

Challenging Questions

1. Linear Equations in Two Variables

- (1) The weight of a bucket is 15 kg, when it is filled with water upto $\frac{3}{5}$ of its capacity. And the weight is 19 kg, if it is filled with water upto $\frac{4}{5}$ of its capacity. Find the weight of bucket, if it is completely filled with water. (4 marks)

Solution:

Let the weight of empty bucket be x kg and the weight of water the bucket can hold to its full capacity be y kg.

According to the first condition,

$$x + \frac{3}{5}y = 15$$

$$\therefore 5x + 3y = 75 \quad \dots(i)$$

According to the second condition,

$$x + \frac{4}{5}y = 19$$

$$\therefore 5x + 4y = 95 \quad \dots(ii)$$

Subtracting (ii) from (i),

$$\begin{array}{r r r r r} 5x & + & 3y & = & 75 \\ 5x & + & 4y & = & 95 \\ (-) & & (-) & & (-) \\ \hline & & -y & = & -20 \end{array}$$

$$\therefore y = 20$$

Substituting $y = 20$ in (i), we get,

$$5x + 3(20) = 75$$

$$\therefore 5x + 60 = 75$$

$$\therefore 5x = 75 - 60$$

$$\therefore 5x = 15$$

$$\therefore x = 3$$

$$\begin{array}{l} \therefore \text{The weight of bucket when it is} \\ \text{completely filled with water} = x + y \\ \phantom{\therefore \text{The weight of bucket when it is}} = 3 + 20 \\ \phantom{\therefore \text{The weight of bucket when it is}} = 23 \text{ kg.} \end{array}$$

- (2) Abdul travelled 300 km by train and 200 km by taxi, it took him 5 hours 30 minutes. But if he travels 260 km by train and 240 km by taxi, he takes 6 minutes longer. Find the speed of the train and that of the taxi. (4 marks)

Solution:

Let the speed of the train be x km/hr and speed of the taxi be y km/hr.

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

$$\text{Time taken to cover 300 km by train} = \frac{300}{x} \text{ hrs.}$$

$$\text{Time taken to cover 200 km by taxi} = \frac{200}{y} \text{ hrs.}$$

According to the first condition,

$$\frac{300}{x} + \frac{200}{y} = 5 \frac{30}{60} \quad [1 \text{ hour} = 60 \text{ min.}]$$

$$\therefore \frac{300}{x} + \frac{200}{y} = \frac{11}{2} \quad \dots(i)$$

Similarly,

$$\text{Time taken to cover 260 km by train} = \frac{260}{x} \text{ hrs.}$$

$$\text{Time taken to cover 240 km by taxi} = \frac{240}{y} \text{ hrs.}$$

According to the second condition,

$$\frac{260}{x} + \frac{240}{y} = 5 \frac{36}{60}$$

$$\therefore \frac{260}{x} + \frac{240}{y} = \frac{28}{5} \quad \dots(ii)$$

Substituting $\frac{1}{x} = m$ and $\frac{1}{y} = n$ in equations (i) and (ii),

$$300m + 200n = \frac{11}{2}$$

$$\text{i.e. } 600m + 400n = 11 \quad \dots(iii)$$

$$\text{Similarly, } 260m + 240n = \frac{28}{5}$$

$$\text{i.e. } 1300m + 1200n = 28 \quad \dots(iv)$$

Multiplying eq. (iii) by 3,

$$1800m + 1200n = 33 \quad \dots(v)$$

Subtracting (iv) from (v),

$$1800m + 1200n = 33$$

$$1300m + 1200n = 28$$

$$\begin{array}{r r r r r} (-) & & (-) & & (-) \\ \hline 500m & & & = & 5 \end{array}$$

$$\therefore m = \frac{5}{500}$$

$$\therefore m = \frac{1}{100}$$

Substituting value of m in equation (iii) we get,

$$600 \times \frac{1}{100} + 400n = 11$$

$$\therefore 6 + 400n = 11$$

$$\therefore 400n = 11 - 6$$

$$\therefore 400n = 5$$

$$\therefore n = \frac{5}{400}$$

$$\therefore n = \frac{1}{80}$$

Resubstituting the values of m and n ,

$$\frac{1}{x} = \frac{1}{100} \text{ and } \frac{1}{y} = \frac{1}{80}$$

$$\therefore x = 100 \text{ and } y = 80$$

The speed of the train is 100 km/hr and the speed of the taxi is 80 km/hr.

- (3) When the son will be as old as his father today, the sum of their ages then will be 126; when the father was as old as his son is today, the sum of their ages then was 38. Find their present ages.

(4 marks)

Solution:

Let the present age of father be x years and son be y years.

Son's age when he will be as old as his father today = x years.

$$\therefore \text{Father's age at that time} = [x + (x - y)] \text{ years}$$

According to the first condition,

$$x + x + (x - y) = 126$$

$$\therefore 3x - y = 126 \quad \dots(i)$$

Father's age when he was as old as son today = y years

$$\therefore \text{Son's age at that time} = [y - (x - y)] \text{ years}$$

According to the second condition,

$$y + y - (x - y) = 38$$

$$\therefore y + y - x + y = 38$$

$$\therefore -x + 3y = 38 \quad \dots(ii)$$

Multiplying (ii) by 3,

$$-3x + 9y = 114 \quad \dots(iii)$$

Adding (i) and (iii), we get,

$$\begin{array}{r} 3x - y = 126 \\ - \quad 3x + 9y = 114 \\ \hline 8y = 240 \end{array}$$

$$\therefore y = \frac{240}{8}$$

$$y = 30$$

Substituting the value of y in equation (i) we get,

$$3x - 30 = 126$$

$$\therefore 3x = 126 + 30$$

$$\therefore 3x = 156$$

$$\therefore x = \frac{156}{3}$$

$$\therefore x = 52$$

The present age of father and son are 52 years and 30 years respectively.

- (4) The forewheel of a carriage makes 6 revolutions more than the rearwheel in going 120 m. If the diameter of the forewheel be increased by $\frac{1}{4}$ its present diameter and the diameter of the rearwheel be increased by $\frac{1}{5}$ of its present diameter, then the forewheel makes 4 revolutions more than the rearwheel in going the same distance. Find the circumference of each wheel of the carriage. (4 marks)

Solution:

Let the circumference of the forewheel be x m and circumference of rearwheel be y m.

$$\text{Number of revolutions} = \frac{\text{Distance covered}}{\text{circumference}}$$

$$\therefore \text{Distance covered by each wheel} = 120 \text{ m}$$

$$\therefore \text{Number of revolutions made by forewheel} = \frac{120}{x}$$

$$\therefore \text{Number of revolutions made by rearwheel} = \frac{120}{y}$$

According to the first condition,

$$\frac{120}{x} = \frac{120}{y} + 6$$

$$\therefore \frac{120}{x} - \frac{120}{y} = 6$$

$$\therefore \frac{20}{x} - \frac{20}{y} = 1 \quad \dots(i)$$

Now, $c = \pi d$

$\therefore$ As diameter increases, circumference also increases.

$$\text{Increase in circumference of forewheel} = \frac{x}{4}$$

$$\begin{aligned} \text{New circumference of forewheel} &= x + \frac{x}{4} \\ &= \frac{4x + x}{4} \\ &= \left(\frac{5x}{4}\right) \text{ m} \end{aligned}$$

$$\begin{aligned} \text{New circumference of rearwheel} &= y + \frac{y}{5} \\ &= \frac{5y + y}{5} \\ &= \left(\frac{6y}{5}\right) \text{ m} \end{aligned}$$

According to the second condition,

$$\begin{aligned}\frac{120}{5x/4} &= \frac{120}{6y/5} + 4 \\ \therefore \frac{120 \times 4}{5x} &= \frac{120 \times 5}{6y} + 4 \\ \therefore \frac{96}{x} &= \frac{100}{y} + 4 \\ \therefore \frac{96}{x} - \frac{100}{y} &= 4 \quad \dots(\text{ii})\end{aligned}$$

Substituting $\frac{1}{x} = a$ and $\frac{1}{y} = b$, we get,

$$\begin{aligned}20a - 20b &= 1 \quad \dots(\text{iii}) \\ 96a - 100b &= 4 \quad \dots(\text{iv})\end{aligned}$$

Multiplying equation (iii) by 5, we get,

$$100a - 100b = 5 \quad \dots(\text{v})$$

Subtracting (v) from (iv),

$$\begin{array}{r} 96a - 100b = 4 \\ 100a - 100b = 5 \\ (-) \quad (+) \quad (-) \\ \hline -4a \quad \quad \quad = -1 \end{array}$$

$$\therefore a = \frac{1}{4}$$

Substituting value of $a = \frac{1}{4}$ in (iii), we get,

$$\begin{aligned}20 \times \frac{1}{4} - 20b &= 1 \\ \therefore 5 - 20b &= 1 \\ \therefore 5 - 1 &= 20b\end{aligned}$$

$$\therefore 4 = 20b$$

$$\therefore b = \frac{4}{20}$$

$$\therefore b = \frac{1}{5}$$

Resubstituting the values of a and b ,

$$a = \frac{1}{x} = \frac{1}{4} \text{ and } b = \frac{1}{y} = \frac{1}{5}$$

$$\therefore x = 4 \text{ and } y = 5$$

$\therefore$ Circumference of forewheel is 4 m and circumference of rearwheel is 5 m.

