

6

Trigonometry

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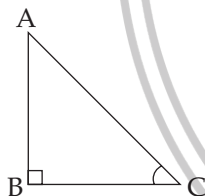
**Points to Remember:**

- Introduction:**

The word 'trigonometry' is derived from Greek words **Tri** meaning **three**, **gona** meaning **sides** and **metron** meaning **measure**.

Thus, trigonometry deals with measurements of sides and angles of a right angled triangle.

In $\triangle ABC$,
 $m\angle ABC = 90^\circ$

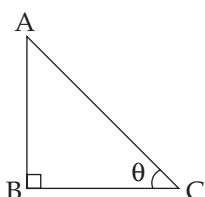


- (1) seg AC is the hypotenuse.
- (2) For $\angle ACB$, seg AB is the opposite side.
- (3) For $\angle ACB$, seg BC is the adjacent side.

- Trigonometric ratios of an acute angle in a right angled triangle:**

For any acute angle in a right angled triangle, the three above mentioned sides, can be arranged two at a time, in six different ratios. These ratios are called **Trigonometric ratios**.

In $\triangle ABC$, $m\angle ABC = 90^\circ$, $m\angle ACB = \theta$



$$\text{Sine ratio of } \theta = \sin \theta = \frac{\text{Opposite side}}{\text{Hypotenuse}} = \frac{AB}{AC}$$

$$\text{Cosine ratio of } \theta = \cos \theta = \frac{\text{Adjacent side}}{\text{Hypotenuse}} = \frac{BC}{AC}$$

$$\text{Tangent ratio of } \theta = \tan \theta = \frac{\text{Opposite side}}{\text{Adjacent side}} = \frac{AB}{BC}$$

$$\text{Cosecant ratio of } \theta = \operatorname{cosec} \theta = \frac{\text{Hypotenuse}}{\text{Opposite side}} = \frac{AC}{AB}$$

$$\text{Secant ratio of } \theta = \sec \theta = \frac{\text{Hypotenuse}}{\text{Adjacent side}} = \frac{AC}{BC}$$

$$\text{Cotangent ratio of } \theta = \cot \theta = \frac{\text{Adjacent side}}{\text{Opposite side}} = \frac{BC}{AB}$$

- Relations between Trigonometric Ratios:**

$$(1) \operatorname{cosec} \theta = \frac{1}{\sin \theta} \quad (2) \sec \theta = \frac{1}{\cos \theta}$$

$$(3) \cot \theta = \frac{1}{\tan \theta} \quad (4) \tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$(5) \cot \theta = \frac{\cos \theta}{\sin \theta}$$

- Trigonometric Identities:**

$$(1) \sin^2 \theta + \cos^2 \theta = 1 \quad (2) 1 + \tan^2 \theta = \sec^2 \theta$$

$$(3) 1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$$

Table of Trigonometric Ratios for Angles 0° , 30° , 45° , 60° and 90°

Trigonometric Ratios	Angle θ				
	0°	30°	45°	60°	90°
$\sin \theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan \theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
$\operatorname{cosec} \theta = \frac{1}{\sin \theta}$	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\sec \theta = \frac{1}{\cos \theta}$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
$\cot \theta = \frac{1}{\tan \theta}$	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

MASTER KEY QUESTION SET - 6

Practice Set - 6.1 (Textbook Page No. 131)

- (1) If $\sin \theta = \frac{7}{25}$ then find $\cos \theta$ and $\tan \theta$. (2 marks)

Solution :

$$\begin{aligned}\sin \theta &= \frac{7}{25} \\ \sin^2 \theta + \cos^2 \theta &= 1 \\ \therefore \left(\frac{7}{25}\right)^2 + \cos^2 \theta &= 1 \\ \therefore \cos^2 \theta &= 1 - \frac{49}{625} \\ \therefore \cos^2 \theta &= \frac{625 - 49}{625} \\ \therefore \cos^2 \theta &= \frac{576}{625} \\ \therefore \cos \theta &= \frac{24}{25} \quad \dots(\text{Taking square roots}) \\ \tan \theta &= \frac{\sin \theta}{\cos \theta} \\ \tan \theta &= \frac{7}{25} \div \frac{24}{25} \\ \therefore \tan \theta &= \frac{7}{25} \times \frac{25}{24} \\ \therefore \tan \theta &= \frac{7}{24}\end{aligned}$$

Problem Set - 6 (Textbook Page No. 138)

- (2) If $\sin \theta = \frac{11}{61}$ find the values of $\cos \theta$ using trigonometric identity.

(2 marks)

Solution :

$$\begin{aligned}\sin^2 \theta + \cos^2 \theta &= 1 \\ \therefore \left(\frac{11}{61}\right)^2 + \cos^2 \theta &= 1 \\ \therefore \cos^2 \theta &= 1 - \frac{121}{3721} \\ \cos^2 \theta &= \frac{3721 - 121}{3721} \\ \therefore \cos^2 \theta &= \frac{3600}{3721} \\ \therefore \cos \theta &= \frac{60}{61} \quad \dots(\text{Taking square roots})\end{aligned}$$

Practice Set - 6.1 (Textbook Page No. 131)

- (2) If $\tan \theta = \frac{3}{4}$ then find the value of $\sec \theta$ and $\cos \theta$. (2 marks)

Solution :

$$\begin{aligned}\tan \theta &= \frac{3}{4} \\ 1 + \tan^2 \theta &= \sec^2 \theta \\ \therefore 1 + \left(\frac{3}{4}\right)^2 &= \sec^2 \theta \\ \therefore 1 + \frac{9}{16} &= \sec^2 \theta \\ \therefore \frac{16 + 9}{16} &= \sec^2 \theta \\ \therefore \sec^2 \theta &= \frac{25}{16} \\ \therefore \sec \theta &= \frac{5}{4} \quad \dots(\text{Taking square roots}) \\ \cos \theta &= \frac{1}{\sec \theta} \\ \therefore \cos \theta &= 1 \div \frac{5}{4} \\ \therefore \cos \theta &= 1 \times \frac{4}{5} \\ \therefore \cos \theta &= \frac{4}{5}\end{aligned}$$

Problem Set - 6 (Textbook Page No. 138)

- (3) If $\tan \theta = 2$ then find values of other trigonometric ratios. (3 marks)

Solution :

$$\tan \theta = 2$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$\therefore 1 + 2^2 = \sec^2 \theta$$

$$\therefore \sec^2 \theta = 1 + 4$$

$$\therefore \sec^2 \theta = 5$$

$$\therefore \boxed{\sec \theta = \sqrt{5}}$$

...(Taking square roots)

$$\cos \theta = \frac{1}{\sec \theta}$$

$$\therefore \boxed{\cos \theta = \frac{1}{\sqrt{5}}}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\therefore \sin \theta = \tan \theta \times \cos \theta$$

$$\therefore \sin \theta = 2 \times \frac{1}{\sqrt{5}}$$

$$\therefore \boxed{\sin \theta = \frac{2}{\sqrt{5}}}$$

$$\operatorname{cosec} \theta = \frac{1}{\sin \theta}$$

$$\therefore \boxed{\operatorname{cosec} \theta = \frac{\sqrt{5}}{2}}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\therefore \boxed{\cot \theta = \frac{1}{2}}$$

Practice Set - 6.1 (Textbook Page No. 131)

- (3) If $\cot \theta = \frac{40}{9}$, find the value of $\operatorname{cosec} \theta$ and $\sin \theta$. (2 marks)

Solution :

$$\cot \theta = \frac{40}{9}$$

$$\operatorname{cosec}^2 \theta = 1 + \cot^2 \theta$$

$$= 1 + \left(\frac{40}{9}\right)^2$$

$$= 1 + \frac{1600}{81}$$

$$= \frac{81 + 1600}{81}$$

$$\therefore \operatorname{cosec}^2 \theta = \frac{1681}{81}$$

$$\therefore \boxed{\operatorname{cosec} \theta = \frac{41}{9}}$$

...(Taking square roots)

$$\sin \theta = \frac{1}{\operatorname{cosec} \theta}$$

$$\therefore \sin \theta = \frac{1}{\frac{41}{9}}$$

$$\therefore \sin \theta = 1 \times \frac{9}{41}$$

$$\therefore \boxed{\sin \theta = \frac{9}{41}}$$

Problem Set - 6 (Textbook Page No. 138)

