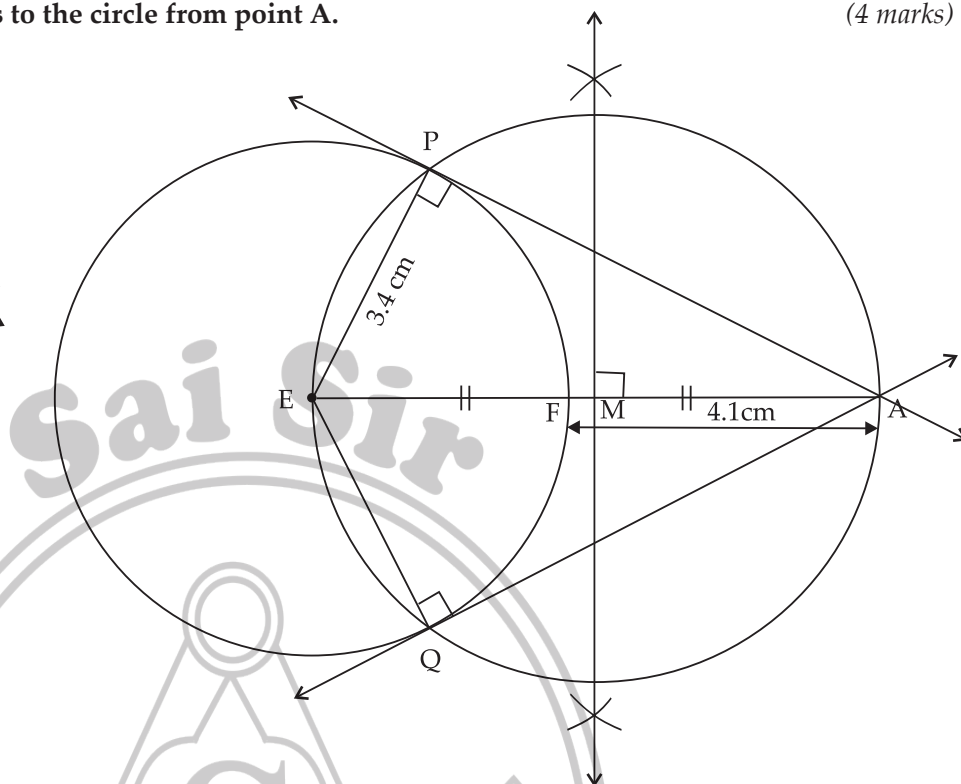
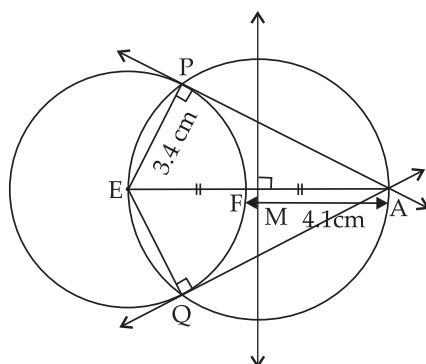


line RA and line RB are the required tangents to the circle at points A and B respectively from point R.

- (6) Draw a circle of radius 3.4 cm and centre E. Take a point F on the circle. Take another point A such that E-F-A and FA = 4.1 cm. Draw tangents to the circle from point A. (4 marks)

Solution :

Analytical figure:



Line AP and line AQ are the required tangents from point A to the circle with centre E.

Problem Set - 4 (Textbook Pg No. 99)

MCQ's

- (1) Select correct alternative for each of the following questions. (1 mark each)
- * (1) The number of tangents that can be drawn to a circle at a point on the circle is
(A) 3 (B) 2 (C) 1 (D) 0
- * (2) The maximum number of tangents that can be drawn to a circle from a point outside it is
(A) 2 (B) 1 (C) one and only one (D) 0
- * (3) $\triangle ABC \sim \triangle PQR$, $\frac{AB}{PQ} = \frac{7}{5}$ then
(A) $\triangle ABC$ is a bigger
(B) $\triangle PQR$ is bigger
(C) Both triangles will be equal
(D) Can not be decided.
- (6) Tangents drawn at the endpoints of a diameter of a circle are
(A) Equal (B) Unequal
(C) Infinite (D) Can't say
- (7) In $\triangle ABC \sim \triangle PQR$, $AB : PQ = 2 : 3$. If $BC = 4$, then $QR =$
(A) 4 (B) 6 (C) 9 (D) 8
- (8) If $AB : BC = 3 : 5$, then how many equal parts seg AC divided to get point B
(a) 3 (b) 5 (c) 8 (d) Can't say
- to a circle from an external point are
(A) Equal (B) Unequal
(C) Infinite (D) Can't say

ANSWERS

- (1) (C) 1 (2) (A) 2 (3) (A) $\triangle ABC$ is a bigger (4) (A) 0
(5) (A) Equal (6) (C) Parallel (7) (B) 6 (8) (C) 8