(5) Find the area of the triangle formed by the following lines and X axis.

$$4x - 3y + 4 = 0 \text{ and } 4x + 3y - 20 = 0 \quad (4 \text{ marks})$$

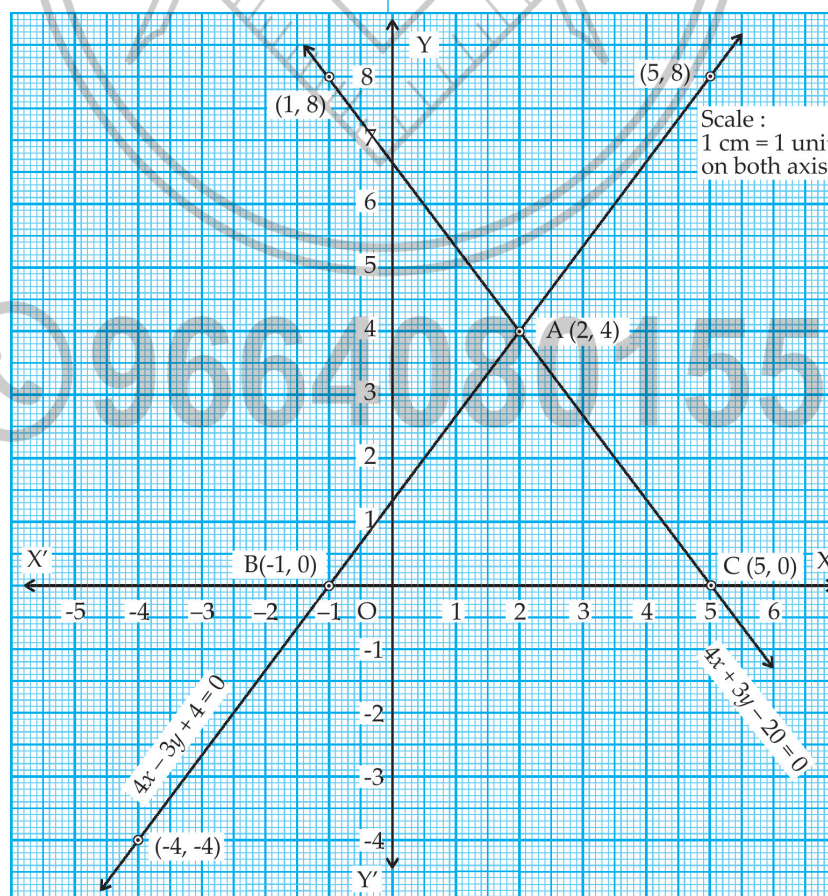
Solution:

$$4x - 3y + 4 = 0 \quad \therefore y = \frac{4x + 4}{3}$$

x	2	5	-4
y	4	8	-4
(x, y)	(2, 4)	(5, 8)	(-4, -4)

$$4x + 3y - 20 = 0 \quad \therefore y = \frac{20 - 4x}{3}$$

x	2	-1	5
y	4	8	0
(x, y)	(2, 4)	(-1, 8)	(5, 0)



For $\triangle ABC$, $A(2, 4)$, $B(-1, 0)$ and $C(5, 0)$

$$\begin{aligned}\therefore A(\triangle ABC) &= \frac{1}{2} \times b \times h \\ &= \frac{1}{2} \times 6 \times 4\end{aligned}$$

$$\therefore A(\triangle ABC) = 12 \text{ sq. units}$$

(6) Solve: $2^x + 3^y = 17$, $2^{x+2} - 3^{y+1} = 5$ (4 marks)

Solution:

$$\begin{aligned}2^x + 3^y &= 17 & \dots(i) \\ 2^{x+2} - 3^{y+1} &= 5\end{aligned}$$

$$\begin{aligned}\therefore 2^x \cdot 2^2 - 3^y \cdot 3^1 &= 5 \\ \therefore 4 \cdot 2^x - 3 \cdot 3^y &= 5 & \dots(ii)\end{aligned}$$

Substituting $2^x = m$ and $3^y = n$, we get,

$$m + n = 17 \quad \dots(iii)$$

$$4m - 3n = 5 \quad \dots(iv)$$

Multiplying eq. (iii) by 3,

$$3m + 3n = 51 \quad \dots(v)$$

Adding (iv) and (v) we get,

$$\begin{array}{rcl}4m - 3n & = & 5 \\ 3m + 3n & = & 51 \\ \hline 7m & = & 56\end{array}$$

$$\therefore m = \frac{56}{7}$$

$$\therefore m = 8$$

Substituting $m = 8$ in (iii),

$$8 + n = 17$$

$$\therefore n = 17 - 8$$

$$\therefore n = 9$$

Resubstituting the value of m and n , we get,

$$\begin{array}{lcl}m = 2^x & & n = 3^y \\ \therefore 8 = 2^x & & 9 = 3^y \\ \therefore 2^3 = 2^x & & 3^2 = 3^y \\ \therefore x = 3 & & y = 2\end{array}$$

$$\therefore x = 3 \text{ and } y = 2 \text{ is the solution.}$$

(7) A person deposits ₹ x in savings bank account at the rate of 5% per annum and ₹ y in fixed deposit at 10% per annum. At the end of one year, he gets ₹ 400 as total interest. If he deposits ₹ y in savings bank account and ₹ x in fixed deposit, he would get ₹ 350 as total interest. Find the total amount he deposited. (4 marks)

Solution:

Amount deposited in savings account = ₹ x

Amount deposited in fixed deposit account = ₹ y

$$\therefore \text{Total amount deposited} = ₹ (x + y)$$

Interest received on Savings account ₹ $\frac{5x}{100}$

Interest received on fixed deposit account ₹ $\frac{10y}{100}$

According to the first condition,

$$\frac{5x}{100} + \frac{10y}{100} = 400$$

$$\therefore 5x + 10y = 40,000 \quad \dots(i)$$

According to the second condition,

$$\frac{10x}{100} + \frac{5y}{100} = 350$$

$$\therefore 5y + 10x = 35,000$$

$$\therefore 10x + 5y = 35,000 \quad \dots(ii)$$

Adding equation (i) and (ii), we get,

$$5x + 10y = 40,000$$

$$10x + 5y = 35,000$$

$$15x + 15y = 75,000$$

$$\therefore x + y = 5,000 \quad [\text{Dividing both sides by 15}]$$

$$\therefore \text{Total amount deposited is ₹ 5000.}$$

(8) The sum of the digits of a number consisting of three digits is 12. The middle digit is equal to half of the sum of the other two. If the order of the digit be reversed, the number diminished by 198. Find the number. (4 marks)

Solution:

Digits	H	T	U
Original number	x	$\frac{1}{2}(x+y)$	y
Reversed number	y	$\frac{1}{2}(y+x)$	x

Let the digit in the hundredth place be x and the digit in the units place be y .

$$\therefore \text{The middle digit} = \frac{1}{2}(x+y)$$

The sum of the digits is 12. (Given)

$$\therefore x + \frac{1}{2}(x+y) + y = 12$$

$$\therefore 2x + x + y + 2y = 24$$

$$\therefore 3x + 3y = 24$$

$$\therefore x + y = 8 \quad \dots(i)$$

$$\text{The original number} = 100x + 10 \times \frac{1}{2}(x+y) + y$$

$$= 100x + 5(x+y) + y$$

$$= 100x + 5x + 5y + y$$

$$= 105x + 6y$$

$$\begin{aligned}
 \text{The reversed number} &= 100y + 10 \times \frac{1}{2}(y+x) + x \\
 &= 100y + 5(y+x) + x \\
 &= 100y + 5y + 5x + x \\
 &= 6x + 105y
 \end{aligned}$$

Now,

$$\text{Reversed number} = \text{Original number} - 198 \quad (\text{Given})$$

$$\begin{aligned}
 \therefore 6x + 105y &= 105x + 6y - 198 \\
 \therefore 6x - 105x + 105y - 6y &= -198 \\
 \therefore -99x + 99y &= -198 \\
 \therefore x - y &= 2 \quad \dots(ii)
 \end{aligned}$$

Adding (i) and (ii), we get,

$$\begin{array}{r}
 x + y = 8 \\
 x - y = 2 \\
 \hline
 2x = 10
 \end{array}$$

$$\therefore x = 5$$

Substituting $x = 5$ in (i),

$$5 + y = 8$$

$$\therefore y = 8 - 5$$

$$\therefore y = 3$$

$$\begin{aligned}
 \therefore \text{The required number} &= 105x + 6y \\
 &= 105 \times 5 + 6 \times 3 \\
 &= 525 + 18 \\
 &= 543
 \end{aligned}$$

$$\therefore \text{The required number is 543}$$

- (9) A train covered a certain distance at a uniform speed. If the train would have been 6 km/hr faster, it would have taken 4 hours less than the schedule time. And, if train were slower by 6 km/hr, it would have taken 6 hours more than the schedule time. Find the length of the journey. (4 marks)