- (4) If $\sec \theta = \frac{13}{12}$, find values of other trigonometric ratios. (3 marks)

Solution :

$$\sec \theta = \frac{13}{12}$$

$$\cos \theta = \frac{1}{\sec \theta}$$

$$\therefore \boxed{\cos \theta = \frac{12}{13}}$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\therefore \sin^2 \theta + \left(\frac{12}{13}\right)^2 = 1$$

$$\therefore \sin^2 \theta = 1 - \frac{144}{169}$$

$$\therefore \sin^2 \theta = \frac{169 - 144}{169}$$

$$\therefore \sin^2 \theta = \frac{25}{169}$$

$$\therefore \boxed{\sin \theta = \frac{5}{13}}$$

...(Taking square roots)

$$\operatorname{cosec} \theta = \frac{1}{\sin \theta}$$

$$\therefore \boxed{\operatorname{cosec} \theta = \frac{13}{5}}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$= \frac{5}{13} \times \frac{12}{13}$$

$$= \frac{5}{13} \times \frac{13}{12}$$

$$\therefore \tan \theta = \frac{5}{12}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\therefore \cot \theta = \frac{12}{5}$$

Practice Set - 6.1 (Textbook Page No. 131)

- (4) If $5 \sec \theta - 12 \operatorname{cosec} \theta = 0$, find the values of $\sec \theta$, $\cos \theta$ and $\sin \theta$. (3 marks)

Solution :

$$5 \sec \theta - 12 \operatorname{cosec} \theta = 0$$

$$\therefore 5 \sec \theta = 12 \operatorname{cosec} \theta$$

$$\therefore \frac{5}{\cos \theta} = \frac{12}{\sin \theta}$$

$$\therefore \frac{\sin \theta}{\cos \theta} = \frac{12}{5}$$

$$\therefore \tan \theta = \frac{12}{5}$$

$$\sec^2 \theta = 1 + \tan^2 \theta$$

$$= 1 + \left(\frac{12}{5}\right)^2$$

$$= 1 + \frac{144}{25}$$

$$= \frac{25 + 144}{25}$$

$$\sec^2 \theta = \frac{169}{25}$$

$$\therefore \sec \theta = \frac{13}{5}$$

...(Taking square roots)

$$\cos \theta = \frac{1}{\sec \theta}$$

$$\therefore \cos \theta = \frac{1}{\frac{13}{5}}$$

$$\therefore \cos \theta = 1 \times \frac{5}{13}$$

$$\therefore \cos \theta = \frac{5}{13}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\therefore \sin \theta = \tan \theta \times \cos \theta$$

$$\therefore \sin \theta = \frac{12}{5} \times \frac{5}{13}$$

$$\therefore \sin \theta = \frac{12}{13}$$

- (5) If $\tan \theta = 1$ then find the value of $\frac{\sin \theta + \cos \theta}{\sec \theta + \operatorname{cosec} \theta}$ (4 marks)

Solution :

$$\tan \theta = 1$$

$$\sec^2 \theta = 1 + \tan^2 \theta$$

$$= 1 + (1)^2$$

$$= 1 + 1$$

$$\therefore \sec^2 \theta = 2$$

$$\therefore \sec \theta = \sqrt{2}$$

...(Taking square roots)

$$\cos \theta = \frac{1}{\sec \theta}$$

$$\therefore \cos \theta = \frac{1}{\sqrt{2}}$$

$$\tan \theta = 1$$

$$\therefore \frac{\sin \theta}{\cos \theta} = 1$$

$$\therefore \sin \theta = \cos \theta$$

$$\therefore \sin \theta = \frac{1}{\sqrt{2}}$$

$$\operatorname{cosec} \theta = \frac{1}{\sin \theta}$$

$$\therefore \operatorname{cosec} \theta = \sqrt{2}$$

$$\frac{\sin \theta + \cos \theta}{\sec \theta + \operatorname{cosec} \theta} = \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \right) \div (\sqrt{2} + \sqrt{2})$$

$$= \frac{2}{\sqrt{2}} \div 2\sqrt{2}$$

$$\therefore \frac{\sin \theta + \cos \theta}{\sec \theta + \operatorname{cosec} \theta} = \frac{2}{\sqrt{2}} \times \frac{1}{2\sqrt{2}}$$

$$\therefore \frac{\sin \theta + \cos \theta}{\sec \theta + \operatorname{cosec} \theta} = \frac{1}{2}$$

(6) Prove that: (2 marks)

(i) $\frac{\sin^2 \theta}{\cos \theta} + \cos \theta = \sec \theta$

Solution :

$$\begin{aligned} \text{Proof: LHS} &= \frac{\sin^2 \theta}{\cos \theta} + \cos \theta \\ &= \frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta} \\ &= \frac{1}{\cos \theta} \quad (\because \sin^2 \theta + \cos^2 \theta = 1) \\ &= \sec \theta \\ &= \text{R.H.S.} \end{aligned}$$

$$\therefore \frac{\sin^2 \theta}{\cos \theta} + \cos \theta = \sec \theta$$

(ii) $\cos^2 \theta (1 + \tan^2 \theta) = 1$ (2 marks)

Solution :

$$\begin{aligned} \text{Proof: LHS} &= \cos^2 \theta (1 + \tan^2 \theta) \\ &= \cos^2 \theta \times \sec^2 \theta \quad (\because 1 + \tan^2 \theta = \sec^2 \theta) \\ &= \cos^2 \theta \times \frac{1}{\cos^2 \theta} \\ &= 1 \\ &= \text{R.H.S.} \end{aligned}$$

$$\therefore \cos^2 \theta (1 + \tan^2 \theta) = 1$$

(iii) $\sqrt{\frac{1 - \sin \theta}{1 + \sin \theta}} = \sec \theta - \tan \theta$ (3 marks)

Solution :

$$\begin{aligned} \text{Proof: LHS} &= \sqrt{\frac{1 - \sin \theta}{1 + \sin \theta}} \\ &= \sqrt{\frac{(1 - \sin \theta)(1 - \sin \theta)}{(1 + \sin \theta)(1 - \sin \theta)}} \\ &= \sqrt{\frac{(1 - \sin \theta)^2}{1 - \sin^2 \theta}} \quad [\because (a + b)(a - b) = a^2 - b^2] \\ &= \sqrt{\frac{(1 - \sin \theta)^2}{1 - \sin^2 \theta}} \\ &= \sqrt{\frac{(1 - \sin \theta)^2}{\cos^2 \theta}} \quad (\because \sin^2 \theta + \cos^2 \theta = 1 \\ &\quad \therefore 1 - \sin^2 \theta = \cos^2 \theta) \\ &= \frac{1 - \sin \theta}{\cos \theta} \end{aligned}$$

$$\begin{aligned} &= \frac{1}{\cos \theta} - \frac{\sin \theta}{\cos \theta} \\ &= \sec \theta - \tan \theta \\ &= \text{R.H.S.} \end{aligned}$$

$$\therefore \sqrt{\frac{1 - \sin \theta}{1 + \sin \theta}} = \sec \theta - \tan \theta$$

(iv) $(\sec \theta - \cos \theta)(\cot \theta + \tan \theta) = \tan \theta \cdot \sec \theta$ (3 marks)

Solution :

$$\begin{aligned} \text{Proof: LHS} &= (\sec \theta - \cos \theta)(\cot \theta + \tan \theta) \\ &= \left(\frac{1}{\cos \theta} - \cos \theta \right) \left(\frac{\cos \theta}{\sin \theta} + \frac{\sin \theta}{\cos \theta} \right) \\ &= \left(\frac{1 - \cos^2 \theta}{\cos \theta} \right) \left(\frac{\cos^2 \theta + \sin^2 \theta}{\cos \theta \cdot \sin \theta} \right) \\ &= \frac{\sin^2 \theta}{\cos \theta} \times \frac{1}{\cos \theta \cdot \sin \theta} \quad (\because \sin^2 \theta + \cos^2 \theta = 1 \\ &\quad \therefore 1 - \cos^2 \theta = \sin^2 \theta) \\ &= \frac{\sin \theta}{\cos \theta} \times \frac{1}{\cos \theta} \\ &= \tan \theta \times \sec \theta \\ &= \text{R.H.S.} \end{aligned}$$

$$\therefore (\sec \theta - \cos \theta)(\cot \theta + \tan \theta) = \tan \theta \cdot \sec \theta$$