PROBLEMS FOR PRACTICE

Based on Practice Set 4.1

- (4) number of tangent/s can be drawn from a point inside the circle.
(A) 0 (B) 1 (C) 2 (D) Infinite
- (5) The lengths of the two tangent segments drawn from a point outside a circle to the points of contact are
(A) Equal (B) Unequal (C) Infinite (D) Can't say
- (1) Draw a line segment $PQ = 8$ cm. Take a point R on it such that $l(PR) : l(RQ) = 3 : 2$. (2 marks)
- (2) $l(AB) : l(BC) = 3 : 2$. Draw seg AB, if $l(AB) = 7.2$ cm. (2 marks)

- (3) $\Delta XYZ \sim \Delta ABC$, $\angle X = 40^\circ$, $\angle Y = 80^\circ$, $XY = 6$ cm. Draw ΔABC , if $AB : XY = 3 : 2$ (3 marks)
- (4) Draw ΔABC with side $BC = 6$ cm, $AB = 5$ cm, and $\angle ABC = 60^\circ$. Also, construct ΔXYZ whose sides are $\frac{3}{4}$ of the corresponding sides of ΔXYZ . (4 marks)
- (5) $\Delta PQR \sim \Delta ABC$, $PQ = 3$ cm, $QR = 4$ cm, $PR = 5$ cm. $A(\Delta PQR) : A(\Delta ABC) = 1 : 4$. Construct both triangles (4 marks)
- (6) $\Delta PQR \sim \Delta PEF$, $m \angle P = 70^\circ$, $PQ = 5$ cm, $PR = 3.5$ cm. Construct ΔPEF , if $PQ : PE = 5 : 7$. (4 marks)
- (7) $\Delta PQR \sim \Delta PAB$, $m \angle P = 60^\circ$, $PQ = 6$ cm, $PR = 4$ cm. Construct ΔPAB , if $PQ : PA = 3 : 2$. (4 marks)
- (8) $\Delta AMT \sim \Delta AHE$, construct ΔAMT such that $MA = 6.3$ cm, $\angle MAT = 120^\circ$, $AT = 4.9$ cm and $\frac{MA}{HA} = \frac{7}{5}$, then construct ΔAHE . (4 marks)
- Based on Practice Set 4.2**
- (9) Draw a tangent to a circle of radius 3 cm and centre 'O' at any point 'K' on the circle. (3 marks)
- (10) Draw a circle with centre 'P' and radius 2.6 cm. Draw a chord AB of length 3.8 cm. Draw tangent to the circle through points A and B. (3 marks)
- (11) Draw a circle with radius 3.4 cm. Draw tangent to the circle, passing through point B on the circle, without using centre. (2 marks)
- (12) Construct a circle with centre 'O' and radius 4.3 cm. Draw a chord AB of length 5.6 cm. Construct the tangents to the circle at point A and B without using centre. (3 marks)
- (13) Draw a circle with centre M and diameter 6 cm. Draw a tangent to the circle from a point N at distance of 9 cm from the centre. (3 marks)
- (14) Draw a circle with 'O' as centre and radius 3.8 cm. Take two points P and Q such that $\angle POQ = 120^\circ$. Draw tangents at P and Q without using centre. (3 marks)
- (15) Draw a circle with 'O' as centre and radius 4 cm. Take a point P at a distance of 7.5 cm from 'O'. Draw tangents to the circle through the point P. (4 marks)

ASSIGNMENT - 4

Time : 1 Hr.

Marks : 20

Q.1. (A) Choose the proper alternative answer for the question given below.

- (1) The number of tangents that can be drawn to a circle at a point on the circle is
(A) 3 (B) 2 (C) 1 (D) 0
- (2) The lengths of the two tangent segments drawn to a circle from an external point are
(A) Equal (B) Unequal (C) Infinite (D) Can't say

Q.1. (B) Attempt any Two of the following:

- (1) Draw seg $PQ = 8$ cm. Divide it in the ratio 3 : 5. (2) Draw $\angle ABC = 120^\circ$ and bisect it
- (3) Draw seg AB of length 6.3 cm and bisect it.

Q.2. Attempt any Two of the following:

- (1) Draw ΔDEF , $EF = 5$ cm, $\angle D = 40^\circ$, $\angle F = 50^\circ$
- (2) Construct a circle with centre 'O' and radius 3.5 cm. Take a point P on it, draw a tangent passing through point P.
- (3) $\Delta ABC \sim \Delta XYZ$, $AB : XY = 3 : 5$. $BC = 9$ cm, $AC = 4.5$ cm. Find YZ and XZ .

Q.3. Attempt any Two of the following:

- (1) Draw a circle with centre 'P' and suitable radius. Draw chord AB of length 5 cm. Draw tangents at points A and B without using centres.
- (2) Draw a circle with centre 'O'. Take two points P and Q on the circle with such that $\angle AOB = 120^\circ$. Draw tangents at points A and B.
- (3) $\Delta DEF \sim \Delta PQR$, $\angle D = 40^\circ$, $\angle F = 60^\circ$, $DF = 6$ cm, $DE : PQ = 3 : 4$. Construct only ΔPQR .

Q.4. Attempt any one of the following:

- (1) Draw a circle with centre 'A' and radius 3.5 cm. Take a point B such that $d(A, B) = 8$ cm. Draw tangents to the circle passing through point B.
- (2) $\Delta ABC \sim \Delta AEF$, $AB : AE = 5 : 2$. $AB = 6$ cm, $BC = 7.5$ cm $AC = 5$ cm. Construct ΔAEF and ΔABC .