Solution:

Let the original speed of train be x km/hr and the actual time taken be y hours.

$$\begin{aligned}
 \text{Then, distance} &= \text{speed} \times \text{time} \\
 &= (xy) \text{ km}
 \end{aligned}$$

According to the first condition,

$$\begin{aligned}
 (x+6)(y-4) &= xy \\
 \therefore xy - 4x + 6y - 24 &= xy \\
 \therefore -4x + 6y &= 24 \\
 \therefore -2x + 3y &= 12 \quad \dots(i)
 \end{aligned}$$

According to the second condition,

$$\begin{aligned}
 (x-6)(y+6) &= xy \\
 \therefore xy + 6x - 6y - 36 &= xy
 \end{aligned}$$

$$\therefore 6x - 6y = 36$$

$$\therefore x - y = 6 \quad \dots(ii)$$

Multiplying (ii) by 2, we get,

$$2x - 2y = 12 \quad \dots(iii)$$

Adding equations (i) and (iii) we get,

$$\begin{array}{r}
 -2x + 3y = 12 \\
 2x - 2y = 12 \\
 \hline
 y = 24
 \end{array}$$

Substituting the value of y in equation (i) we get,

$$x - 24 = 6$$

$$\therefore x = 6 + 24$$

$$\therefore x = 30$$

$\therefore$ The original speed = 30 km/hr and time = 24 hours.

$$\begin{aligned}
 \text{Hence, length of the journey} &= \text{speed} \times \text{time} \\
 &= 30 \times 24 \\
 &= 720 \text{ km.}
 \end{aligned}$$

$$\therefore \text{Length of the journey is 720 km.}$$

- (10) Solve : $(a-b)x + (a+b)y = a^2 - 2ab - b^2$ and $(a+b)(x+y) = a^2 + b^2$ (4 marks)

Solution:

$$(a-b)x + (a+b)y = a^2 - 2ab - b^2 \quad \dots(i)$$

$$(a+b)(x+y) = a^2 + b^2$$

$$\therefore (a+b)x + (a+b)y = a^2 + b^2 \quad \dots(ii)$$

Subtracting (ii) from (i),

$$\begin{array}{r}
 (a-b)x + (a+b)y = a^2 - 2ab - b^2 \\
 (a+b)x + (a+b)y = a^2 + b^2 \\
 (-) \quad \quad \quad (-) \quad \quad \quad (-) \quad \quad \quad (-) \\
 \hline
 [a-b-(a+b)]x = -2ab - 2b^2
 \end{array}$$

$$\therefore (a-b-a-b)x = -2b(a+b)$$

$$\therefore -2bx = -2b(a+b)$$

$$\therefore x = \frac{-2b(a+b)}{-2b}$$

$$\therefore x = (a+b)$$

Substituting value of x in (i), we get,

$$(a-b)(a+b) + (a+b)y = a^2 - 2ab - b^2$$

$$\therefore a^2 - b^2 + (a+b)y = a^2 - 2ab - b^2$$

$$\therefore (a+b)y = -2ab$$

$$\therefore y = \frac{-2ab}{a+b}$$

$$\therefore x = (a+b) \text{ and } y = \frac{-2ab}{a+b} \text{ is the solution of the given equations.}$$

2. Quadratic Equations

- (1) Two trains leave a railway station at the same time. The first train travels due west and the second train due north. The first train travels 5 km/hr faster than second train. If after two hours, they are 50 km apart, find the speed of each train. (4 marks)

Solution:

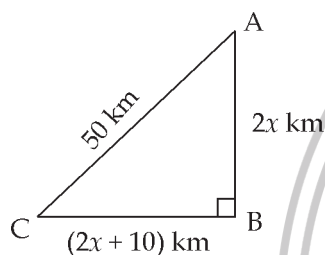
Let the speed of second train be x km/hr.

$\therefore$ Speed of first train = $(x + 5)$ km/hr

Distance = speed $\times$ time

Distance covered by second train = $x \times 2 = 2x$ km

Distance covered by first train = $(x + 5) \times 2$
= $(2x + 10)$ km



In $\triangle ABC$, $m\angle ABC = 90^\circ$

$AB^2 + BC^2 = AC^2$... [Pythagoras theorem]

$$\therefore (2x)^2 + (2x + 10)^2 = (50)^2$$

$$\therefore 4x^2 + 4x^2 + 40x + 100 = 2500$$

$$\therefore 8x^2 + 40x + 100 - 2500 = 0$$

$$\therefore 8x^2 + 40x - 2400 = 0$$

$$\therefore x^2 + 5x - 300 = 0 \quad \dots \text{(Dividing both sides by 8)}$$

$$\therefore x^2 + 20x - 15x - 300 = 0$$

$$\therefore x(x + 20) - 15(x + 20) = 0$$

$$\therefore (x + 20)(x - 15) = 0$$

$$\therefore x + 20 = 0 \text{ or } x - 15 = 0$$

$$\therefore x = -20 \text{ or } x = 15$$

$x \neq -20$ as speed cannot be negative.

$$\therefore x = 15 \text{ and } x + 5 = 15 + 5 = 20$$

The speeds of the trains are 20 km/hr and 15 km/hr respectively.

- (2) If the sum of the roots of the quadratic equation $ax^2 + bx + c = 0$ is equal to the sum of the squares of their reciprocals, then prove that $2a^2c = c^2b + b^2a$. (4 marks)

Solution:

Let α and β be the roots of the given quadratic equation.

$$\therefore \alpha + \beta = \frac{-b}{a} \text{ and } \alpha\beta = \frac{c}{a}$$

As per given condition,

$$\alpha + \beta = \frac{1}{\alpha^2} + \frac{1}{\beta^2}$$

$$\therefore \alpha + \beta = \frac{\alpha^2 + \beta^2}{\alpha^2\beta^2}$$

$$\therefore \alpha + \beta = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{(\alpha\beta)^2}$$

$$\therefore \frac{-b}{a} = \frac{\left(\frac{-b}{a}\right)^2 - 2\left(\frac{c}{a}\right)}{\left(\frac{c}{a}\right)^2}$$

$$\therefore \frac{-b}{a} = \left[\frac{b^2}{a^2} - \frac{2c}{a}\right] \div \frac{c^2}{a^2}$$

$$\therefore \frac{-b}{a} = \left[\frac{b^2 - 2ac}{a^2}\right] \div \frac{c^2}{a^2}$$

$$\therefore \frac{-b}{a} = \frac{b^2 - 2ac}{a^2} \times \frac{a^2}{c^2}$$

$$\therefore \frac{-b}{a} = \frac{b^2 - 2ac}{c^2}$$

$$\therefore bc^2 = a(b^2 - 2ac)$$

$$\therefore bc^2 = ab^2 - 2a^2c$$

$$\therefore 2a^2c = c^2b + b^2a \quad \dots \text{Hence proved.}$$

- (3) If the roots of the quadratic equation $ax^2 + cx + c = 0$ are in the ratio $p : q$, then show that

$$\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{c}{a}} = 0 \quad (4 \text{ marks})$$

Solution:

Let α and β be the roots of the given quadratic equation.

$$\therefore \alpha + \beta = \frac{-c}{a} \text{ and } \alpha\beta = \frac{c}{a}$$

$$\text{Also, } \frac{\alpha}{\beta} = \frac{p}{q} \quad \dots \text{(Given)}$$

$$\begin{aligned} \therefore \sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{c}{a}} &= \sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} + \sqrt{\alpha\beta} \\ &= \frac{\sqrt{\alpha}}{\sqrt{\beta}} + \frac{\sqrt{\beta}}{\sqrt{\alpha}} + \sqrt{\alpha\beta} \\ &= \frac{\alpha + \beta}{\sqrt{\alpha\beta}} + \sqrt{\alpha\beta} \end{aligned}$$

$$= \frac{\alpha + \beta + \alpha\beta}{\sqrt{\alpha\beta}}$$

$$= \frac{-\frac{c}{a} + \frac{c}{a}}{\sqrt{\alpha\beta}}$$

$$= \frac{0}{\sqrt{\alpha\beta}}$$

$$= 0$$

$$\therefore \sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{c}{a}} = 0$$

- (4) A businessman bought some items for ₹ 600. Keeping 10 items for himself he sold the remaining items at a profit of ₹ 5 per item. From the amount received in this deal he could buy 15 more items. Find the original price of each item. (4 marks)

Solution:

Total cost price of some items = ₹ 600 (given)

Let the original cost price of 1 item = ₹ x

$$\therefore \text{Number of items purchased} = \frac{600}{x}$$

$$\begin{aligned} \text{Number of items sold after keeping 10 items} \\ = \frac{600}{x} - 10 \end{aligned}$$

$$\text{Selling price of each item} = ₹ (x + 5)$$

$$\text{Total selling price of all items} = ₹ (x + 5) \left(\frac{600}{x} - 10 \right)$$

Profit = S.P. - C.P.