(v) $\cot \theta + \tan \theta = \operatorname{cosec} \theta \cdot \sec \theta$ (2 marks)

Solution :

$$\begin{aligned} \text{LHS} &= \cot \theta + \tan \theta \\ &= \frac{\cos \theta}{\sin \theta} + \frac{\sin \theta}{\cos \theta} \\ &= \frac{\cos^2 \theta + \sin^2 \theta}{\sin \theta \cdot \cos \theta} \\ &= \frac{1}{\sin \theta \cdot \cos \theta} \quad (\because \sin^2 \theta + \cos^2 \theta = 1) \\ &= \operatorname{cosec} \theta \cdot \sec \theta \\ &= \text{R.H.S.} \end{aligned}$$

$$\therefore \cot \theta + \tan \theta = \operatorname{cosec} \theta \cdot \sec \theta$$

(vi) $\frac{1}{\sec \theta - \tan \theta} = \sec \theta + \tan \theta$ (2 marks)

Solution :

$$\begin{aligned} \text{LHS} &= \frac{1}{\sec \theta - \tan \theta} \\ &= \frac{1 \times (\sec \theta + \tan \theta)}{(\sec \theta - \tan \theta)(\sec \theta + \tan \theta)} \\ &= \frac{\sec \theta + \tan \theta}{\sec^2 \theta - \tan^2 \theta} \\ &= \frac{\sec \theta + \tan \theta}{1} \quad \left(\begin{array}{l} \sec^2 \theta = 1 + \tan^2 \theta \\ \therefore \sec^2 \theta - \tan^2 \theta = 1 \end{array} \right) \end{aligned}$$

$$= \text{R.H.S.}$$

$$\therefore \frac{1}{\sec \theta - \tan \theta} = \sec \theta + \tan \theta$$

(vii) $\sin^4 \theta - \cos^4 \theta = 1 - 2\cos^2 \theta$ (3 marks)

Solution :

$$\begin{aligned} \text{Proof: LHS} &= \sin^4 \theta - \cos^4 \theta \\ &= (\sin^2 \theta)^2 - (\cos^2 \theta)^2 \\ &= (\sin^2 \theta + \cos^2 \theta)(\sin^2 \theta - \cos^2 \theta) \\ &= 1 (\sin^2 \theta - \cos^2 \theta) \left(\begin{array}{l} \sin^2 \theta + \cos^2 \theta = 1 \\ \therefore \sin^2 \theta = 1 - \cos^2 \theta \end{array} \right) \\ &= 1 - \cos^2 \theta - \cos^2 \theta \\ &= 1 - 2\cos^2 \theta \\ &= \text{R.H.S.} \\ \therefore \sin^4 \theta - \cos^4 \theta &= 1 - 2\cos^2 \theta \end{aligned}$$

(viii) $\sec \theta + \tan \theta = \frac{\cos \theta}{1 - \sin \theta}$ (3 marks)

Solution :

$$\begin{aligned} \text{Proof: LHS} &= \sec \theta + \tan \theta \\ &= \frac{1}{\cos \theta} + \frac{\sin \theta}{\cos \theta} \\ &= \frac{1 + \sin \theta}{\cos \theta} \\ &= \frac{(1 + \sin \theta)(1 - \sin \theta)}{\cos \theta (1 - \sin \theta)} \\ &= \frac{1 - \sin^2 \theta}{\cos \theta (1 - \sin \theta)} \\ &= \frac{\cos^2 \theta}{\cos \theta (1 - \sin \theta)} \left(\begin{array}{l} \sin^2 \theta + \cos^2 \theta = 1 \\ \therefore \cos^2 \theta = 1 - \sin^2 \theta \end{array} \right) \\ &= \frac{\cos \theta}{1 - \sin \theta} \\ &= \text{R.H.S.} \end{aligned}$$

$$\therefore \sec \theta + \tan \theta = \frac{\cos \theta}{1 - \sin \theta}$$

(ix) If $\tan \theta + \frac{1}{\tan \theta} = 2$ then show that prove

$$\tan^2 \theta + \frac{1}{\tan^2 \theta} = 2 \quad (2 \text{ marks})$$

Solution :

$$\begin{aligned} \text{Proof: } \tan \theta + \frac{1}{\tan \theta} &= 2 \\ \text{squaring both sides,} \\ \left(\tan \theta + \frac{1}{\tan \theta} \right)^2 &= 4 \\ \therefore \tan^2 \theta + 2 \tan \theta \cdot \frac{1}{\tan \theta} + \frac{1}{\tan^2 \theta} &= 4 \end{aligned}$$

$$\therefore \tan^2 \theta + 2 + \frac{1}{\tan^2 \theta} = 4$$

$$\therefore \tan^2 \theta + \frac{1}{\tan^2 \theta} = 4 - 2$$

$$\therefore \tan^2 \theta + \frac{1}{\tan^2 \theta} = 2$$

(x) $\frac{\tan A}{(1 + \tan^2 A)^2} + \frac{\cot A}{(1 + \cot^2 A)^2} = \sin A \cos A$. (3 marks)

Solution :

$$\begin{aligned} \text{Proof: LHS} &= \frac{\tan A}{(1 + \tan^2 A)^2} + \frac{\cot A}{(1 + \cot^2 A)^2} \\ &= \frac{\tan A}{(\sec^2 A)^2} + \frac{\cot A}{(\operatorname{cosec}^2 A)^2} \left(\begin{array}{l} \because 1 + \tan^2 A = \sec^2 A \\ \text{and } 1 + \cot^2 A = \operatorname{cosec}^2 A \end{array} \right) \\ &= \frac{\tan A}{\sec^4 A} + \frac{\cot A}{\operatorname{cosec}^4 A} \\ &= \tan A \cdot \cos^4 A + \cot A \cdot \sin^4 A \\ &= \frac{\sin A}{\cos A} \cos^4 A + \frac{\cos A}{\sin A} \sin^4 A \\ &= \sin A \cdot \cos^3 A + \cos A \cdot \sin^3 A \\ &= \sin A \cdot \cos A (\cos^2 A + \sin^2 A) \\ &= \sin A \cdot \cos A \cdot 1 \quad [\because \sin^2 A + \cos^2 A = 1] \\ &= \sin A \cdot \cos A \\ &= \text{R.H.S.} \end{aligned}$$

$$\therefore \frac{\tan A}{(1 + \tan^2 A)^2} + \frac{\cot A}{(1 + \cot^2 A)^2} = \sin A \cos A$$

(xi) $\sec^4 A (1 - \sin^4 A) - 2 \tan^2 A = 1$. (3 marks)

Solution :

$$\begin{aligned} \text{Proof: LHS} &= \sec^4 A (1 - \sin^4 A) - 2 \tan^2 A \\ &= \sec^4 A (1 + \sin^2 A) (1 - \sin^2 A) - 2 \tan^2 A \\ &= \sec^4 A (1 + \sin^2 A) \cos^2 A - 2 \tan^2 A \\ &\quad \left(\begin{array}{l} \sin^2 \theta + \cos^2 \theta = 1 \\ \therefore 1 - \sin^2 \theta = \cos^2 \theta \end{array} \right) \\ &= \frac{1}{\cos^4 A} (1 + \sin^2 A) \cos^2 A - \frac{2 \sin^2 A}{\cos^2 A} \\ &= \frac{1 + \sin^2 A}{\cos^2 A} - \frac{2 \sin^2 A}{\cos^2 A} \\ &= \frac{1 + \sin^2 A - 2 \sin^2 A}{\cos^2 A} \\ &= \frac{1 - \sin^2 A}{\cos^2 A} \left(\begin{array}{l} \sin^2 A + \cos^2 A = 1 \\ \therefore 1 - \sin^2 A = \cos^2 A \end{array} \right) \\ &= \frac{\cos^2 A}{\cos^2 A} \\ &= 1 \\ &= \text{R.H.S.} \\ \therefore \sec^4 A (1 - \sin^4 A) - 2 \tan^2 A &= 1 \end{aligned}$$

$$(xii) \quad \frac{\tan \theta}{\sec \theta - 1} = \frac{\tan \theta + \sec \theta + 1}{\tan \theta + \sec \theta - 1} \quad (3 \text{ marks})$$