$\therefore$ Net profit made in the deal

$$= ₹ (x + 5) \left(\frac{600}{x} - 10 \right) - 600$$

But, profit is equal to cost price of 15 items = ₹ $15x$

As per given condition,

$$(x + 5) \left(\frac{600}{x} - 10 \right) - 600 = 15x$$

$$\therefore x \left(\frac{600}{x} - 10 \right) + 5 \left(\frac{600}{x} - 10 \right) - 600 = 15x$$

$$\therefore 600 - 10x + \frac{3000}{x} - 50 - 600 - 15x = 0$$

$$\therefore -10x + \frac{3000}{x} - 50 - 15x = 0$$

$$\therefore -25x + \frac{3000}{x} - 50 = 0$$

$$\therefore -25x^2 + 3000 - 50x = 0$$

...(Multiplying both sides by x)

$$\therefore x^2 - 120 + 2x = 0 \quad \text{...(Dividing both sides by } -25)$$

$$\therefore x^2 + 2x - 120 = 0$$

$$\therefore x^2 + 12x - 10x - 120 = 0$$

$$\therefore x(x + 12) - 10(x + 12) = 0$$

$$\therefore (x + 12)(x - 10) = 0$$

$$\therefore x + 12 = 0 \text{ or } x - 10 = 0$$

$$\therefore x = -12 \text{ or } x = 10$$

$$\therefore x \neq -12 \text{ as cost price cannot be negative.}$$

$$\therefore x = 10$$

$$\therefore \text{Original price of each item is ₹ 10.}$$

- (5) A man travels by boat 36 km down a river and back in 8 hours. If the speed of his boat in still water is 12 km/hr, find the speed of the river current. (4 marks)

Solution:

Let the speed of river current = x km/hr

Speed of boat in still water = 12 km/hr

$$\therefore \text{Speed of boat up the river} = (12 - x) \text{ km/hr}$$

Speed of boat down the river = $(12 + x)$ km/hr

$$\text{Time} = \frac{\text{Distance}}{\text{speed}}$$

$$\therefore \text{Time taken by boat to travel 36 km down the river}$$

$$= \left(\frac{36}{12 + x} \right) \text{ hrs.}$$

$$\therefore \text{Time taken by boat to travel 36 km up the river}$$

$$= \left(\frac{36}{12 - x} \right) \text{ hrs.}$$

As per given condition,

$$\frac{36}{12 + x} + \frac{36}{12 - x} = 8$$

$$36 \left(\frac{1}{12 + x} + \frac{1}{12 - x} \right) = 8$$

$$\therefore \frac{12 - x + 12 + x}{(12 + x)(12 - x)} = \frac{8}{36}$$

$$\therefore \frac{24}{144 - x^2} = \frac{2}{9}$$

$$\therefore 108 = 144 - x^2$$

$$\therefore x^2 = 144 - 108$$

$$\therefore x^2 = 36$$

$$\therefore x = \pm 6$$

$\therefore x \neq -6$ as speed cannot be negative.

$\therefore x = 6.$

$\therefore$ **The speed of river current = 6 km/hr.**

- (6) A number consists of two digits whose product is 56. When 9 is subtracted from the number, the digits interchange their places. Find the number. (4 marks)

Solution:

Let the ten's digit = x and unit's digit = y

So, the number = $10x + y$

Product of the digits = 56

$\therefore xy = 56$... (i)

On subtracting 9 from the number, the digits are interchanged.

$\therefore (10x + y) - 9 = 10y + x$

$\therefore 10x - x + y - 10y = 9$

$\therefore 9x - 9y = 9$

$\therefore x - y = 1$

$\therefore x - \frac{56}{x} = 1$

... [From (i)]

$\therefore x^2 - 56 = x$

$\therefore x^2 - x - 56 = 0$

$\therefore x^2 - 8x + 7x - 56 = 0$

$\therefore x(x - 8) + 7(x - 8) = 0$

$\therefore (x - 8)(x + 7) = 0$

$\therefore x - 8 = 0$ or $x + 7 = 0$

$\therefore x = 8$ or $x = -7$

$\therefore x \neq -7$ as it is a digit and cannot be negative.

$\therefore x = 8.$

Now, $8 \times y = 56$

... [From (i)]

$\therefore y = \frac{56}{8}$

$\therefore y = 7$

$\therefore$ **Hence, the number is $10 \times 8 + 7 = 87$.**

- (7) Two pipes running together can fill a cistern in $3\frac{1}{13}$ minutes. If one pipe takes 3 minutes more than the other to fill it, find the time in which each pipe would fill the cistern. (4 marks)

Solution:

Let the time taken by other pipe to fill the cistern be x minutes.

$\therefore$ Time taken by first pipe to fill the cistern = $(x + 3)$ minutes

Portion of cistern filled by other pipe in one minute = $\frac{1}{x}$ and portion of cistern filled by first pipe in one minute = $\frac{1}{x+3}$

Position of cistern filled by both the pipes in one

minute = $\frac{1}{3\frac{1}{13}}$

= $\frac{1}{\frac{40}{13}}$

= $\frac{13}{40}$

As per given condition,

$\frac{1}{x} + \frac{1}{x+3} = \frac{13}{40}$

$\therefore \frac{x+3+x}{x(x+3)} = \frac{13}{40}$

$\therefore \frac{2x+3}{x^2+3x} = \frac{13}{40}$

$\therefore 40(2x+3) = 13(x^2+3x)$

$\therefore 80x+120 = 13x^2+39x$

$\therefore 13x^2-41x-120 = 0$

$\therefore 13x^2-65x+24x-120 = 0$

$\therefore 13x(x-5)+24(x-5) = 0$

$\therefore (x-5)(13x+24) = 0$

$\therefore x-5 = 0$ or $13x+24 = 0$

$\therefore x = 5$ or $x = \frac{-24}{13}$

$\therefore x \neq \frac{-24}{13}$ as time cannot be negative.

$\therefore x = 5$ and $x+3 = 5+3 = 8$

$\therefore$ Time taken by two pipes to fill the cistern separately are 5 minutes and 8 minutes.

- (8) If the sum of roots of the quadratic equation is

$\frac{1}{x+p} + \frac{1}{x+q} = \frac{1}{r}$ is zero, show that the product

of the roots is $-\left(\frac{p^2+q^2}{2}\right)$. (4 marks)

Solution

$\frac{1}{x+p} + \frac{1}{x+q} = \frac{1}{r}$

$$\therefore \frac{x+q+x+p}{(x+p)(x+q)} = \frac{1}{r}$$

$$\therefore \frac{2x+q+p}{x^2+qx+px+pq} = \frac{1}{r}$$

$$\therefore r(2x+p+q) = x^2+qx+px+pq$$

$$\therefore 2rx+pr+qr = x^2+qx+px+pq$$

$$\therefore 0 = x^2+qx+px+pq-2rx-pr-qr$$

$$\therefore x^2+qx+px-2rx+pq-pr-qr = 0$$

$$\therefore x^2+x(q+p-2r)+pq-pr-qr = 0$$

$$\therefore x^2+(p+q-2r)x+(pq-pr-qr) = 0 \quad \dots(i)$$

Comparing with $ax^2+bx+c=0$, we get,

$$a=1, b=p+q-2r, c=pq-pr-qr$$

Let α and β be the roots of equation (i).

$$\text{Let } \alpha + \beta = \frac{-b}{a}$$

$$= \frac{-(p+q-2r)}{1}$$

$$\alpha + \beta = -p-q+2r$$

$$\text{But } \alpha + \beta = 0$$

$$\therefore -p+q-2r = 0$$

$$\therefore 2r = p+q$$

$$\therefore r = \frac{p+q}{2}$$

$$\text{Also, } \alpha\beta = \frac{c}{a}$$

$$= \frac{pq-pr-qr}{1}$$

$$= pq-pr-qr$$

$$= pq-r(p+q)$$

$$= pq - \left(\frac{p+q}{2}\right)(p+q) \quad \dots [r = \frac{p+q}{2}]$$

$$= pq - \frac{(p+q)^2}{2}$$

$$= \frac{2pq - (p+q)^2}{2}$$

$$= \frac{2pq - (p^2 + 2pq + q^2)}{2}$$

$$= \frac{2pq - p^2 - 2pq - q^2}{2}$$

$$= \frac{-p^2 - q^2}{2}$$

$$\therefore \alpha\beta = -\left(\frac{p^2+q^2}{2}\right)$$

(9) Solve : $\sqrt{x^2-16} - \sqrt{x^2-8x+16} = \sqrt{x^2-5x+4}$ (4 marks)

Solution

$$\sqrt{x^2-16} - \sqrt{x^2-8x+16} = \sqrt{x^2-5x+4}$$

$$\therefore \sqrt{(x+4)(x-4)} - \sqrt{(x-4)^2} = \sqrt{(x-4)(x-1)}$$

$$\therefore \sqrt{(x+4)(x-4)} - \sqrt{(x-4)^2} - \sqrt{(x-4)(x-1)} = 0$$

$$\therefore \sqrt{x-4} [\sqrt{x+4} - \sqrt{(x-4)} - \sqrt{x-1}] = 0$$

$$\therefore \sqrt{x-4} = 0 \text{ or } \sqrt{x+4} - \sqrt{x-4} - \sqrt{x-1} = 0$$

$$\therefore x-4=0 \text{ or } \sqrt{x+4} - \sqrt{x-4} = \sqrt{x-1}$$

$$\therefore x=4 \text{ or } \sqrt{x+4} - \sqrt{x-4} = \sqrt{x-1}$$

$$\text{Now, } \sqrt{x+4} - \sqrt{x-4} = \sqrt{x-1} \quad \dots(i)$$

$$\therefore (\sqrt{x+4} - \sqrt{x-4})^2 = x-1 \quad \dots(\text{Squaring both sides})$$

$$\therefore x+4-2\sqrt{(x+4)(x-4)}+x-4=x-1$$

$$\therefore 2x-2\sqrt{x^2-16}=x-1$$

$$\therefore 2x-x+1=2\sqrt{x^2-16}$$

$$\therefore x+1=2\sqrt{x^2-16}$$

$$\therefore (x+1)^2 = 4(x^2-16) \quad \dots(\text{Squaring both sides})$$

$$\therefore x^2+2x+1=4x^2-64$$

$$\therefore 4x^2-x^2-2x-64-1=0$$

$$\therefore 3x^2-2x-65=0$$

$$\therefore 3x^2-15x+13x-65=0$$

$$\therefore 3x(x-5)+13(x-5)=0$$

$$\therefore (3x+13)(x-5)=0$$

$$\therefore 3x+13=0 \text{ or } x-5=0$$

$$\therefore x = \frac{-13}{3} \text{ or } x=5$$

Now, $x \neq \frac{-13}{3}$ as it does not satisfy the (i)

$\therefore$

Hence, the roots of the given equation are 4 and 5.

3. Arithmetic Progression

- (1) 200 logs of wood are stacked in the following manner: 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on. In how many rows 200 logs are placed and how many logs are there in the top row? (4 marks)

Solution:

The arrangement of logs

20, 19, 18, ... forms an A.P. with $a = 20$, $d = -1$

Let 200 logs be arranged in n rows.

$$S_n = 200$$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$\therefore 200 = \frac{n}{2} [2 \times 20 + (n-1)(-1)]$$

$$\therefore 400 = n [40 - n + 1]$$

$$\therefore 400 = n [41 - n]$$

$$\therefore 400 = 41n - n^2$$

$$\therefore n^2 - 41n + 400 = 0$$

$$\therefore n^2 - 25n - 16n + 400 = 0$$

$$\therefore n(n-25) - 16(n-25) = 0$$

$$\therefore (n-25)(n-16) = 0$$

$$\therefore n-25 = 0 \text{ or } n-16 = 0$$

$$\therefore n = 25 \text{ or } n = 16$$

If $n = 25$

$$t_n = a + (n-1)d$$

$$\therefore t_{25} = 20 + (25-1)(-1)$$

$$= 20 + 24(-1)$$

$$= 20 - 24$$

$$t_{25} = -4$$

No. of logs in the 25th row cannot be negative

$$\therefore n \neq 25 \quad \therefore n = 16$$

$$t_n = a + (n-1)d$$

$$\therefore t_{16} = 20 + (16-1)(-1)$$

$$= 20 + 15(-1)$$

$$= 20 - 15$$

$$t_{16} = 5$$

$\therefore$ 200 logs are placed in 16 rows and there are 5 logs in the top row.

- (2) If sum of m terms is n and sum of n terms is m , then show that the sum of $(m+n)$ terms is $-(m+n)$. (4 marks)

Solution:

$$\text{Given: } S_m = n, S_n = m$$

$$\text{To prove: } S_{m+n} = -(m+n)$$

Proof:

$$S_m = \frac{m}{2} [2a + (m-1)d]$$

$$\therefore n = \frac{m}{2} [2a + (m-1)d] \quad \dots(i)$$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$m = \frac{n}{2} [2a + (n-1)d] \quad \dots(ii)$$

Subtracting (ii) from (i)

$$\therefore n - m = \frac{m}{2} [2a + (m-1)d] - \frac{n}{2} [2a + (n-1)d]$$

$$\therefore 2(n-m) = m[2a + (m-1)d] - n[2a + (n-1)d]$$

$$\therefore 2(n-m) = m[2a + md - d] - n[2a + nd - d]$$

$$\therefore 2(n-m) = 2am + m^2d - md - 2an - n^2d + nd$$

$$\therefore 2(n-m) = 2am - 2an + m^2d - n^2d - md + nd$$

$$\therefore 2(n-m) = 2a(m-n) + d(m^2 - n^2) - d(m-n)$$

$$\therefore -2(m-n) = 2a(m-n) + d(m+n)(m-n) - d(m-n)$$

Dividing throughout by $m-n$,

$$\therefore -2 = 2a + d(m+n) - d$$

$$\therefore -2 = 2a + d(m+n-1) \quad \dots(iii)$$

$$\text{Now, } S_{m+n} = \frac{m+n}{2} [2a + d(m+n-1)]$$

$$\therefore S_{m+n} = \frac{m+n}{2} [-2] \quad \dots[\text{From (iii)}]$$

$$\therefore S_{m+n} = m+n(-1)$$

$$\therefore S_{m+n} = -(m+n)$$

- (3) How many terms of the A.P. 16, 14, 12, ... are needed to give the sum 60? Explain why we get two answers. (4 marks)

Solution:

For the A.P. 16, 14, 12, ...

$$a = 16, d = 14 - 16 = -2$$

$$\text{Let } S_n = 60$$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$\therefore 60 = \frac{n}{2} [2 \times 16 + (n-1)(-2)]$$

$$\therefore 120 = n [32 - 2n + 2]$$

$$\therefore 120 = n [34 - 2n]$$

$$\therefore 120 = 34n - 2n^2$$

$$\therefore 2n^2 - 34n + 120 = 0$$

$$\therefore n^2 - 17n + 60 = 0 \dots [\text{Dividing both sides by 2}]$$

$$\therefore n^2 - 12n - 5n + 60 = 0$$

$$\therefore n(n - 12) - 5(n - 12) = 0$$

$$\therefore (n - 12)(n - 5) = 0$$

$$\therefore n - 12 = 0 \text{ or } n - 5 = 0$$

$$\therefore n = 12 \text{ or } n = 5$$

$\therefore$ The number of terms required to give a sum of 60 are 12 or 5.

The reason for getting two answers is the common difference is -2 and the progression is towards the negative side.

The A. P. is as follows:

16, 14, 12, 10, 8, 6, 4, 2, 0, -2, -4, -6, ...

$$S_5 = 16 + 14 + 12 + 10 + 8 = 60$$

$$S_{12} = 16 + 14 + 12 + 10 + 8 + 6 + 4 + 2 + 0 - 2 - 4 - 6 = 60$$

(4) If p^{th} , q^{th} and r^{th} terms of an A. P. are l , m , n respectively, show that

$$(q - r)l + (r - p)m + (p - q)n = 0 \quad (4 \text{ marks})$$

Solution:

$$\text{Given : } t_p = l, t_q = m, t_r = n$$

Let the 1st term of given A.P. be a and d be the common difference.

$$t_n = a + (n - 1)d$$

$$\therefore t_p = a + (p - 1)d$$

$$\therefore l = a + (p - 1)d \quad \dots(i)$$

$$\therefore t_q = a + (q - 1)d$$

$$\therefore m = a + (q - 1)d \quad \dots(ii)$$

$$\therefore t_r = a + (r - 1)d$$

$$\therefore n = a + (r - 1)d \quad \dots(iii)$$

Subtracting (ii) from (i), we get

$$l = a + (p - 1)d$$

$$- m = a + (q - 1)d$$

$$\begin{array}{r} (-) \quad (-) \quad (-) \\ l - m = (p - 1)d - (q - 1)d \end{array}$$

$$\therefore l - m = (p - 1 - q + 1)d$$

$$\therefore l - m = (p - q)d$$

$$\therefore \frac{l - m}{d} = (p - q) \quad \dots(iv)$$

Subtracting (iii) from (ii), we get

$$m = a + (q - 1)d$$

$$n = a + (r - 1)d$$

$$\begin{array}{r} (-) \quad (-) \quad (-) \\ m - n = (q - 1)d - (r - 1)d \end{array}$$

$$\therefore m - n = (q - 1 - r + 1)d$$

$$\therefore m - n = (q - 1 - r + 1)d$$

$$\therefore m - n = (q - r)d$$

$$\therefore \frac{m - n}{d} = (q - r) \quad \dots(v)$$

Subtracting (i) from (iii), we get

$$n = a + (r - 1)d$$

$$l = a + (p - 1)d$$

$$\begin{array}{r} (-) \quad (-) \quad (-) \\ n - l = (r - 1)d - (p - 1)d \end{array}$$

$$\therefore n - l = (r - 1 - p + 1)d$$

$$\therefore n - l = (r - 1 - p + 1)d$$

$$\therefore n - l = (r - p)d$$

$$\therefore \frac{n - l}{d} = (r - p) \quad \dots(vi)$$

Now, $(q - r)l + (r - p)m + (p - q)n$

$$= \left(\frac{m - n}{d} \right) l + \left(\frac{n - l}{d} \right) m + \left(\frac{l - m}{d} \right) n$$

...[From (iv), (v), (vi)]

$$= \frac{1}{d} [lm - ln + mn - lm + ln - mn]$$

$$= 0$$

Hence proved.