Solution :

$$\begin{aligned} \text{Proof:} \quad 1 + \tan^2 \theta &= \sec^2 \theta \\ \therefore \tan^2 \theta &= \sec^2 \theta - 1 \\ \therefore \tan \theta \tan \theta &= (\sec \theta + 1)(\sec \theta - 1) \\ \therefore \frac{\tan \theta}{\sec \theta - 1} &= \frac{\sec \theta + 1}{\tan \theta} \\ \therefore \frac{\tan \theta}{\sec \theta - 1} &= \frac{\tan \theta + \sec \theta + 1}{\sec \theta - 1 + \tan \theta} \quad (\text{Theorem on equal ratios}) \\ \therefore \frac{\tan \theta}{\sec \theta - 1} &= \frac{\tan \theta + \sec \theta + 1}{\tan \theta + \sec \theta - 1} \end{aligned}$$

Problem Set - 6 (Textbook Page No. 138)

(5) Prove the following:

$$(i) \quad \sec \theta (1 - \sin \theta) (\sec \theta + \tan \theta) = 1 \quad (2 \text{ marks})$$

Solution :

$$\begin{aligned} \text{Proof: LHS} &= \sec \theta (1 - \sin \theta) (\sin \theta + \tan \theta) \\ &= \frac{1}{\cos \theta} (1 - \sin \theta) \times \left(\frac{1}{\cos \theta} + \frac{\sin \theta}{\cos \theta} \right) \\ &= \frac{(1 - \sin \theta)}{\cos \theta} \frac{(1 + \sin \theta)}{\cos \theta} \\ &= \frac{1 - \sin^2 \theta}{\cos^2 \theta} \quad \dots [\because (a + b)(a - b) = a^2 - b^2] \\ &= \frac{\cos^2 \theta}{\cos^2 \theta} \quad \left(\because \sin^2 \theta + \cos^2 \theta = 1 \right) \\ &= 1 \\ &= \text{R.H.S.} \\ \therefore \sec \theta (1 - \sin \theta) (\sec \theta + \tan \theta) &= 1 \end{aligned}$$

$$(ii) \quad (\sec \theta + \tan \theta) (1 - \sin \theta) = \cos \theta \quad (2 \text{ marks})$$

Solution :

$$\begin{aligned} \text{Proof: LHS} &= (\sec \theta + \tan \theta) (1 - \sin \theta) \\ &= \left(\frac{1}{\cos \theta} + \frac{\sin \theta}{\cos \theta} \right) (1 - \sin \theta) \\ &= \frac{(1 + \sin \theta)}{\cos \theta} (1 - \sin \theta) \\ &= \frac{1 - \sin^2 \theta}{\cos \theta} \quad \dots [\because (a + b)(a - b) = a^2 - b^2] \\ &= \frac{\cos^2 \theta}{\cos \theta} \quad \left(\because \sin^2 \theta + \cos^2 \theta = 1 \right) \\ &= \cos \theta \\ &= \text{R.H.S.} \\ \therefore (\sec \theta + \tan \theta) (1 - \sin \theta) &= \cos \theta \end{aligned}$$

$$(iii) \quad \sec^2 \theta + \operatorname{cosec}^2 \theta = \sec^2 \theta \times \operatorname{cosec}^2 \theta \quad (2 \text{ marks})$$

Solution :

$$\begin{aligned} \text{Proof: LHS} &= \sec^2 \theta + \operatorname{cosec}^2 \theta \\ &= \frac{1}{\cos^2 \theta} + \frac{1}{\sin^2 \theta} \\ &= \frac{\sin^2 \theta + \cos^2 \theta}{\cos^2 \theta \times \sin^2 \theta} \\ &= \frac{1}{\cos^2 \theta \times \sin^2 \theta} \quad \dots [\because \sin^2 \theta + \cos^2 \theta = 1] \\ &= \sec^2 \theta \cdot \operatorname{cosec}^2 \theta \\ &= \text{R.H.S.} \\ \therefore \sec^2 \theta + \operatorname{cosec}^2 \theta &= \sec^2 \theta \times \operatorname{cosec}^2 \theta \end{aligned}$$

$$(iv) \quad \cot^2 \theta - \tan^2 \theta = \operatorname{cosec}^2 \theta - \sec^2 \theta \quad (2 \text{ marks})$$

Solution :

$$\begin{aligned} \text{Proof: LHS} &= \cot^2 \theta - \tan^2 \theta \quad (\because 1 + \cot^2 \theta = \operatorname{cosec}^2 \theta) \\ &= (\operatorname{cosec}^2 \theta - 1) - (\sec^2 \theta - 1) \quad \dots (\because \sec^2 \theta = 1 + \tan^2 \theta) \\ &= \operatorname{cosec}^2 \theta - 1 - \sec^2 \theta + 1 \quad \dots (\because \tan^2 \theta = \sec^2 \theta - 1) \\ &= \operatorname{cosec}^2 \theta - \sec^2 \theta \\ &= \text{R.H.S.} \\ \therefore \cot^2 \theta - \tan^2 \theta &= \operatorname{cosec}^2 \theta - \sec^2 \theta \end{aligned}$$

$$(v) \quad \tan^4 \theta + \tan^2 \theta = \sec^4 \theta - \sec^2 \theta \quad (2 \text{ marks})$$

Solution :

$$\begin{aligned} \text{Proof: LHS} &= \tan^4 \theta + \tan^2 \theta \\ &= \tan^2 \theta (\tan^2 \theta + 1) \quad \left(\because 1 + \tan^2 \theta = \sec^2 \theta \right) \\ &= (\sec^2 \theta - 1) (\sec^2 \theta) \quad \dots (\because \tan^2 \theta = \sec^2 \theta - 1) \\ &= \sec^4 \theta - \sec^2 \theta \\ &= \text{R.H.S.} \\ \therefore \tan^4 \theta + \tan^2 \theta &= \sec^4 \theta - \sec^2 \theta \end{aligned}$$

$$(vi) \quad \frac{1}{1 - \sin \theta} + \frac{1}{1 + \sin \theta} = 2 \sec^2 \theta \quad (3 \text{ marks})$$

Solution :

$$\begin{aligned} \text{Proof: LHS} &= \frac{1}{1 - \sin \theta} + \frac{1}{1 + \sin \theta} \\ &= \frac{1 + \sin \theta + 1 - \sin \theta}{(1 - \sin \theta)(1 + \sin \theta)} \\ &= \frac{2}{1 - \sin^2 \theta} \\ &= \frac{2}{\cos^2 \theta} \quad \left(\because \sin^2 \theta + \cos^2 \theta = 1 \right) \\ &= 2 \sec^2 \theta \quad \left(\because \cos^2 \theta = 1 - \sin^2 \theta \right) \end{aligned}$$

$$\begin{aligned}
 &= 2 \sec^2 \theta \\
 &= \text{R.H.S.} \\
 \therefore \frac{1}{1 - \sin \theta} + \frac{1}{1 + \sin \theta} &= 2 \sec^2 \theta
 \end{aligned}$$

(vii) $\sec^6 x - \tan^6 x = 1 + 3 \sec^2 x \times \tan^2 x$ (3 marks)

Solution :

Proof: LHS = $\sec^6 x - \tan^6 x$

$$\begin{aligned}
 &= (\sec^2 x)^3 - (\tan^2 x)^3 \\
 &= (\sec^2 x - \tan^2 x)^3 + 3 \sec^2 x \tan^2 x (\sec^2 x + \tan^2 x) \\
 &\quad \dots [\because a^3 - b^3 = (a - b)^3 + 3ab(a + b)] \\
 &= (1)^3 + 3 \sec^2 x \cdot \tan^2 x (1) \\
 &\quad \left(\begin{array}{l} 1 + \tan^2 x = \sec^2 x \\ \therefore \sec^2 x - \tan^2 x = 1 \end{array} \right) \\
 &= 1 + 3 \sec^2 x \cdot \tan^2 x \\
 &= \text{R.H.S.}
 \end{aligned}$$

(viii) $\frac{\tan \theta}{\sec \theta + 1} = \frac{\sec \theta - 1}{\tan \theta}$ (4 marks)

Solution :