(5) Jinal saves ₹ 1600 during the first year, ₹ 2100 in the second year, ₹ 2600 in the third year, If she continues her saving in this pattern, in how many years will she save ₹ 38,500? (4 marks)

Solution:

Pattern of her saving is ₹ 1600, ₹ 2100, ₹ 2600, ...

It is an A.P. with $a = 1600$, $d = 2100 - 1600 = 500$

Also, $S_n = 38500$

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

$$\therefore 38500 = \frac{n}{2} [2 \times 1600 + (n - 1)500]$$

$$\therefore 77000 = n [3200 + 500n - 500]$$

$$\therefore 77000 = n [2700 + 500n]$$

$$\therefore 77000 = 2700n + 500n^2$$

$$\therefore 500n^2 + 2700n - 77000 = 0$$

$$\therefore 5n^2 + 27n - 770 = 0$$

...[Dividing both sides by 100]

$$\therefore 5n^2 + 27n - 50n - 770 = 0$$

$$\therefore n(5n + 27) - 10(5n + 77) = 0$$

$$\therefore (n - 10)(5n + 77) = 0$$

$$\therefore n - 10 = 0 \text{ or } 5n + 77 = 0$$

$$\therefore n = 10 \text{ or } n = \frac{-77}{5}$$

Now $n = \frac{-77}{5}$ (Not possible as n is natural number)

$$\therefore n = 10$$

Jinal will save ₹ 38500 in 10 years.

- (6) Find the middle term of sequence formed by all three digit numbers which leave a remainder 3 when divided by 4. Also find sum of all numbers on both sides of the middle term. (4 marks)

Solution:

The sequence formed by the given numbers is 103, 107, 111, 115, ..., 999.

This is an A.P. with $a = 103$, $d = 107 - 103 = 4$ and $t_n = 999$

$$t_n = a + (n-1)d$$

$$\therefore 999 = 103 + (n-1)4$$

$$\therefore 896 = 4n - 4$$

$$\therefore 4n = 900$$

$$\therefore n = \frac{900}{4}$$

$$\therefore n = 225$$

$$\therefore \text{Middle term} = \left(\frac{n+1}{2} \right)^{\text{th}} \text{ term}$$

$$= \left(\frac{225+1}{2} \right)^{\text{th}} \text{ term}$$

$$= \frac{226}{2}$$

$$= 113^{\text{th}} \text{ term.}$$

$$t_n = a + (n-1)d$$

$$\therefore t_{113} = 103 + (113-1)4$$

$$\therefore = 103 + 112 \times 4$$

$$= 103 + 448$$

$$t_{113} = 551$$

$$\text{Now, } t_{112} = 551 - 4 = 547$$

So, we have to find S_{112} and $(S_{225} - S_{113})$

$$S_n = \frac{n}{2} [t_1 + t_n]$$

$$\therefore S_{112} = \frac{112}{2} [103 + 547]$$

$$\therefore = \frac{112}{2} \times 650$$

$$\therefore S_{112} = 36400$$

Similarly,

$$S_{225} - S_{113} = \frac{225}{2} (103 + 999) - \frac{113}{2} (103 + 551)$$

$$= (225 \times 551) - (113 \times 327)$$

$$= 123975 - 36951$$

$$= 87024$$

$\therefore$ Sum of all members on LHS of middle term is 36400 and sum of all numbers on RHS of the middle term is 87024.

4. Financial Planning

- (1) Mr. Modi invested ₹ 30,120 in equity shares of FV ₹10, when the market value was ₹ 60. After receiving dividend on them at 90%, he sold them at MV of ₹ 55. In each transaction he paid 0.4%. What was the total gain or loss in this transaction? (4 marks)

Solution:

Rate of GST is not given, so it is not considered.

Shares Purchased : FV = ₹ 10, MV = ₹ 60

$$\text{Brokerage per share} = \frac{0.4}{100} \times 60$$

$$= ₹ 0.24$$

$$\therefore \text{Cost of one share} = 60 + 0.24$$

$$= ₹ 60.24$$

$$\text{Number of shares purchased} = \frac{30120}{60.24}$$

$$= 500$$

Shares sold : FV = ₹ 10, MV = ₹ 55

$$\therefore \text{Brokerage per share} = \frac{0.4}{100} \times 55$$

$$= ₹ 0.22$$

$$\therefore \text{Selling price per share} = 55 - 0.22$$

$$= ₹ 54.78$$

$$\therefore \text{Selling price of 500 shares} = 500 \times 54.78$$

$$= ₹ 27,390$$

Dividend received 90%

$$\therefore \text{Dividend per share} = \frac{90}{100} \times 10$$

$$= ₹ 9$$

$$\therefore \text{Dividend of 500 shares} = 9 \times 500$$

$$= ₹ 4,500$$

Mr. Modi's income = ₹ 27,390 + ₹ 4,500

$$= ₹ 31,890$$

Sum invested = ₹ 30,120

$$\therefore \text{Profit} = 31,890 - 30,120 = ₹ 1,770$$

$\therefore$ **Mr. Modi made a profit of ₹ 1770 in the whole transaction.**

- (2) Usha Joshi invested an equal amount in two companies by purchasing equity shares with MV ₹ 145 and ₹ 160 each. The FV is same and it is ₹ 100 for both the shares. At the end of the year, both companies declared the dividends at 20% and 30% each; In which company was her investment profitable? (4 marks)

Solution:

Rate of GST and brokerage is not given, so it is not considered.

Suppose the amount invested by Usha Joshi in both the companies is ₹ 23,200 (i.e. ₹ 145 × ₹ 160)

	Company 1	Company 2
Amount Invested	₹ 23,200	₹ 23,200
MV per share	₹ 145	₹ 160
Brokerage per share [∵ Brokerage is not mentioned]	₹ 0	₹ 0
∴ cost of one share	₹ 145	₹ 160
No. of shares purchased	$\frac{23200}{145} = 160$	$\frac{23200}{160} = 145$
Dividend declared	20%	30%
FV per share	₹ 100	₹ 100
Dividend per share	$= \frac{20}{100} \times 100$ $= ₹ 20$	$= \frac{30}{100} \times 100$ $= ₹ 30$
Dividend on shares =	160×20 $= ₹ 3200$	145×30 $= ₹ 4350$

∴ Company 2 gives more amount of dividend than Company 1.

∴ **Investment in company 2 is more profitable.**

- (3) Mr. Deepak Pal invested ₹ 1,00,354 in shares of FV ₹ 100, when the market value is ₹ 50. Rate of brokerage is 0.3% and rate of GST on brokerage is 18%, then how many shares were purchased?

(4 marks)

Solution:

FV = ₹ 100, MV = ₹ 50

∴ Brokerage per share = 0.3% of ₹ 50

$$= \frac{0.3}{100} \times 50$$

$$= ₹ 0.15$$

∴ GST per share = 18 % of ₹ 0.15

$$= \frac{18}{100} \times 0.15$$

$$= ₹ 0.027$$

$$\text{Cost of 1 share} = \text{MV} + \text{Brokerage} + \text{GST}$$

$$= 50 + 0.15 + 0.027$$

$$= ₹ 50.177$$

$$\therefore \text{Number of shares} = \frac{\text{Total investment}}{\text{Cost of 1 share}}$$

$$= \frac{1,00,354}{50.177}$$

$$= 2000$$

∴ **Mr. Deepak Pal purchased 2000 shares.**

- (4) Star Pharma purchased some chemicals for ₹ 8,000 (with GST) and sold it to the M/s. Pooja Chemicals for ₹ 10,000 (with GST). Rate of GST is 18%. Find the amount of CGST and SGST to be paid by Star Pharma. (4 marks)

Solution:

Total value (including GST) = Taxable value + GST

∴ The ratio of $\frac{\text{Total Value}}{\text{Taxable Value}}$ is constant as the

rate of GST is same.

- (i) For total value of ₹ 118, the taxable value is ₹ 100 and for total value of ₹ 8000, let taxable value be ₹ x.

$$\therefore \frac{x}{8000} = \frac{100}{118}$$

$$\therefore x = \frac{8000 \times 100}{118}$$

$$\therefore x = ₹ 6779.66$$

∴ GST paid at the time of the purchase

$$= 8000 - 6779.66$$

$$= ₹ 1220.34$$

∴ Input tax = ₹ 1220.34

∴ ITC = ₹ 1220.34

...(i)

- (ii) For total value of ₹ 10,000, let taxable value be ₹ y.

$$\therefore \frac{y}{10000} = \frac{100}{118}$$

$$\therefore y = \frac{10000 \times 100}{118}$$

$$\therefore y = ₹ 8474.58$$

∴ Output Tax = 10000 - 8474.58

$$= ₹ 1525.42$$

...(ii)

∴ GST payable = Output tax - ITC

$$= ₹ 1525.42 - ₹ 1220.34$$

[From i, ii]

$$= ₹ 305.08$$

∴ Payable CGST = Payable SGST

$$= \frac{305.08}{2}$$

$$= ₹ 152.54$$

∴ Star Pharma has to pay ₹ 152.54 CGST and ₹ 152.54 SGST.

- (5) A manufacturer sold electric goods for a taxable value of ₹ 40,000 to the wholeseller. Wholeseller sold it to the retailer for ₹ 48,000 (taxable value). Retailer sold it to a customer for ₹ 52,000 (taxable value). Rate of GST is 18%. Find the CGST and SGST payable at each stage of trading. Also show statement of GST payable at each stage of trading. (4 marks)

Solution:

$$\text{Output tax of manufacturer} = 12\% \text{ of } 40,000$$

$$= \frac{12}{100} \times 40,000$$

$$= ₹ 4800$$

$$\therefore \text{GST payable by manufacturer} = ₹ 4800$$

$$\text{Output tax of wholeseller} = 12\% \text{ of } 48,000$$

$$= \frac{12}{100} \times 48,000$$

$$= ₹ 5760$$

$$\therefore \text{GST payable by wholeseller} = \text{Output tax} - \text{ITC}$$

$$= 5760 - 4800$$

$$= ₹ 960$$

$$\text{Output tax of Retailer} = 12\% \text{ of } 52,000$$

$$= \frac{12}{100} \times 52,000$$

$$= ₹ 6240$$

$$\therefore \text{GST payable by Retailer} = \text{Output tax} - \text{ITC}$$

$$= 6240 - 5760$$

$$= ₹ 480$$

Statement of GST payable at each stage of trading.

Persons	Payable GST	Payable CGST	Payable SGST
Manufacturer	₹ 4800	₹ 2400	₹ 2400
Wholeseller	₹ 960	₹ 480	₹ 480
Retailer	₹ 480	₹ 240	₹ 240
Total Tax	₹ 6240	₹ 3120	₹ 3120

- (6) Mr. Joshi purchased 250 shares of FV ₹ 100 for MV of ₹ 500. Find the sum invested. After taking 40% dividend, he sold all the shares when market value was ₹ 400. He paid 0.1% brokerage for each trading done. Find the percentage of profit or loss in the share trading. (4 marks)

Solution:

$$\text{Shares Purchased : FV} = ₹ 100, \text{ MV} = ₹ 500$$

$$\text{No. of shares} = 250$$

$$\text{Brokerage per share} = \frac{0.1}{100} \times 500$$

$$= ₹ 0.5$$

$$\therefore \text{Cost of one share} = \text{MV} + \text{Brokerage}$$

$$= 500 + 0.5$$

$$= ₹ 500.5$$

$$\therefore \text{Cost of 250 shares} = 250 \times 500.5$$

$$= ₹ 1,25,125$$

Sum invested by Mr. Joshi is ₹ 1,25,125.

$$\text{Shares sold : FV} = ₹ 100, \text{ MV} = ₹ 400$$

$$\text{Brokerage per share} = \frac{0.1}{100} \times 400$$

$$= ₹ 0.4$$

$$\therefore \text{Selling price of per share} = \text{MV} - \text{Brokerage}$$

$$= 400 - 0.4$$

$$= ₹ 399.6$$

$$\therefore \text{Selling price of 250 shares} = 250 \times 399.6$$

$$= ₹ 99,900$$

$$\text{Dividend received } 40\%.$$

$$\therefore \text{Dividend per share} = \frac{40}{100} \times 100$$

$$= ₹ 40$$

$$\therefore \text{Dividend on 250 shares} = 40 \times 250$$

$$= ₹ 10,000$$

$$\therefore \text{Mr. Joshi's income} = ₹ 99,900 + ₹ 10,000$$

$$= ₹ 1,09,900$$

$$\text{But, sum invested by Mr. Joshi} = ₹ 1,25,125$$

$$\therefore \text{Loss} = 1,25,125 - 1,09,900$$

$$= ₹ 15,225$$

$$\text{Loss Percent} = \frac{\text{Loss}}{\text{Amount invested}} \times 100$$

$$= \frac{15,225}{1,25,125} \times 100$$

$$= 12.17$$

∴ **Mr. Joshi incurred a loss of 12.17% in this trading.**

5. Probability

- (1) Two customers Sumit and Amit are visiting a particular shop in the same week (Tuesday to Saturday). Each is equally likely to visit the shop on any day as on another day. What is the probability that both will visit the shop on:
- (i) the same day (ii) different days
(iii) Consecutive days. (4 marks)

Solution:

Total possible ways of visiting shop by them

$$= 5 \times 5 = 25$$

$$\therefore n(S) = 25$$

- (i) Let A be the event of visiting the shop by them on the same day.

$$\therefore n(A) = 5$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A) = \frac{5}{25} = \frac{1}{5}$$

- (ii) Let B be the event of visiting the shop by them on the consecutive days.

Sumit : Tuesday	Wednesday	Thursday	Friday
Amit : Wednesday	Thursday	Friday	Saturday

Amit : Tuesday	Wednesday	Thursday	Friday
Sumit : Wednesday	Thursday	Friday	Saturday

$$\therefore n(B) = 8$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{8}{25}$$

- (iii) Let C be the event of visiting the shop by them on different days.

$$\therefore n(C) = 25 - 5 = 20$$

$$P(C) = \frac{n(C)}{n(S)}$$

$$P(V) = \frac{20}{25} = \frac{4}{5}$$

- (2) What is the probability than an ordinary year has 53 Sundays? (4 marks)

Solution:

An ordinary year has 365 days.

i.e. 52 weeks and 1 extra day.

52 weeks will have 52 Sundays.

The sample space for one extra day is

$S = \{\text{Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday}\}$

$$\therefore n(S) = 7$$

Let A be the event that the extra day is a Sunday.

$$A = \{\text{Sunday}\}$$

$$\therefore n(A) = 1$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{7}$$

$\therefore$ The probability that an ordinary year has 53 Sundays is $\frac{1}{7}$.

- (3) What is the probability that a leap year has 53 Sundays? (4 marks)

Solution:

A leap year has 366 days, which is equivalent to 52 weeks and 2 days.

52 weeks will have 52 Sundays.

The sample space for remaining two days can be as follows:

$\therefore S = \{\text{Sunday-Monday, Monday-Tuesday, Tuesday-Wednesday, Wednesday-Thursday, Thursday-Friday, Friday-Saturday}\}$

$$\therefore n(S) = 7$$

Let A be the event of getting 53rd Sunday in remaining 2 days.

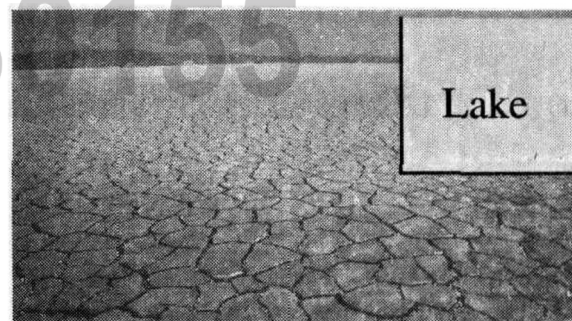
$$A = \{\text{Saturday-Sunday, Sunday-Monday}\}$$

$$\therefore n(A) = 2$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{2}{7}$$

$\therefore$ The probability that a leap year has 53 Sundays is $\frac{2}{7}$.

- (4) A missing helicopter is reported to have crashed somewhere in the rectangular region as shown in figure. What is the probability that it crashed inside the lake shown in the figure? (4 marks)



Solution:

For rectangular region:

Length = 9 km, Breadth = 4.5 km

Area of rectangular region = Length $\times$ Breadth

$$= 9 \times 4.5$$

$$= 40.5 \text{ sq. km}$$

For rectangular Lake:

$$\text{length} = 9 - 6 = 3 \text{ km}$$

$$\text{breadth} = 4.5 - 2 = 2.5 \text{ km}$$

$$\begin{aligned} \text{Area of rectangular lake} &= \text{length} \times \text{breadth} \\ &= 3 \times 2.5 \\ &= 7.5 \text{ sq. km} \end{aligned}$$

Let A be the event that the helicopter crashed inside the lake.

$$\begin{aligned} P(A) &= \frac{\text{Area of lack}}{\text{Area of rectangular region}} \\ &= \frac{7.5}{40.5} \\ &= \frac{70}{405} \\ P(A) &= \frac{5}{27} \end{aligned}$$

- (5) Each coefficient in equation $ax^2 + bx + c = 0$ is obtained by throwing an ordinary die. Find the probability that the equation has real roots. (4 marks)

Solution:

$$ax^2 + bx + c = 0$$

a, b and c can be selected from 1, 2, 3, 4, 5, 6 in 6^3 ways.

$$\therefore n(S) = 216$$

For the equation to have real roots,

$$b^2 - 4ac > 0$$

$$b^2 \geq 4ac$$

The values of a, b and c satisfying the above condition can be tabulated:

b	a and c	No. of favourable cases
2	(1, 1)	1
3	(1, 1), (1, 2), (2, 1)	3
4	(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (3, 1), (4, 1)	8
5	(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (4, 1), (5, 1) (6, 1)	14
6	(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (4, 1), (4, 2), (5, 1) (6, 1)	17

There are 43 favourable cases.