Proof: LHS = $\frac{\tan \theta}{\sec \theta + 1}$

$$\begin{aligned}
 &= \frac{\tan \theta}{\sec \theta + 1} \cdot \frac{\sec \theta - 1}{\sec \theta - 1} \\
 &= \frac{\tan \theta (\sec \theta - 1)}{\sec^2 \theta - 1} \quad [\because (a + b)(a - b) = a^2 - b^2] \\
 &= \frac{\tan \theta (\sec \theta - 1)}{\tan^2 \theta} \quad [\because 1 + \tan^2 \theta = \sec^2 \theta \\
 &\quad \therefore \sec^2 \theta - 1 = \tan^2 \theta] \\
 &= \frac{\sec \theta - 1}{\tan \theta} \\
 &= \text{R.H.S.} \\
 \therefore \frac{\tan \theta}{\sec \theta + 1} &= \frac{\sec \theta - 1}{\tan \theta}
 \end{aligned}$$

(ix) $\frac{\tan^3 \theta - 1}{\tan \theta - 1} = \sec^2 \theta + \tan \theta$ (3 marks)

Solution :

Proof: LHS = $\frac{\tan^3 \theta - 1}{\tan \theta - 1}$

$$\begin{aligned}
 &= \frac{\tan^3 \theta - 1^3}{\tan \theta - 1} \\
 &= \frac{(\tan \theta - 1)(\tan^2 \theta + \tan \theta + 1)}{(\tan \theta - 1)} \\
 &= \tan^2 \theta + 1 + \tan \theta \quad (\because \tan^2 \theta + 1 = \sec^2 \theta)
 \end{aligned}$$

$$\begin{aligned}
 &= \sec^2 \theta + \tan \theta \\
 &= \text{R.H.S.} \\
 \therefore \frac{\tan^3 \theta - 1}{\tan \theta - 1} &= \sec^2 \theta + \tan \theta
 \end{aligned}$$

(x) $\frac{\sin \theta - \cos \theta + 1}{\sin \theta + \cos \theta - 1} = \frac{1}{\sec \theta - \tan \theta}$ (4 marks)

Solution :

Proof: R.H.S. = $\frac{1}{\sec \theta - \tan \theta}$

$$\begin{aligned}
 &= 1 \left[\frac{1}{\cos \theta} - \frac{\sin \theta}{\cos \theta} \right] \\
 &= 1 \left[\frac{1 - \sin \theta}{\cos \theta} \right] \\
 &= \frac{\cos \theta}{1 - \sin \theta} \quad \dots(i)
 \end{aligned}$$

$$\begin{aligned}
 \sin^2 \theta + \cos^2 \theta &= 1 \\
 \therefore \cos^2 \theta &= 1 - \sin^2 \theta \\
 \therefore \cos \theta \cos \theta &= (1 - \sin \theta)(1 + \sin \theta)
 \end{aligned}$$

$$\begin{aligned}
 \therefore \frac{\cos \theta}{1 - \sin \theta} &= \frac{1 + \sin \theta}{\cos \theta} \\
 \therefore \frac{\cos \theta}{1 - \sin \theta} &= \frac{1 + \sin \theta - \cos \theta}{\cos \theta - (1 - \sin \theta)} \quad \dots(\text{By theorem on equal ratios})
 \end{aligned}$$

$$\therefore \frac{\cos \theta}{1 - \sin \theta} = \frac{1 + \sin \theta - \cos \theta}{\cos \theta - 1 + \sin \theta} \quad \dots(ii)$$

From (i) and (ii)

$$\therefore \frac{\sin \theta - \cos \theta + 1}{\sin \theta + \cos \theta - 1} = \frac{1}{\sec \theta - \tan \theta}$$

Points to Remember:

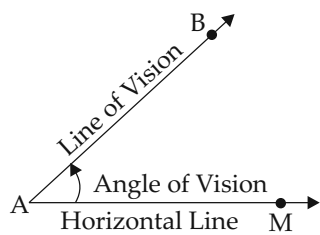
- **Application of trigonometry:**
Height and Distances:

Many times, we require to find the height of a tower, building, tree or distance of a ship from the lighthouse or width of the river etc. We cannot measure them actually, we can find the heights and distances with the help of trigonometric ratios.

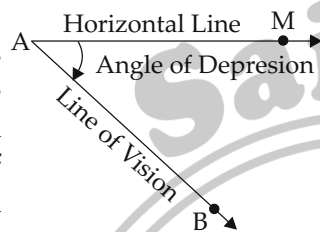
- Line of vision:** The line connecting the eye of the observer and the objects is called the **Line of vision**.

(ii) **Angle of Elevation:**

If A, B are two points such that B is at higher level than A and AM is horizontal line through A, then $\angle MAB$ is the **angle of elevation** of B with respect to A.



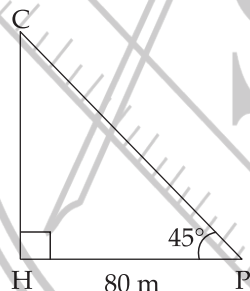
(iii) **Angle of Depression:** If A, B are two points such that B is at lower level than A and AM is the horizontal line through A, then $\angle MAB$ is the **angle of depression** of B with respect to A.

**Practice Set - 6.2 (Textbook Page No. 137)**

- (1) A person is standing at a distance of 80m from a church looking at its top. The angle of elevation is of 45° . Find the height of the church. (3 marks)

Solution :

CH represents the height of the church and C represents its top. P is the position of the person at a distance of 80 m from the church.



$$\therefore PH = 80 \text{ m}$$

$\angle CPH$ is the angle of the elevation

$$\therefore \angle CPH = 45^\circ$$

$$\text{In } \triangle PHC, \angle CHP = 90$$

$$\therefore \tan \angle CPH = \frac{CH}{PH} \quad \dots (\text{By definition})$$

$$\therefore \tan 45 = \frac{CH}{80}$$

$$\therefore 1 = \frac{CH}{80}$$

$$\therefore CH = 80 \text{ m}$$

$$\therefore \text{Height of the church is 80 m}$$

Problem Set - 6 (Textbook Page No. 139)

- (6) A boy standing at a distance of 48 meters from a building observes the top of the building and makes an angle of elevation of 30° . Find the height of the building. (3 marks)

Solution :

AB represents the height of the building. C represents the position of the boy at a distance of 48 m from the building.

$\angle ACB$ is the angle of elevation

$$\therefore \angle ACB = 30^\circ,$$

$$\text{In } \triangle ABC, \angle ABC = 90$$

$$\therefore \tan \angle ACB = \frac{AB}{BC}$$

$$\therefore \tan 30 = \frac{AB}{48}$$

$$\therefore \frac{1}{\sqrt{3}} = \frac{AB}{48}$$

$$\therefore AB = \frac{48}{\sqrt{3}}$$

$$\therefore AB = \frac{48 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}$$

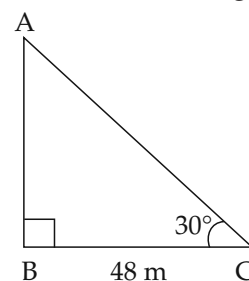
$$\therefore AB = \frac{48\sqrt{3}}{3}$$

$$\therefore AB = 16\sqrt{3}$$

$$\therefore AB = 16 \times 1.73$$

$$\therefore AB = 27.68 \text{ m}$$

$$\therefore \text{The height of the building is 27.68 m.}$$



...(By definition)

Practice Set - 6.2 (Textbook Page No. 137)

- (2) From the top of a lighthouse, an observer looking at a ship makes angle of depression of 60° . If the height of the lighthouse is 90 metre, then find how far the ship is from the lighthouse. ($\sqrt{3} = 1.73$) (3 marks)

Solution :

AB represents the height of the lighthouse.

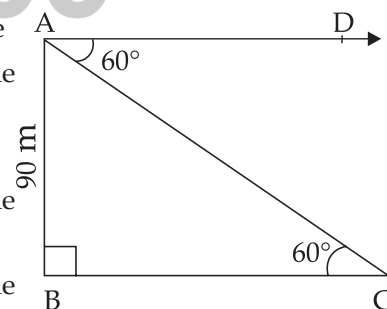
$$AB = 90 \text{ m}$$

C represents the position of ship.

$\angle DAC$ is the angle of depression

$$\therefore \angle DAC = 60^\circ,$$

$$\therefore \angle DAC = \angle ACB = 60^\circ \quad \dots (\text{Alternate angle theorem})$$



In $\triangle ABC$, $\angle ABC = 90^\circ$

$$\therefore \tan 60 = \frac{AB}{BC} \quad \dots(\text{By definition})$$

$$\therefore \sqrt{3} = \frac{90}{BC}$$

$$\therefore BC = \frac{90}{\sqrt{3}}$$

$$\therefore BC = \frac{90\sqrt{3}}{\sqrt{3} \times \sqrt{3}}$$

$$\therefore BC = \frac{90\sqrt{3}}{3}$$

$$\therefore BC = 30\sqrt{3}$$

$$\therefore BC = 30(1.73)$$

$$\therefore BC = 51.9 \text{ m}$$

The distance of the ship from the lighthouse is 51.90 m.

Problem Set - 6 (Textbook Page No. 139)

- (7) From the top of a lighthouse, an observer looks at a ship and finds the angle of depression to be 30° . If the height of the lighthouse is 100 m, then find how far is that ship from the lighthouse. (3 marks)

Solution :

AB represents the height of the lighthouse.

$$\therefore AB = 100 \text{ m}$$

C represents the position of ship.

$\angle DAC$ is the angle of depression.

$$\therefore \angle DAC = \angle ACB = 30^\circ \quad \dots(\text{BA alternate angle theorem})$$

In $\triangle ABC$, $\angle ABC = 90^\circ$

$$\therefore \tan \angle ACB = \frac{AB}{BC} \quad \dots(\text{By definition})$$

$$\therefore \tan 30 = \frac{100}{BC}$$

$$\therefore \frac{1}{\sqrt{3}} = \frac{100}{BC}$$

$$\therefore BC = 100\sqrt{3}$$

$$\therefore BC = 100(1.73)$$

$$\therefore BC = 173 \text{ m}$$

The distance of the ship from the lighthouse is 173 m.

Practice Set - 6.2 (Textbook Page No. 137)

- (3) Two buildings are facing each other on either side of a road of width 12 m. From the top of the first building, which is 10 m. high, the angle of elevation of the top of the second is 60° . What is the height of the second building? (4 marks)

Solution :

AB and CD represents the height of the two buildings, on either side of a road.

$$AB = 10 \text{ m}, BD = 12 \text{ m}$$

$\angle CAE$ is the angle of the elevation.

$$\therefore \angle CAE = 60^\circ,$$

$\square ABDE$ is a rectangle $\dots(\text{By definition})$

$$AB = DE = 10 \text{ m} \quad (\text{Opposite sides of a rectangle})$$

$$AE = BD = 12 \text{ m}$$

In $\triangle AEC$, $\angle AEC = 90^\circ$

$$\therefore \tan \angle CAE = \frac{CE}{AE} \quad \dots(\text{By definition})$$

$$\therefore \tan 60 = \frac{CE}{12}$$

$$\therefore \sqrt{3} = \frac{CE}{12}$$

$$\therefore CE = 12\sqrt{3}$$

$$\therefore CE = 12(1.73)$$

$$\therefore CE = 20.76 \text{ m}$$

$$CD = CE + DE \quad \dots(C - E - D)$$

$$= 20.76 + 10$$

$$CD = 30.76 \text{ m}$$

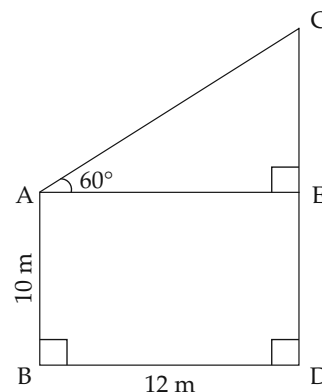
Height of the second building is 30.76 m.

Problem Set - 6 (Textbook Page No. 139)

- (8) Two buildings are in front of each other on a road of width 15 meters. From the top of the first building, having a height of 12 meter, the angle of elevation of the top of the second building is 30° . What is the height of the second building? (4 marks)

Solution :

AB and CD represents the height of two buildings at distance of 15 m, i.e. $BC = 15 \text{ m}$,



AB = 12 m

$\angle DAE$ is the angle of elevation.

$$\therefore \angle DAE = 30^\circ$$

$\square ABCE$ is a rectangle.

...(By definition)

$$\therefore AB = EC = 12 \text{ m}$$

$$BC = AE = 15 \text{ m}$$

...(Opposite sides of rectangle)

In $\triangle AED$, $\angle AED = 90^\circ$

$$\therefore \tan \angle DAE = \frac{DE}{AE}$$

...(By definition)

$$\therefore \tan 30^\circ = \frac{DE}{15}$$

$$\therefore \frac{1}{\sqrt{3}} = \frac{DE}{15}$$

$$\therefore DE = \frac{15}{\sqrt{3}}$$

$$= \frac{15 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}$$

$$= \frac{15\sqrt{3}}{3}$$

$$= 5\sqrt{3}$$

$$= 5 \times 1.73$$

$$\therefore DE = 8.65 \text{ m}$$

$$CD = CE + DE$$

...(C - E - D)

$$= 12 + 8.65$$

$$CD = 20.65 \text{ m}$$

$\therefore$ Height of the second building is 20.65 m.

Practice Set - 6.2 (Textbook Page No. 137)

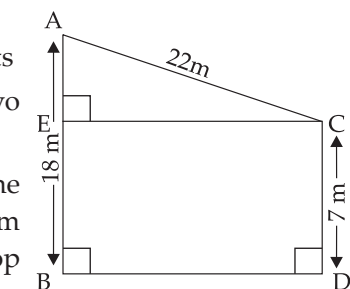
- (4) Two poles of heights 18 metre and 7 metre are erected on a ground. The length of the wire fastened at their tops in 22 metre. Find the angle made by the wire with the horizontal. (4 marks)

Solution :

AB and CD represents the height of the two poles.

AC is the length of the wire of length 22 m joining top A and top C of two poles.

$$AC = 22 \text{ m}$$



$\angle ACE$ is the angle made by the wire with the horizontal.

$\square EBDC$ is a rectangle (By definition)

$$BE = CD = 7 \text{ m}$$

...(Opposite sides of rectangle)

$$AB = AE + BE$$

...(A - E - B)

$$\therefore 18 = AE + 7$$

$$\therefore 18 - 7 = AE$$

$$\therefore AE = 11 \text{ m}$$

In $\triangle AEC$, $\angle AEC = 90^\circ$

$$\therefore \sin \angle ACE = \frac{AE}{AC}$$

...(By definition)

$$\therefore \sin \angle ACE = \frac{11}{22}$$

$$\therefore \sin \angle ACE = \frac{1}{2}$$

$$\text{But, } \sin 30^\circ = \frac{1}{2}$$

$$\therefore \sin \angle ACE = \sin 30^\circ$$

$$\therefore \angle ACE = 30^\circ$$

$\therefore$ The angle made by the wire with the horizontal is 30° .

- (5) A storm broke a tree and the treetop rested 20 m from the base of the tree, making an angle of 60° with the horizontal. Find the height of the tree. (4 marks)

Solution :

AB represents the height of the tree.

Tree breaks at D.

AD represents the broken part of the tree which takes the position DC.

$$\therefore AD = DC$$

$$\angle DCB = 60^\circ$$

$$BC = 20 \text{ m}$$

In $\triangle DBC$, $\angle DBC = 90^\circ$

$$\therefore \tan 60^\circ = \frac{DB}{BC}$$

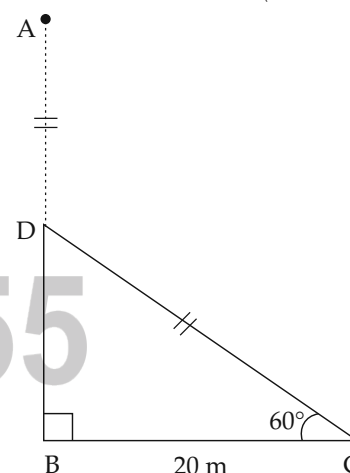
...(By definition)

$$\therefore \sqrt{3} = \frac{DB}{20}$$

$$\therefore DB = 20\sqrt{3}$$

$$= 20(1.73)$$

$$\therefore DB = 34.60 \text{ m}$$



$$\begin{aligned}\cos 60 &= \frac{BC}{DC} && \dots(\text{By definition}) \\ \therefore \frac{1}{2} &= \frac{20}{DC} \\ \therefore DC &= 40 \text{ m} \\ \therefore AD = DC &= 40 \text{ m} \\ \therefore AB = AD + DB &&& \dots(A - D - B) \\ \therefore AB &= 40 + 34.60 \\ \therefore AB &= 74.60 \text{ m} \\ \therefore \text{The height of the tree is } 74.60 \text{ m.}\end{aligned}$$

- (6) A kite is flying at a height of 60 m above the ground. The string attached to the kite is tied at the ground. It makes an angle of 60° with the ground. Assuming that the string is straight, find the length of the string. ($\sqrt{3} = 1.73$) (3 marks)

Solution :

'K' is the position of kite in the sky, 60 m above the ground level, KG represents the length of the string.