Let A be the event that the value of a, b, c selected in such a way that the roots of equation are real.

$$\therefore n(A) = 43$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{43}{216}$$

$$\therefore P(A) = \frac{43}{216}$$

- (6) A box contains 12 balls, out of which x are black, (i) If one ball is drawn at random, what is the probability that it will be a black ball? (ii) If 6 more black balls are put in the bag, the probability of drawing a black ball will double than that in (i), find x . (4 marks)

Solution:

- (i) Total number of balls = 12

$$\text{Number of black balls} = x$$

$$\therefore \text{Probability of getting a black ball} = \frac{x}{12}$$

- (ii) If 6 more black balls added in the box, then total number of black balls = $x + 6$

$$\text{Total no. of balls in the box} = 12 + 6 = 18$$

$$\therefore \text{Probability of getting a black balls} = \frac{x+6}{18}$$

According to given condition,

$$\frac{x+6}{18} = 2 \times \left(\frac{x}{12} \right)$$

$$\therefore \frac{x+6}{18} = \frac{x}{6}$$

$$\therefore 6x + 36 = 18x$$

$$\therefore 36 = 18x - 6x$$

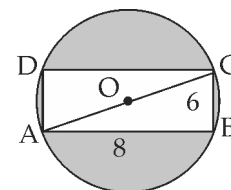
$$\therefore 36 = 12x$$

$$\therefore x = \frac{36}{12}$$

$$\therefore x = 3$$

Number of black balls are 3.

- (7) In the adjoining figure, a dart thrown lands in the interior of the circle. What is the probability that the dart will land in the shaded region. (4 marks)



Solution:

□ ABCD is a rectangle.

$$\therefore m \angle ABC = 90^\circ \quad [\text{Angle of a rectangle}]$$

In $\triangle ABC$,

$$AC^2 = AB^2 + BC^2 \quad [\text{Pythagoras theorem}]$$

$$= 8^2 + 6^2$$

$$= 64 + 36$$

$$\therefore AC^2 = 100$$

$$\therefore AC = 10 \text{ units}$$

$$\therefore \text{radius of circle} = \frac{10}{2} = 5 \text{ units.}$$

$$\begin{aligned} \text{Area of circle} &= \pi r^2 \\ &= 3.14 \times 5^2 \\ &= 3.14 \times 25 \\ &= 78.5 \text{ sq. units} \end{aligned}$$

$$\begin{aligned} \text{Area of rectangle ABCD} &= l \times b \\ &= 8 \times 6 \\ &= 48 \text{ sq. units} \end{aligned}$$

$$\begin{aligned} \therefore \text{Area of shaded region} &= \text{Area of circle} - \\ &\quad \text{Area of rectangle} \\ &= 78.5 - 48 \\ &= 30.5 \text{ sq. units} \end{aligned}$$

Let A be the event that dart lands in the shaded region.

$$\begin{aligned} P(A) &= \frac{\text{Area of Shaded region}}{\text{Area of circle}} \\ &= \frac{30.5}{78.5} \\ &= \frac{305}{785} \\ \therefore P(A) &= \frac{61}{157} \end{aligned}$$

6. Statistics

- (1) The mean of the following frequency distribution is 50. Find the value of f : (4 marks)

Class interval	10 - 20	20 - 40	40 - 60	60 - 80	80 - 100
Frequency	14	f	32	24	19

Solution:

Class intervals	Class marks (x_i)	frequency (f_i)	$f_i x_i$
0 - 20	10	17	170
20 - 40	30	f	$30f$
40 - 60	50	32	1600
60 - 80	70	24	1680
80 - 100	90	19	1710
Total		Σf_i	$\Sigma f_i x_i = 5160 + 30f$

$$\text{Mean } (\bar{x}) = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$\therefore 50 = \frac{5160 + 30f}{92 + f}$$

$$\begin{aligned} \therefore 50(92 + f) &= 5160 + 30f \\ \therefore 4600 + 50f &= 5160 + 30f \\ \therefore 50f - 30f &= 5160 - 4600 \\ \therefore 20f &= 560 \\ \therefore f &= \frac{560}{20} = 28 \\ \therefore \boxed{f = 28} \end{aligned}$$

- (2) An incomplete frequency distribution is given as follows:

Class interval	Frequency
10 - 20	12
20 - 30	30
30 - 40	?
40 - 50	65
50 - 60	?
60 - 70	25
70 - 80	18
Total	229

Given that median value is 46, determine the missing frequencies using the medians formula. (4 marks)

Solution:

Median is 46 it lies in the class 40 - 50 and the corresponding frequency is 65.

Classes	Frequency	Cumulative frequency (c.f.)
10 - 20	12	12
20 - 30	30	42
30 - 40	f_1	$42 + f_1$
40 - 50	$65 \rightarrow f$	$42 + f_1 + 65 = 107 + f_1$
50 - 60	f_2	$107 + f_1 + f_2$
60 - 70	25	$107 + f_1 + f_2 + 25 = 132 + f_1 + f_2$
70 - 80	18	$132 + f_1 + f_2 + 18 = 150 + f_1 + f_2 = 229$
Total	229	

From the last c.f.

$$150 + f_1 + f_2 = 229$$

$$\therefore f_1 + f_2 = 229 - 150$$

$$\therefore f_1 + f_2 = 79$$

$$\therefore f_2 = 79 - f_1 \quad \dots(i)$$

$$\text{Median} = L + \left(\frac{N}{2} - c.f. \right) \frac{h}{f}$$

Where $L = 40$, $N = 229$, $c.f. = 42 + f_1$, $h = 10$, $f = 65$

$$\text{Median} = 40 + \left(\frac{229}{2} - (42 + f_1) \right) \frac{10}{65}$$

$$\therefore 46 = 40 + \left(\frac{229}{2} - (42 + f_1) \right) \frac{2}{13}$$

$$\therefore 46 - 40 = \left(\frac{229}{2} - 42 - f_1 \right) \frac{2}{13}$$

$$\therefore 6 \times \frac{13}{2} = \frac{229}{2} - 42 - f_1$$

$$\therefore 39 = \frac{229}{2} - 42 - f_1$$

$$\therefore 78 = 229 - 84 - 2f_1$$

$$\therefore 2f_1 = 229 - 84 - 78$$

$$\therefore 2f_1 = 67$$

$$\therefore f_1 = \frac{67}{2}$$

$$\therefore f_1 = 33.5 = 34$$

$$\therefore f_2 = 79 - f_1 \quad [\text{From (i)}]$$

$$\therefore f_2 = 79 - 34$$

$$\therefore f_2 = 45$$

Hence, $f_1 = 34$ and $f_2 = 45$

- (3) The following data gives the information on the observed life time (in hour) of 225 electrical components:

Lifetime (in hours)	Frequency
0 - 20	10
20 - 40	35
40 - 60	52
60 - 80	61
80 - 100	38
100 - 120	29

Determine the modal life time of the components. (3 marks)

Solution:

The class having maximum frequency is 60 - 80 is the modal class.

Lifetime (in hour)	Frequency
0 - 20	10
20 - 40	35
40 - 60	52 $\rightarrow f_1$
60 - 80	61 $\rightarrow f_0$
80 - 100	38 $\rightarrow f_2$
100 - 120	29

The modal class is 60 - 80 and $L = 60$, $f_1 = 52$, $f_0 = 61$, $f_2 = 38$, $h = 20$

$$\text{Mode} = L + \left(\frac{f_0 - f_1}{2f_0 - f_1 - f_2} \right) h$$

$$= 60 + \left(\frac{61 - 52}{2(61) - 52 - 38} \right) 20$$

$$= 60 + \left(\frac{9}{122 - 90} \right) 20$$

$$= 60 + \left(\frac{9}{32} \right) 20$$

$$\therefore \text{Mode} = 60 + 5.625 = 65.625$$

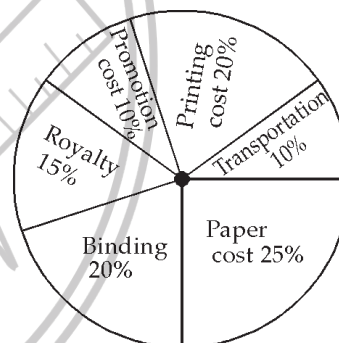
$$\therefore \text{Mode} = 65.625 \text{ hours.}$$

- (4) The adjoining pie-diagram shows the percentage distribution of the expenditure incurred in publishing a book. Study the diagram and answer the questions:

(a) If for certain quantity of books, the publisher has to pay ₹ 30,600 as printing, then what will be the amount of royalty to be paid for these books?

(b) What is the central angle of the sector corresponding to the expenditure incurred on Royalty? (3 marks)

Solution:



- (a) Let the total cost for certain quantity of books be ₹ 'x'

Printing cost = 20% of x

$$30600 = \frac{20}{100} \times x$$

$$\frac{30600 \times 100}{20} = x$$

$$x = ₹ 1,53,000$$

Royalty paid for these books = 15% of total cost

$$= \frac{15}{100} \times 153000$$

$$= 15 \times 1530$$

$$= ₹ 22,950$$

- (b) Measure of central angle for Royalty

$$= \frac{15}{100} \times 360 = 54^\circ$$