$\angle KGM$ is the angle between string and the ground $\angle KGM = 60^\circ$

In $\triangle KMG$, $\angle KMG = 90$

$$\begin{aligned}\therefore \sin \angle KGM &= \frac{KM}{GK} && \dots(\text{By definition}) \\ \therefore \sin 60 &= \frac{60}{GK} \\ \therefore \frac{\sqrt{3}}{2} &= \frac{60}{GK} \\ \therefore GK &= \frac{60 \times 2}{\sqrt{3}} \\ \therefore GK &= \frac{120}{\sqrt{3}} \\ \therefore GK &= \frac{120 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} \\ \therefore GK &= \frac{120\sqrt{3}}{3} \\ \therefore GK &= 40\sqrt{3} \\ \therefore GK &= 40 \times 1.73 \\ \therefore GK &= 69.20 \text{ m}\end{aligned}$$

$\therefore$ Length of the string is 69.20 m.

Problem Set - 6 (Textbook Page No. 139)

- (9) A ladder on the platform of a fire brigade van can be elevated at an angle of 70° to the maximum. The length of the ladder can be extended upto 20 m. If the platform is 2 m above the ground, find the maximum height from the ground upto which the ladder can reach. ($\sin 70^\circ = 0.94$)

(5 marks)

Solution :

GD is ground level. BC is base of the ladder of the fire brigades van at a height of 2 m from ground level.

'T' is top of ladder of the fire brigades van at the maximum height

$\angle TBC = 70^\circ$... (Angle of elevation)

BT is the length of the ladder

BT = 20 m, BG = 2 m

$\square BGDC$ is a rectangle ... (By definition)

BG = CD = 2 m ... (Opposite sides of a rectangle)

In $\triangle BCT$, $\angle BCT = 90$

$\therefore \sin \angle TBC = \frac{TC}{TB}$... (By definition)

$$\therefore \sin 70 = \frac{TC}{20}$$

$$\therefore 0.94 = \frac{TC}{20}$$

$$\therefore TC = 0.94 \times 20$$

$$\therefore TC = 18.80 \text{ m}$$

$$TD = TC + CD \quad \dots(T - C - D)$$

$$\therefore TD = 18.80 + 2$$

$$\therefore TD = 20.80 \text{ m}$$

$\therefore$ Other end of the ladder can reach 20.80 m above the ground ladder.

- * (10) While landing at an airport, a pilot made an angle of depression of 20° . Average speed of the plane was 200 km/hr. The plane reached the ground after 54 seconds. Find the height at which the plane was when it started landing.

($\sin 20^\circ = 0.342$)

(5 marks)

Solution :

A represents the position of the plane above the ground.

'C' is the landing point of the plane on the ground

AB represents the height of the plane from the ground.

$\angle DAC$ is the angle of depression

$$\angle DAC = \angle ACB = 20^\circ$$

$$\text{Distance (AC)} = \text{speed} \times \text{time}$$

$$= 200 \text{ km/hr} \times 54 \text{ sec}$$

$$= 200 \text{ km/hr} \times \frac{54}{3600} \text{ hr}$$

$$(\because 1 \text{ hr} = 3600 \text{ sec})$$

$$= 200 \times \frac{54}{3600}$$

$$= 3 \text{ km}$$

$$\therefore AC = 3000 \text{ m}$$

In $\triangle ABC$, $\angle ABC = 90^\circ$

$$\therefore \sin \angle ACB = \frac{AB}{AC} \quad \dots (\text{By definition})$$

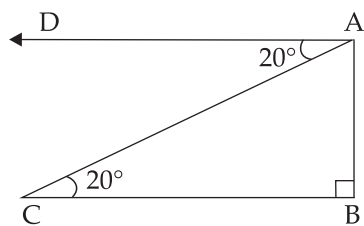
$$\therefore \sin 20^\circ = \frac{AB}{3000}$$

$$\therefore 0.342 = \frac{AB}{3000}$$

$$\therefore AB = 0.342 \times 3000$$

$$\therefore AB = 1026 \text{ km.}$$

$\therefore$ Plane was at a height of 1026 km, when it started landing.



(4) When we see at a higher level from the horizontal line, angle formed is

- (A) Angle of Elevation (B) Angle of Depression
(C) 0 (D) Straight angle

Additional MCQ's

(5) If $\sin \theta = \frac{4}{5}$ and $\cos \theta = \frac{3}{5}$, then $\tan \theta =$

- (A) $\frac{4}{3}$ (B) $\frac{3}{4}$
(C) $\frac{12}{25}$ (D) can not be calculated

(6) If $\operatorname{cosec} \theta = \frac{61}{60}$, $\sec \theta = \frac{61}{11}$, then $\cot \theta =$

- (A) $\frac{61^2}{660}$ (B) $\frac{60}{11}$
(C) $\frac{11}{60}$ (D) can not be calculated

(7) If $\sin \theta = \frac{24}{25}$, then $\cos \theta =$

- (A) $\frac{\sqrt{24}}{5}$ (B) $\frac{25}{24}$ (C) $\frac{25}{7}$ (D) $\frac{7}{25}$

(8) If $\tan \theta = 1$, then $\sec \theta =$

- (A) 1 (B) $\sqrt{2}$ (C) 2 (D) 0

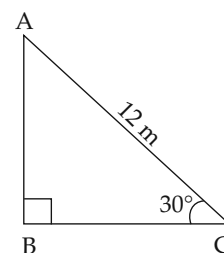
(9) If $\cot \theta = \frac{3}{4}$, then $\tan \theta =$

- (A) $\frac{4}{3}$ (B) $\frac{9}{16}$ (C) $\frac{16}{9}$ (D) $\frac{5}{4}$

(10) If $\operatorname{cosec} \theta = \frac{2}{\sqrt{3}}$, then $\theta =$

- (A) 0 (B) 45 (C) 30 (D) 60

(11) In the adjoining figure, if $\angle B = 90^\circ$, $\angle C = 30^\circ$, $AC = 12 \text{ m}$. then $AB =$



- (A) $12\sqrt{3} \text{ m}$ (B) $6\sqrt{3} \text{ m}$ (C) 12 m (D) 6 m

(12) If $(\sec \theta - 1)(\sec \theta + 1) = \frac{1}{3}$, then $\cos \theta =$

- (A) $\frac{1}{2}$ (B) $\frac{1}{\sqrt{2}}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{\sqrt{2}}{3}$

(13) If $\sin \theta + \cos \theta = a$, and $\sin \theta - \cos \theta = b$, then = ...

- (A) $a^2 + b^2 = 1$ (B) $a^2 - b^2 = 1$
(C) $a^2 + b^2 = 2$ (D) $a^2 - b^2 = 2$

Problem Set - 6 (Textbook Page No. 138)**MCQ's**

Choose the correct alternative answer for the following questions.

(1) $\sin \theta \cdot \operatorname{cosec} \theta =$

- (A) 1 (B) 0 (C) $\frac{1}{2}$ (D) $\sqrt{2}$

(2) $\operatorname{cosec} 45^\circ = ?$

- (A) $\frac{1}{\sqrt{2}}$ (B) $\sqrt{2}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{2}{\sqrt{3}}$

(3) $1 + \tan^2 \theta = ?$

- (A) $\cot^2 \theta$ (B) $\operatorname{cosec}^2 \theta$ (C) $\sec^2 \theta$ (D) $\tan^2 \theta$

- (14) If $\sin \theta = 1$, then find $\cot \theta = \dots\dots\dots$
 (A) 0 (B) 1 (C) $\sqrt{3}$ (D) $\frac{1}{\sqrt{3}}$

ANSWERS

- (1) (A) 1 (2) (B) $\sqrt{2}$ (3) (C) $\sec^2 \theta$
 (4) (A) Angle of Elevation (5) (A) $\frac{4}{3}$ (6) (C) $\frac{11}{60}$
 (7) (D) $\frac{7}{25}$ (8) (B) $\sqrt{2}$ (9) (A) $\frac{4}{3}$ (10) (D) 60
 (11) (D) 6 cm (12) (C) $\frac{\sqrt{3}}{2}$ (13) (C) $a^2 + b^2 = 2$
 (14) (A) 0

PROBLEMS FOR PRACTICE**Based on Practice Set 6.1**

- (1) If $\tan \theta = 2$, find the values of other trigonometric ratios using the identities. (3 marks)
 (2) If $\cot \theta = \frac{7}{24}$, find the values of other trigonometric ratios using the identity. (3 marks)
 (3) $3 \sin \theta - 4 \cos \theta = 0$, then find the values of all trigonometric ratios. (3 marks)
 (4) If $\sqrt{3} \tan \theta = 3 \sin \theta$, find the value of $\sin^2 \theta - \cos^2 \theta$. (3 marks)
 (5) Simplify : $\sin \theta (\operatorname{cosec} \theta - \sin \theta)$. (2 marks)
 (6) Prove: (3 marks each)
 (i) $\cos^2 \theta + \frac{1}{1 + \cot^2 \theta} = 1$
 (ii) $\frac{1}{1 + \sin \theta} + \frac{1}{1 - \sin \theta} = 2 \sec^2 \theta$
 (iii) $(1 + \tan^2 \theta) (1 + \sin \theta) (1 - \sin \theta) = 1$
 (iv) $(1 + \cot^2 \theta) (1 + \cos \theta) (1 - \cos \theta) = 1$
 (v) $\cot^2 \theta - \frac{1}{\sin^2 \theta} = -1$
 (vi) $\sin^4 \theta - \cos^4 \theta = 1 - 2 \cos^2 \theta$
 (vii) $\sec \theta + \tan \theta = \frac{1}{\sec \theta - \tan \theta}$
 (viii) $\frac{\cos \theta}{1 + \sin \theta} = \sec \theta - \tan \theta$
 (ix) $\frac{\tan^3 A - 1}{\tan A - 1} = \sec^2 A + \tan A$
 (x) $\frac{\sin \theta + \tan \theta}{\cos \theta} = \tan \theta (1 + \sec \theta)$
 (xi) $\operatorname{cosec}^2 A - \cos^2 A = \frac{\sec^2 A - \sin^2 A}{\tan^2 A}$
 (xii) $\left(\frac{1}{\cos \theta} + \frac{1}{\cot \theta} \right) = (\sec \theta - \tan \theta) = 1$

- (xiii) $\frac{\cos^2 A + \tan^2 A - 1}{\sin^2 A} = \tan^2 A$
 (xiv) $\frac{\tan A + \sec A - 1}{\tan A - \sec A + 1} = \frac{1 + \sin A}{\cos A}$
 (xv) $\frac{\cos^2 \theta}{1 - \tan \theta} + \frac{\sin^3 \theta}{\sin \theta - \cos \theta} = 1 + \sin \theta$

Based on Practice Set 6.2

- (7) For a person standing at a distance of 80 m from a temple, the angle of elevation of its top is 45° . Find the height of the church. (3 marks)
 (8) From the top of a lighthouse, an observer looks at a ship and finds the angle of depression to be 60° . If the lighthouse is 90 m, then find how far is that ship from the lighthouse? ($\sqrt{3} = 1.73$) (4 marks)
 (9) A building is $200\sqrt{3}$ metres high. Find the angle of elevation if its top is 200 m away from its foot. (2 marks)
 (10) A straight road leads to the foot of a tower of height 50 m. From the top of the tower, the angle of depression of two cars standing on the road are 30° and 60° . What is the distance between the two cars? (4 marks)
 (11) A ship of height 24 m is sighted from a lighthouse. From the top of the lighthouse, the angle of depression to the top of the mast and base of the ship is 30° and 45° respectively. How far is the ship from the lighthouse? ($\sqrt{3} = 1.73$) (4 marks)
 (12) From a point on the roof of a house, 11 m high, it is observed that the angles of depression of the top and foot of a lamp post are 30° and 60° respectively. What is the height of the lamp post? (4 marks)

ANSWERS

- | | $\sin \theta$ | $\cos \theta$ | $\tan \theta$ | $\cot \theta$ | $\sec \theta$ | $\operatorname{cosec} \theta$ |
|------|---|----------------------|---------------------|----------------|----------------|-------------------------------|
| (1) | $\frac{2}{\sqrt{5}}$ | $\frac{1}{\sqrt{5}}$ | 2 | $\frac{1}{2}$ | $\sqrt{5}$ | $\frac{\sqrt{5}}{2}$ |
| (2) | $\frac{24}{25}$ | $\frac{7}{25}$ | $\frac{24}{7}$ | $\frac{7}{24}$ | $\frac{25}{7}$ | $\frac{25}{24}$ |
| (3) | $\frac{4}{5}$ | $\frac{3}{5}$ | $\frac{4}{3}$ | $\frac{3}{4}$ | $\frac{5}{3}$ | $\frac{5}{4}$ |
| (4) | $\sin^2 \theta - \cos^2 \theta = \frac{1}{2}$ | | (5) $\cos^2 \theta$ | | | |
| (7) | 80 m | (8) | 51.9 m | (9) | 60 | |
| (10) | $\frac{100}{\sqrt{3}}$ | (11) | 56.76 m | (12) | 7.33 m | |

ASSIGNMENT – 6

Time : 1 Hr.

Marks : 20

Q.1. (A) Choose the correct alternative answer for the following:

(2)

- (1) $\sin \theta \cdot \operatorname{cosec} \theta = \dots\dots\dots$
 (A) 1 (B) 0 (C) $\frac{1}{2}$ (D) $\sqrt{2}$
- (2) If $\sin \theta = \frac{4}{5}$ and $\cos \theta = \frac{3}{5}$, then $\tan \theta = \dots\dots\dots$
 (A) $\frac{4}{3}$ (B) $\frac{3}{4}$ (C) $\frac{12}{25}$ (D) can not be calculated

Q.1. (B) Solve the following:

(2)

- (1) If $\sin \theta = \frac{\sqrt{3}}{2}$, then find θ .
- (2) Find the value of $\tan 40^\circ \times \tan 50^\circ$.

Q.2. Perform any one of the following activities

(2)

- (1) If $\tan \theta = 1$, then complete the following activity to find $\cos \theta$.

Sol.

$$1 + \tan^2 \theta = \boxed{} \dots(\text{Identity})$$

$$\therefore 1 + (1)^2 = \boxed{}$$

$$\therefore 2 = \boxed{}$$

Taking square roots

$$\therefore \boxed{} = \sqrt{2}$$

$$\therefore \cos \theta = \frac{1}{\boxed{}}$$

$$\therefore \cos \theta = \frac{1}{\sqrt{2}}$$

- (2) A boy is at a distance of 60 m from a tree, makes an angle of elevation of 60° with the top of the tree. What is the height of the tree?
- (3) Prove that $\frac{\sin^2 A}{\cos A} + \cos A = \sec A$.

Q.3. Solve the following:

(6)

- (1) If $x = r \cos \theta$ and $y = r \sin \theta$, then prove $x^2 + y^2 = r^2$
- (2) Using Pythagoras theorem, prove that $1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$,
- (3) Two poles of height 18 m and 7 m are erected on the ground. A wire of length 22 m tied to the top of the poles. Find the angle made by the wire with the horizontal.

Q.4. Solve any two of the following questions:

(8)

- (1) Prove that $\left(1 + \frac{1}{\tan^2 A}\right) \left(1 + \frac{1}{\cot^2 A}\right) = \frac{1}{\sin^2 A - \sin^4 A}$
- (2) In a right angled $\triangle ABC$, $\angle A = 90^\circ$ and $\frac{5\sin^2 B + 7\cos^2 C + 4}{3 + 8\tan^2 60^\circ} = \frac{7}{27}$. If $AC = 3$, find the perimeters of $\triangle ABC$.
- (3) Prove: $\frac{\cos^2 \theta}{1 - \tan \theta} + \frac{\sin^2 \theta}{\sin \theta - \cos \theta}$

